

# Free Vibration Analysis of Gabled Frame Considering Elastic Supports and Semi-Rigid Connections

A. Shooshtari, A. R. Masoodi, S. Heyrani Moghaddam

**Abstract**—Free vibration analysis of a gabled frame with elastic support and semi-rigid connections is performed by using a program in OpenSees software. Natural frequencies and mode shape details of frame are obtained for two states, which are semi-rigid connections and elastic supports, separately. The members of this structure are analyzed as a prismatic nonlinear beam-column element in software. The mass of structure is considered as two equal lumped masses at the head of two columns in horizontal and vertical directions. Note that the degree of freedom, allocated to all nodes, is equal to three. Furthermore, the mode shapes of frame are achieved. Conclusively, the effects of connections and supports flexibility on the natural frequencies and mode shapes of structure are investigated.

**Keywords**—Natural frequency, mode shape, gabled frame, semi-rigid connection, elastic support, OpenSees software.

## I. INTRODUCTION

STEEL frame structures perform an efficient role in bearing various live, snow, wind, earthquake and crane loads in gabled frame. Steel frame structure is typically composed of parts, having variable cross-sections and rigid or semi-rigid connections. These parts are simultaneously subjected to axial, shear forces and bending moments.

One of the assumptions in dynamic analysis of frames is the consideration of rigid and hinge connections. These connections include support and nodal connections. Although, the application of this method simplifies the analysis but it doesn't reveal the true behavior of the structure. In the past, so many researches have been done about semi-rigid frame analysis. A numerical solution was presented for linear and non-linear free vibration analysis of the frames with semi-rigid connections by Chan and Ho. They modeled connections with rotational springs and introduced stiffness matrix [1]. In 1989, the influence of non-linear semi-rigid connections on static and dynamic responses of spatial frames is examined by Shi and Atluri. They analyzed semi-rigid frames by using geometric stiffness matrix of the semi-rigid member [2]. In 1998, Rodrigues et al. investigated the behavior of steel frames with and without lateral brace, having nodal semi-rigid connections. They employed FEM in the analysis frame [3]. Also, seismic analysis of braced steel frame with semi-rigid connections was investigated by Fu et al. in 1998 [4]. In recent

decade, seismic analysis researches have been performed about semi-rigid steel frames. Nguyen and Kim, based on beam-column theory, introduced an effective solution for nonlinear elastic analysis of three dimensional steel structures with semi-rigid connections. They utilized Newton-Raphson method and Newmark integration for solving nonlinear dynamic equation [5]. In 2002, the influence of flexibility and damping of connections on dynamic responses of a steel frame was examined by Sekulovic et al. They worked on nonlinear impact of nodal connections behavior of frame members. Finally, they found out a numerical pattern considering second-order nonlinear effects and behavior of connections [6]. Ozyigit performed free and forced in-plane and out-of-plane vibrations of frames are investigated in 2009. A lumped mass is also located at different points of the frame with different mass ratios [7].

In this research, free vibration analysis of a gabled frame with elastic support and semi-rigid connections is carried out. The members of this structure are modeled in a prismatic shape. Moreover, the effect of various support conditions is assessed. For investigation of the influence of the flexibility of the nodal and support connections, the rotational and axial springs are used. For analysis of this frame, OpenSees 2.4 software is employed. The conclusions are separately reported for various states of support and semi-rigid connections. Besides, the natural frequencies and mode shapes of gabled frame are reported for different states of connections and support. Ultimately, the effects of connections and supports flexibility are investigated.

## II. MODELING OF GABLED FRAME

In recent years, it is more common to use gabled steel frame rather than truss in large bays. The current study investigates the behavior of one kind of these symmetrical frames. For a detailed examination of seismic behavior of this frame, its supports and connections are modeled in state of semi-rigid. The members of this frame are assumed to be prismatic with equal moment of inertia. The general model of frame is illustrated in Fig. 1. The span of the frame in all cases is supposed to be 12.0m. The rigidity of connections in the frame is shown by symbol  $R$ . The frame columns height is equal to 5.0m. Also, the height coefficient of slope roof, which is defined by letter  $n$ , is considered to be 0.4. Table I reports the properties of geometric cross-sections of the structure members. The unit of reported values in Table I is  $cm$ .

