Energy Separation Mechanism in Uni-Flow Vortex Tube Using Compressible Vortex Flow

Hiroshi Katanoda, Mohd Hazwan bin Yusof

Abstract—A theoretical investigation from the view point of gas-dynamics and thermodynamics was carried out, in order to clarify the energy separation mechanism in a viscous compressible vortex, as a primary flow element in a uni-flow vortex tube. The mathematical solutions of tangential velocity, density and temperature in a viscous compressible vortical flow were used in this study. It is clear that a total temperature in the vortex core falls well below that distant from the vortex core in the radial direction, causing a region with higher total temperature, compared to the distant region, peripheral to the vortex core.

Keywords—Energy separation mechanism, theoretical analysis, vortex tube, vortical flow.

I. INTRODUCTION

A vortex tube is a simple fluid-dynamic device which can separately discharge cold and hot gases from a pressurized gas at room temperature. This device is also called a Ranque-Hilsch vortex tube since its invention by G. Ranque [1] and its investigation by Hilsch [2].

The VT is generally categorized into two types, (a) uni-flow type and (b) counter flow type, as shown in Fig. 1. In the uni-flow type VT, compressed air enters the VT through a single tangential nozzle or multiple tangential nozzles and produces a high-speed vortical flow in the vortex chamber. The rotational flow follows the tube wall to the opposite end. Then, the core flow exits the VT at a lower temperature, the peripheral flow exits at a higher temperature. In the counter flow VT, the peripheral flow exits with a higher temperature, and the core is forced back to the vortex chamber by a control valve, and exits the VT as a cold flow.

At the present time, several theories have been proposed for the thermal energy separation mechanism in the VT by several researchers [3], [4].It is generally accepted from those theories that, the cold flow temperature reduction is caused by an adiabatic expansion of the compressed gas. However, details of the physics explaining the cold flow generation, from the fluid dynamic view point still remain unclear.

In this study, a viscous compressible vortex model based on the Navier-Stokes equations is examined in order to clarify the physical reasons for the total energy separation phenomena which occurs in VT. Since it is difficult to obtain a mathematical solution for theNavier-Stokes equations with

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axial reverse flow, which occurs in the counter flow VT, a uni-flow type VT is the focus in this study. Another reason for analyzing the uni-flow type in this study is that the flow structure in the uni-flow VT is simpler compared to that in the counter flow type. Therefore, first the flow mechanism of the uni-flow VT should be clarified.
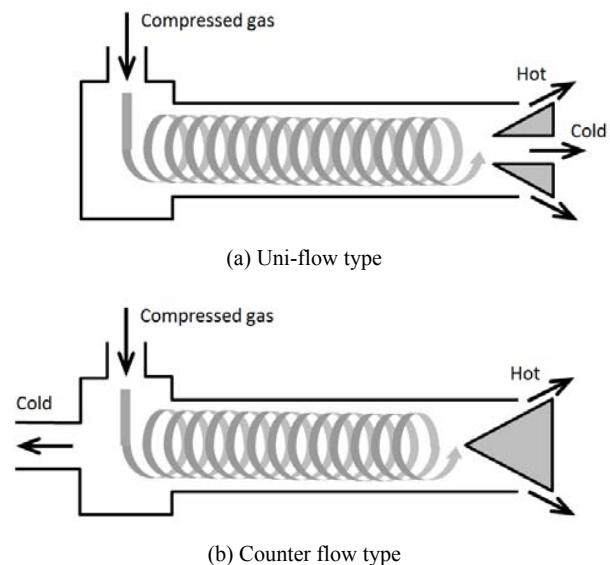

(a) Uni-flow type

(b) Counter flow type

Fig. 1 Schematic diagram of vortex tube

II. VORTEX MODEL [5]

The basic equations for compressible axisymmetric vortex are the equations of mass continuity, Navier-Stokes, and energy in cylindrical coordinate (r, θ, z), and are written as;

$$\frac{\partial \rho}{\partial t} + \frac{1}{r}\frac{\partial(\rho V_r r)}{\partial r} + \frac{1}{r}\frac{\partial(\rho V_\theta)}{\partial \theta} + \frac{\partial(\rho V_z)}{\partial z} = 0 \tag{1}$$

$$\frac{DV_r}{Dt} - \frac{V_\theta^2}{r} = \frac{1}{\rho}\left(\frac{\partial \sigma_r}{\partial r} + \frac{\sigma_r}{r} - \frac{\sigma_\theta}{r} + \frac{\partial \tau_{r\theta}}{r\partial \theta} + \frac{\partial \tau_{zr}}{\partial z}\right) \tag{2}$$

$$\frac{DV_\theta}{Dt} + \frac{V_r V_\theta}{r} = \frac{1}{\rho}\left(\frac{\partial \sigma_\theta}{r\partial \theta} + \frac{\partial \tau_{r\theta}}{\partial r} + \frac{\tau_{r\theta}}{r} + \frac{\tau_{\theta r}}{r} + \frac{\partial \tau_{z\theta}}{\partial z}\right) \tag{3}$$

$$c_p\frac{DT}{Dt} - \frac{1}{\rho}\frac{Dp}{Dt} = \frac{k}{\rho}\nabla^2 T + \frac{\Phi}{\rho} \tag{4}$$

where, t is the time, ρ is the density, p is the pressure, T is the temperature, (V_r, V_θ, V_z) is the velocity of (r, θ, z) component, c_p

is the specific heat at constant pressure, k is the thermal conductivity, respectively. The normal stress σ and tangential stress τ are expressed as;

$$\sigma_r = -p + 2\mu \frac{\partial V_r}{\partial r} - \frac{2}{3}\mu \cdot div\mathbf{V} \tag{5}$$

$$\sigma_\theta = -p + 2\mu\left(\frac{1}{r}\frac{\partial V_\theta}{\partial \theta} + \frac{V_r}{r}\right) - \frac{2}{3}\mu \cdot div\mathbf{V} \tag{6}$$

$$\tau_{r\theta} = \tau_{\theta r} = \mu\left\{r\frac{\partial}{\partial r}\left(\frac{V_\theta}{r}\right) + \frac{1}{r}\frac{\partial V_r}{\partial \theta}\right\} \tag{7}$$

$$\tau_{z\theta} = \mu\left(\frac{1}{r}\frac{\partial V_z}{\partial \theta} + \frac{\partial V_\theta}{\partial z}\right) \tag{8}$$

$$\tau_{zr} = \mu\left(\frac{\partial V_r}{\partial z} + \frac{\partial V_z}{\partial r}\right) \tag{9}$$

where, μ is the coefficient of viscosity, Φ is the dissipation function expressed as;

$$\Phi = 2\mu\left\{\left(\frac{\partial V_r}{\partial r}\right)^2 + \left(\frac{1}{r}\frac{\partial V_\theta}{\partial \theta} + \frac{V_r}{r}\right)^2 + \left(\frac{\partial V_z}{\partial z}\right)^2\right\}$$
$$+ \mu\left\{r\frac{\partial}{\partial}\left(\frac{V_\theta}{r}\right) + \frac{1}{r}\frac{\partial V_r}{\partial \theta}\right\}^2 + \mu\left(\frac{1}{r}\frac{\partial V_z}{\partial \theta} + \frac{\partial V_\theta}{\partial z}\right)^2 \tag{10}$$
$$+ \mu\left(\frac{\partial V_r}{\partial z} + \frac{\partial V_z}{\partial r}\right)^2$$

The mathematical operators used in (2)-(6) are;

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + V_r\frac{\partial}{\partial r} + \frac{V_\theta}{r}\frac{\partial}{\partial \theta} + V_z\frac{\partial}{\partial z} \tag{11}$$

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r}\frac{\partial^2}{\partial \theta^2} + \frac{\partial^2}{\partial z^2} \tag{12}$$

$$div\mathbf{V} = \frac{1}{r}\frac{\partial(rV_r)}{\partial r} + \frac{1}{r}\frac{\partial V_\theta}{\partial \theta} + \frac{\partial V_z}{\partial z} \tag{13}$$

The physical properties used in the above equations are normalized as follows;

$$\left.\begin{array}{l} r' = \dfrac{r}{r_c}, \ z' = \dfrac{z}{r_c} \\[2mm] V_r' = \dfrac{V_r}{V_{\theta c}}, \ V_\theta' = \dfrac{V_\theta}{V_{\theta c}}, \ V_z' = \dfrac{V_z}{V_{\theta c}} = h'(r')z' \\[2mm] p' = \dfrac{p}{\rho_\infty V_{\theta c}^2}, \ \rho' = \dfrac{\rho}{\rho_\infty}, \ T' = \dfrac{T}{T_\infty}, \ T_0' = \dfrac{T_0}{T_\infty} \end{array}\right\} \tag{14}$$

where, r_c is the core radius of the vortex. $V_{\theta c}$ is the tangential velocity at $r = r_c$, which is written as;

$$V_{\theta c} = \frac{\Gamma_\infty}{2\pi r_c} \tag{15}$$

where, Γ_∞ is the vortex circulation. The subscript ∞ shows the properties distant from the vortex center.