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TABLE I  
THE CROSS-SECTION PROPERTIES OF GABLED FRAME MEMBERS

section properties	d (cm)	hw (cm)	tf (cm)	tw (cm)	b (cm)	A (cm <sup>2</sup> )	I (cm <sup>4</sup> )
	30	27.6	1.2	0.8	30	94.08	16340.1984

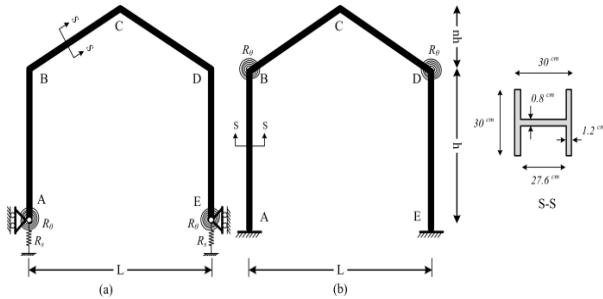


Fig. 1 The model of steel gabled frame with uniform I-section

The free vibration analysis of gabled frame is performed by using OpenSees software. Making model in the software is performed in two-dimensional case and each node has three degree of freedom. The effects of P-Δ are ignored. The materials, which are used in structure members, are supposed to be bilinear axial steel. It should be added, the yield stress and the module of elasticity are equal to  $2400 \text{ Kg}/\text{cm}^2$  and  $2.05 \times 10^6 \text{ Kg}/\text{cm}^2$ , respectively. The stress-strain behavior of materials is shown in Fig. 2. The members of this frame are modeled as a nonlinear beam-column element. The modified Newton-Raphson algorithm, which is used for solving nonlinear equation, is employed in this analysis [8].

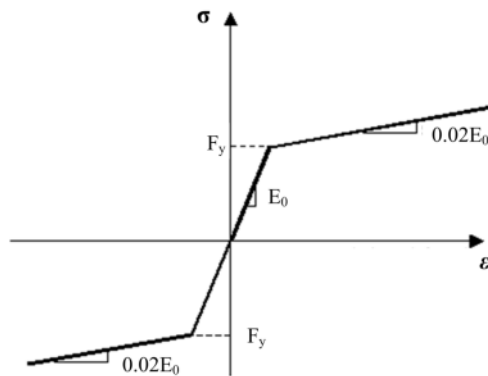


Fig. 2 The stress-strain curve of material Steel01

### III. CONNECTIONS BEHAVIOR PATTERN

It is common that the behavior of a connection is defined by its movement-curvature. In this study, in order to model of semi-rigid connections, a linear pattern is used. The simplest connection pattern is the linear pattern which is shown as follows:

$$\begin{aligned} M &= R.\theta \\ F &= K.\Delta \end{aligned} \quad (1)$$

The connection stiffness ( $R$ ) could be introduced by the stiffness of initial connection or tendinous stiffness. The dimensionless coefficient  $r$  and  $r_s$  are defined as flexibility ratio of connections and supports. The range of  $r$  and  $r_s$  is from zero to one. The relation between flexibility factor and connection stiffness is shown in (2) [9]. Moreover, the relation of support stiffness ( $K$ ) is suggested as below.

$$r = \frac{1}{1 + \frac{3EI}{RL}}, \quad r_s = \frac{1}{1 + \frac{3EI}{KL^3}} \quad (2)$$

$$R = \frac{3EI}{L} \left( \frac{r}{1-r} \right), \quad K = \frac{3EI}{L^3} \left( \frac{r_s}{1-r_s} \right)$$

where the connection stiffness is  $R$  and the flexibility factor is  $r$ . Besides, the transmission spring stiffness is  $K$  and its flexibility factor is  $r_s$ . In this research, only the initial stiffness of the connections is considered. It should be noted, if  $r$  tends to zero, the aforementioned connection is hinge and if  $r$  tends to one, the connection is fixed. In this study, the elastic module ( $E$ ) is equal to  $205 \text{ GPa}$ . The height of columns in is considered  $5.0 \text{ m}$ . The specifications of column cross-section are discussed in Table I. The connection stiffness versus their flexibility factor ( $r$ ) is reported in Table II.

TABLE II  
THE CONNECTIONS STIFFNESS VERSUS FLEXIBILITY FACTOR  $r, r_s$

$r, r_s$	$R$	$K$
0	0	0
0.2	5024611	200984
0.4	13398963	535958
0.6	30147666	1205906
0.8	80393776	3215751
1.0	$\infty$	$\infty$

### IV. RESULTS AND DISCUSSION

For free vibration analysis of structure, distribution of mass should be assigned. It should be added, the total mass which is included the sum of live and dead load is equal to  $15000 \text{ kg}$ . It should be understood that the live load applied in total mass includes the snow load equaling to  $150 \text{ kg}/\text{m}^2$ . Therefore, the pattern of mass distribution in this frame is considered as lumped masses at three nodes of frame which is illustrated in Fig. 3.