In order to derive the solutions of the basic equations, the following assumptions are used; 1) The flow is steady and axisymmetric, 2) V_r' and $h' \ll V_\theta'$. Then, (1)-(4) are simplified as follows using the simplification based on the same order magnitude consideration of the governing equations outlined in [6], [7];

$$\frac{1}{r'}\frac{d(\rho'V_r'r')}{dr'} + \rho'h' = 0 \tag{16}$$

$$\rho'\frac{V_\theta'^2}{r'} = \frac{dp'}{dr'} \tag{17}$$

$$\rho'R_e\frac{V_r'}{r'}\frac{d(V_\theta'r')}{dr'} = \frac{d}{dr'}\left\{\frac{1}{r'}\frac{d(V_\theta'r')}{dr'}\right\} \tag{18}$$

$$\rho'P_rR_eV_r'\frac{dT'}{dr'} - P_rR_e(\gamma-1)M_o^2V_r'\frac{dp'}{dr'} =$$
$$\frac{1}{r'}\frac{d}{dr'}\left(r'\frac{dT'}{dr'}\right) + P_r(\gamma-1)M_o^2F' \tag{19}$$

where,

$$F' = r'^2\left\{\frac{d}{dr'}\left(\frac{V_\theta'}{r'}\right)\right\}^2 \tag{20}$$

$$R_e = \frac{\rho_\infty V_{\theta c}r_c}{\mu} \tag{21}$$

$$M_o = \frac{V_{\theta c}}{\sqrt{\gamma RT_\infty}} \tag{22}$$

$$P_r = \frac{\mu c_p}{k} \tag{23}$$

The equation of state for a calorically perfect gas in nondimensional form is written as;

$$p' = \frac{\rho'T'}{\gamma M_o^2} \tag{24}$$

or

$$\frac{p}{p_\infty} = \rho T' \tag{25}$$

The required boundary conditions are
1) at $r' = 0$;
$$V_\theta'(r') = 0, \quad V_r'(r') = 0, \quad \frac{d\rho'(r')}{dr'} = 0, \quad \frac{dT'(r')}{dr'} = 0$$
2) at $r' \to \infty$;
$$V_\theta'r' \to 1, \quad T'(r') \to 1, \quad \rho'(r') \to 1, \quad \gamma M_o^2 p' \to 1$$

A set of most practical solutions of (16)-(18), under the boundary conditions of 1) and 2), are

$$V_\theta' = \frac{r'}{\sqrt{1 + r'^4}} \tag{26}$$

$$V_r' = -\frac{1}{R_e \rho'} \cdot \frac{6r'^3}{1+r'^4} \qquad (27)$$

$$V_z' = \frac{z'}{R_e \rho'} \cdot \frac{24r'^2}{\left(1+r'^4\right)^2} \qquad (28)$$

The normalized temperature, which satisfies (19) with temperature boundary conditions, can be written as;

$$T' = 1 + \frac{\gamma-1}{6} M_o^2 \left\{ \tan^{-1}(r'^2) - \frac{r'^2}{1+r'^4} - \frac{\pi}{2} \right\} \qquad (29)$$

In this case, the Prandtl number $P_r = 2/3$ is assumed, to obtain (29). The normalized density is obtained by (17), (26), (24) and (29) as;

$$\rho' = \frac{\exp\left[\gamma M_o^2 \left(\int_0^{r'} \frac{V_\theta'^2}{r'T'} dr' - \int_0^\infty \frac{V_\theta'^2}{r'T'} dr'\right)\right]}{T'} \qquad (30)$$

The tangential velocity calculated by (26), along with that of the Rankine vortex, is shown in Fig. 2. The figure shows that V_θ' of the Vatistas and Aboelkassem (VA) model has a rounded peak compared to the Rankine vortex sharp peak at $r' = 1$.

The static pressure distributions are calculated using (25), (29), (30) and are shown in Fig. 3 with solid curves for $M_o = 0.6$, 0.8 and 1.0, along with the static pressure distributions of the Rankine vortex using broken lines. Fig. 3 shows that the static pressure of the VA model does not decrease as sharply as the Rankine vortex in the core region $0 < r' < 1$.