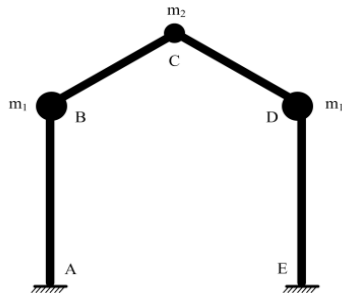


Fig. 3 The pattern of mass distribution in the gabled frame

According to Fig. 3, the values of  $m_1$  and  $m_2$  are considered to be 6000kg and 3000kg, respectively. By using this pattern, the mode shapes of structure are obtained. Based on the outputs of OpenSees software, main three shape modes of gabled frame in the state of semi-rigid connections are achieved. The general view of three shape modes related to gabled frame in two cases of semi-rigid connections and elastic rotational supports in the state of  $r_s = 1.0$  is illustrated as follows:

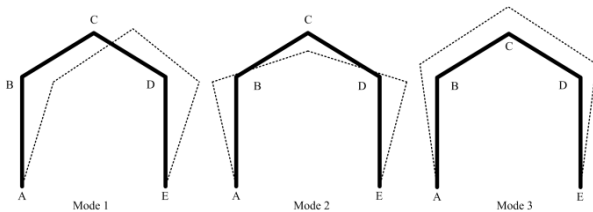


Fig. 4 The shape modes of gabled frame with semi-rigid connections

Although the flexibility of supports and connections influenced on the natural frequency of structure, the sequence of shape modes obtained for the gabled frame with semi-rigid connection is often based on the Fig. 4. In addition, the general view of three shape modes related to gabled frame with elastic transmission supports is illustrated below.

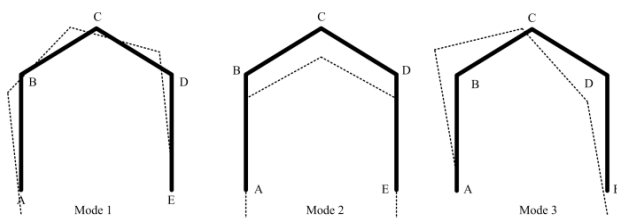


Fig. 5 The shape modes of gabled frame with elastic supports

Also, the sequence of shape modes obtained for the gabled frame with elastic transmission supports is based on the Fig. 5. By utilizing outputs of OpenSees software, the natural frequencies of semi-rigid gabled frame structure are depicted for different states of supports and connections flexibility. The natural frequencies of the gabled frame with different states of elastic supports are reported in Table III.

TABLE III  
THE NATURAL FREQUENCY OF GABLED FRAME WITH ELASTIC SUPPORTS

$r_s$	$r$	Natural frequency (Hz)		
		mode 1	mode 2	mode 3
0.2	0.0	0.589	0.822	1.692
	0.2	0.808	0.822	1.852
	0.4	0.822	0.926	2.037
	0.6	0.822	0.995	2.227
	0.8	0.822	1.040	2.427
	1.0	0.822	1.070	2.625
0.4	0.0	0.822	1.335	1.980
	0.2	1.083	1.335	2.075
	0.4	1.252	1.335	2.193
	0.6	1.335	1.361	2.342
	0.8	1.337	1.433	2.506
	1.0	1.337	1.479	2.688
0.6	0.0	0.962	1.976	2.532
	0.2	1.266	1.980	2.571
	0.4	1.499	1.980	2.625
	0.6	1.686	1.984	2.695
	0.8	1.832	1.984	2.786
	1.0	1.942	1.988	2.899
0.8	0.0	1.044	3.049	3.802
	0.2	1.368	3.067	3.817
	0.4	1.634	3.086	3.831
	0.6	1.869	3.096	3.846
	0.8	2.083	3.115	3.876
	1.0	2.278	3.125	3.891
1.0	0.0	1.094	4.149	35.714
	0.2	1.422	4.274	35.714
	0.4	1.701	4.405	35.714
	0.6	1.949	4.525	35.714
	0.8	2.179	4.673	35.714
	1.0	2.392	4.808	35.714

Moreover, Table IV expresses the natural frequency results of gabled frame in different cases of semi-rigid connections. The unit of natural frequency reported in the Tables III and IV is Hz. It should be mentioned, the state of  $r_s = 0$  does not exist because in this case, the structure is not stable and the external stability of structure is missing.

TABLE IV  
THE NATURAL FREQUENCY OF GABLED FRAME WITH SEMI-RIGID CONNECTIONS

$r$	Natural frequency (Hz)		
	mode 1	mode 2	mode 3
0.0	1.647	2.488	35.714
0.2	1.931	3.356	35.714
0.4	2.110	3.891	35.714
0.6	2.232	4.274	35.714
0.8	2.326	4.566	35.714
1.0	2.392	4.808	35.714

Based on the results, the natural frequency of structure with rigid connections reported in Table IV and rigid supports given in Table III are the same. Conclusively, the accuracy and correctness of gabled frame modeling with semi-rigid connections is approved. Furthermore, in the all states of

$r_s = 1.0$ , the natural frequency of mode 3 is the same. Therefore, the distinction of natural frequency in mode 3 is just influenced by the flexibility of transmission supports ( $r_s$ ). In order to compare the effect of semi-rigid connections and elastic supports on the natural frequency of structure, the following charts are available. The effects of support flexibility on the natural frequency are indicated for various shape modes in Figs. 6, 7 and 8. Noted, the vertical axis of graphs shows the natural frequency in the unit of Hz. Moreover, the horizontal axis of graph shows the rotational flexibility of supports. In addition, the effects of connection flexibility on the natural frequency of gabled frame are represented for different shape modes in Fig. 9. In this case, since the third shape mode of frame is constant, it can be ignored and is not reported.

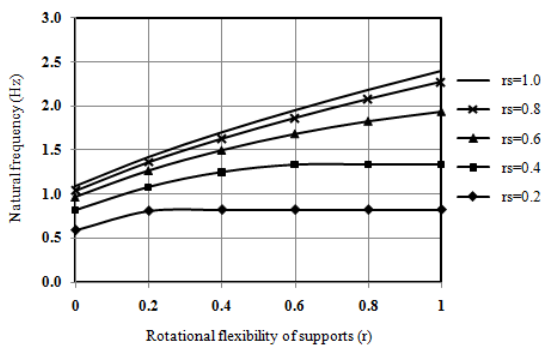


Fig. 6 The first natural frequency of gabled frame with elastic supports

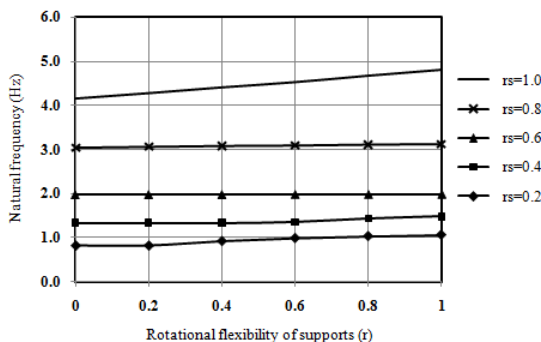


Fig. 7 The second natural frequency of gabled frame with elastic supports

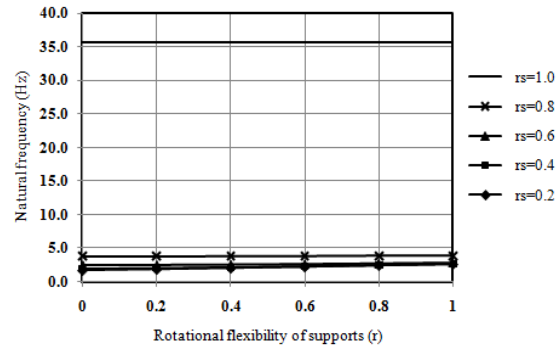


Fig. 8 The third natural frequency of gabled frame with elastic supports

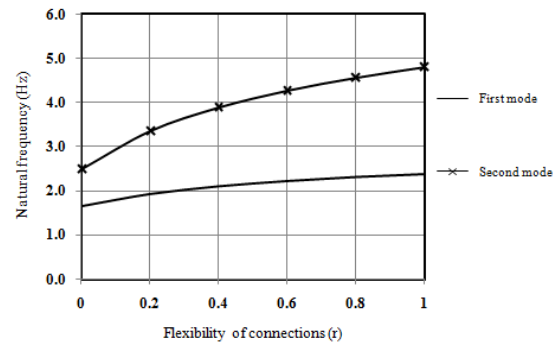


Fig. 9 The natural frequency of gabled frame with semi-rigid connections

## V. CONCLUSION

In this paper, free vibration analysis was studied on the gabled frame with various flexibilities of connections and supports. Moreover, the natural frequency and shape modes of frame were obtained in different cases of support and connection flexibility. Conclusively, the influences of these factors on the natural frequency of frame were depicted for various modes in separated diagrams.

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