The static temperature distributions are calculated using (29) and are shown in Fig. 4. The figure shows that the static temperature decreases below T_∞ ($T' < 1$) in the radial inward direction. In addition, the larger the Mach number M_o is, the smaller the nondimensional temperature T' is.
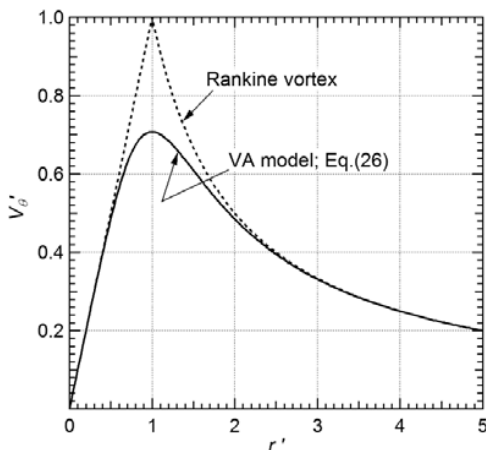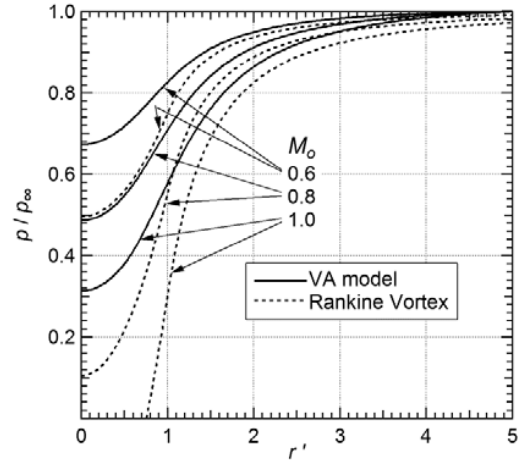

Fig. 2 Normalized tangential velocity

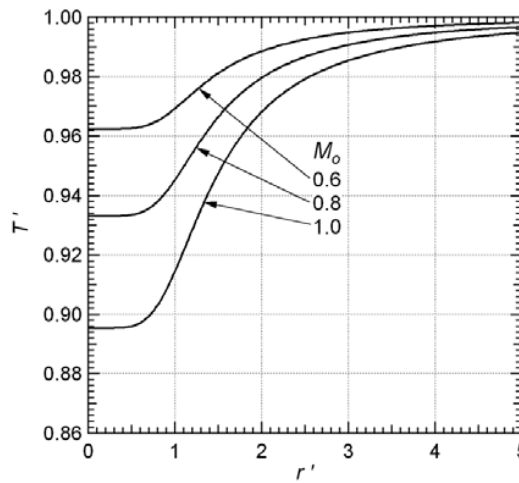Fig. 3 Normalized static pressure

Fig. 4 Normalized static temperature

III. RESULTS AND DISCUSSION

The authors believe that understanding the total enthalpy or total temperature distribution in a compressible vortex helps to clarify the energy separation mechanism in a uni-flow vortex tube. When discussing the thermodynamic performance of a VT, the total temperature is one of the most important physical properties. A local total temperature in nondimensional form can be written as;

$$T_0' = T' + \frac{\gamma-1}{2} M_o^2 V_\theta'^2 \qquad (31)$$

The solid line in Fig. 5 shows the nondimensional total temperature distribution, in the radial direction calculated by (31) along with (26) and (29) at $M_o = 1.0$ and $\gamma = 1.40$. The dotted lines in the figure show the first and second terms in (31). From (31), T_0' has a minimum value at $r' = 0$, and is derived as;

$$T_{0,min}' = T_{min}' = 1 - \frac{\pi}{2}\frac{\gamma-1}{6} M_o^2 \qquad (r' = 0) \qquad (32)$$

Likewise, the maximum value of T_0' can be expressed as;

$$T_{0,\max}' = 1 + 0.343 \frac{\gamma-1}{6} M_o^2 \qquad (r' = \sqrt[4]{3} \cong 1.32) \qquad (33)$$

Equations (32) and (33) clearly show that if the flow is compressible, that is $M_o > 0$, the total (or static) temperature is smaller than T_∞ at the center of the vortex, and the total temperature is greater than T_∞ at outside the vortex core, $r' = \sqrt[4]{3}$, regardless the values of γ and M_o. In addition, the greater the Mach number M_o is, the smaller $T_{0,\min}'$ is, and also the greater $T_{0,\max}'$ is. If T_∞ is regarded as a room temperature in the actual uni-flow VT case, it can be said that the phenomenon of total temperature separation occurs essentially due to the existence of the gas flow compressibility.

In order to discuss the effect of viscosity on the total temperature distribution in the compressible vortex, the value of a dissipation function, Φ, is examined. The nondimensional form of the function, (10), is simplified as;

$$\Phi' \equiv \frac{\Phi}{\frac{\mu V_{\theta c}^2}{r_c^2}} = r'^2 \left\{ \frac{d}{dr'} \left(\frac{V_\theta'}{r'} \right) \right\}^2 = 4 \frac{r'^8}{\left(1 + r'^4\right)^3} \qquad (34)$$

Fig. 6 shows the result calculated by (34). The maximum value of the function is obtained at $r' = \sqrt[4]{2} \cong 1.19$.This value is close to $r' = \sqrt[4]{3} \cong 1.32$, where the total temperature reaches the maximum value. This result implies that the energy dissipation due to viscosity has a contribution to the total temperature increase over T_∞ outside the vortex core.
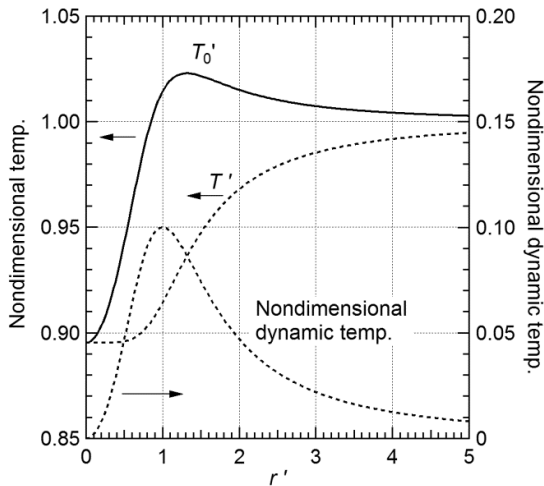

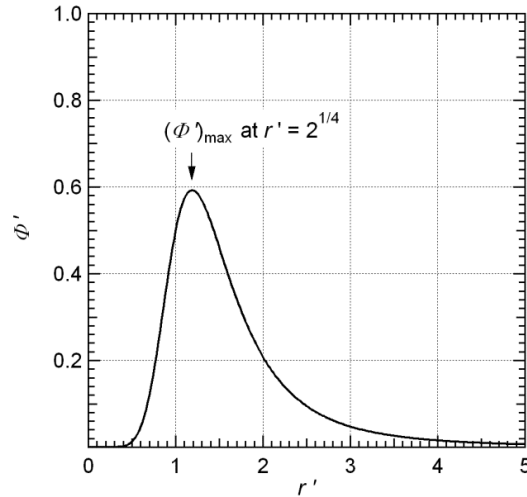Fig. 5 Normalized total/static/dynamic temperatures at M_o=1.0

Fig. 6 Normalized dissipation function

IV. CONCLUSIONS

A viscous compressible vortex model was examined in order to clarify the physical reason for total energy separation phenomena which occurs in a uni-flow VT. The results obtained in this study are summarized as follows;

1) If the flow is compressible, the total temperature at the center of the vortex is smaller than T_∞ , the static temperature distant from the vortex center, and is greater than T_∞ at the outside the of vortex core.

2) The greater the representative Mach number M_o is, the smaller the total temperature at the center is, and the greater the maximum total temperature is over T_∞ .

3) The value of the dissipation function reaches its maximum value at a radial point close to the point where the total temperature is at maximum.

REFERENCES

[1] G. J. Ranque, "Experiences sur la de ´tente giratoire avec productions simultanees dun echappement dair chaud et dun echappement dair froid",*J. Phys. Radium.*, vol.7, no. 4,pp. 112-114, 1933.
[2] R. Hilsch, "The use of expansion of gases in a centrifugal field as cooling process", *Rev. Sci. Instrum.*, vol. 18(2), 108-113, Feb.1947.
[3] C.U. Linderstrom-Lang, "The three-dimensional distributions of tangential velocity and total-temperature in vortex tubes", *J. Fluid Mech.*, vol. 45, no. 1, pp. 161–187, Jan.1971.
[4] R.Z. Alimov, "Flow friction and heat and mass transfer in a swirled flow", *J. Eng.Phys. Thermophys*, vol. 10, no. 4, pp. 251–257, Apr.1966.
[5] G. H. Vatistas, and Y. Aboelkassem, "Extension of the incompressible *n*=2 vortex into compressible", *AIAA J.*, vol. 44, no. 8, pp. 1912-1915, Jan.2006.
[6] Y. Aboelkassem, and G. H. Vatistas, "New Model for Compressible Vortices", *Trans. ASME, J. Fluids Eng.*, vol. 129, no. 8, pp. 1073-1079, Feb.2007.
[7] G. H. Vatistas, and Y. Aboelkassem, "Time Decay of n Family of Votrices", *AIAA J.*, vol. 43, no. 6, pp. 1389-1391, Jan.2005.