MHD Non-Newtonian Nanofluid Flow over a Permeable Stretching Sheet with Heat Generation and Velocity Slip

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Abstract—The problem of magnetohydrodynamics boundary layer flow and heat transfer on a permeable stretching surface in a second grade nanofluid under the effect of heat generation and partial slip is studied theoretically. The Brownian motion and thermophoresis effects are also considered. The boundary layer equations governed by the PDE's are transformed into a set of ODE's with the help of local similarity transformations. The differential equations are solved by variational finite element method. The effects of different controlling parameters on the flow field and heat transfer characteristics are examined. The numerical results for the dimensionless velocity, temperature and nanoparticle volume fraction as well as the reduced Nusselt and Sherwood number have been presented graphically. The comparison confirmed excellent agreement. The present study is of great interest in coating and suspensions, cooling of metallic plate, oils and grease, paper production, coal water or coal-oil slurries, heat exchangers technology, materials processing exploiting.

Keywords—Viscoelastic nanofluid, partial slip, stretching sheet, heat generation/absorption, MHD flow, FEM.

I. INTRODUCTION

CONVECTIVE heat transfer can be enhanced passively by changing the flow geometry, boundary conditions, or by enhancing thermal conductivity of the fluid. Various techniques have been proposed to enhance the heat transfer performance of fluids. Researchers have also tried to increase the thermal conductivity of base fluids by suspending micro- or larger-sized solid particles in fluids, since the thermal conductivity of solid is typically higher than that of liquid. Modern nanotechnology provides new opportunities to process and produce materials with average crystallite sizes below 50 nm. Fluids with nanoparticles suspended in them are called nanofluids, a term first proposed by Choi [1]. Choi et al. [2], and Masuda et al. [3] have shown that a very small amount of nanoparticles (usually less than 5%), when dispersed uniformly and suspended stably in base fluids, can provide dramatic improvements in the thermal conductivity and in the heat transfer coefficient of the base fluid. Nanofluid is a suspension of nanoparticles in the base fluid.

A comprehensive survey of convective transport in nanofluids was made by Buongiorno [4]. Khan and Pop [5] have used the model of Kuznetsov and Nield [6] to study the fundamental work on the boundary layer flow of nanofluid over a stretching sheet. Makinde and Aziz [7] extended the work of Khan and Pop [5] for convective boundary conditions.

It is now well accepted fact that many fluids of industrial and geophysical importance are non-Newtonian. Due to much attention in many industrial applications, the research on boundary layer behaviour of a viscoelastic fluid over a continuously stretching surface keeps going. McCormack and Crane [8] have provided comprehensive discussion on boundary layer flow caused by stretching of an elastic flat sheet moving in its own plane with a velocity varying linearly with distance. Several researchers viz. Gupta and Gupta [9], Dutta et al. [10], Chen and Char [11] extended the work of Crane [8] by including the effects of heat and mass transfer under different situations. Later on, Rajagopal et al. [12] and Chang [13] presented an analysis on flow of viscoelastic fluid over a stretching sheet. The above sources all utilize the no-slip condition. Wang [14] discussed the partial slip effects on the planar stretching flow. Off late, Noghrehabadi et al. [15] investigated the development of the slip effects on the boundary layer flow and heat transfer over a stretching sheet.

A study of utilizing heat source or sink in moving fluids assumes a greater significant in all situations which dealing with exothermic or endothermic chemical reaction and those concerned with dissociating fluids. Sparrow and Cess [16] investigated the steady stagnation point flow and heat transfer in the presence of temperature dependent heat absorption. Later, Azim et al. [17] discussed the effect of viscous Joule heating on MHD-conjugate heat transfer for a vertical flat plate in the presence of heat generation. One of the latest work is the study of the heat transfer characteristic in the mixed convection flow of a nanofluid along a vertical plate with heat source/sink was studied by Rana and Bhargava [18].

In real situations in Nanofluids, the base fluid do not satisfy the properties of Newtonian fluids, hence it is more justified to consider them as viscoelastic fluids. In the present paper, the base fluid is taken as second grade fluid. To our best of knowledge, no studies have far been investigated to analyze the partial slip effect on the boundary layer flow of viscoelastic nanofluid over a permeable stretching sheet under the effect of MHD and heat generation. The objective of the present paper is therefore to extend the work of Noghrehabadi [15] by taking base fluid as second grade fluid. A similarity solution is presented and used to predict the heat and mass transfer characteristics of the flow. The effects of the embedded flow controlling parameters on the fluid velocity,

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temperature, nanoparticle concentration, heat transfer rate, and the nanoparticle volume fraction rate have been demonstrated graphically and discussed. A comparative study is also present.

II. MATHEMATICAL FORMULATION

Consider two-dimensional, steady, incompressible, laminar flow of non-Newtonian nanofluid past a stretching sheet in a quiescent fluid. The velocity of the stretching sheet is $u_w = U = cx$. The x-axis is taken along the plate in the vertically upward direction and the y-axis is taken normal to the plate. A transverse magnetic field of strength B_o is applied parallel to the y-axis. The surface of plate is maintained at uniform temperature and concentration, T_w and C_w, respectively, and these values are assumed to be greater than the ambient temperature and concentration, T_∞ and C_∞, respectively. Moreover, it is assumed that both the fluid phase and nanoparticles are in thermal equilibrium state. The thermo physical properties of the nanofluid are assumed to be constant. The pressure gradient and external forces are neglected. The governing equations are:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad (1)$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \upsilon\frac{\partial^2 u}{\partial y^2} - \frac{\sigma}{\rho_f}B_o^2 u + \frac{\alpha_1}{\rho_f}\left[\frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial y^2} + u\frac{\partial^3 u}{\partial x \partial y^2} + \frac{\partial u}{\partial y}\frac{\partial^2 v}{\partial y^2} + v\frac{\partial^3 u}{\partial y^3}\right] \qquad (2)$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha_m \nabla^2 T + \frac{Q_o}{(\rho C)_f}(T - T_\infty) + \tau\left[D_B\frac{\partial C}{\partial y}\cdot\frac{\partial T}{\partial y} + \frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial y}\right)^2\right] \qquad (3)$$

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D_B\frac{\partial^2 C}{\partial y^2} + \frac{D_T}{T_\infty}\cdot\frac{\partial^2 T}{\partial y^2} \qquad (4)$$

The boundary conditions for the velocity, temperature, and concentration fields are given as follows:

$$u = U + \kappa\upsilon\frac{\partial u}{\partial y}, v = v_w, T = T_w, C = C_w \ at \ y = 0 \qquad (5a)$$

$$u = 0, T = T_\infty, C = C_\infty \ as \ y \to \infty \qquad (5b)$$

where u and v are the velocity component along the x and y directions, respectively, p is the pressure, ρ_f is the density of base fluid, ρ_p is the nanoparticle density, μ is the absolute viscosity of the base fluid, υ is the kinematic viscosity of the base fluid, σ is the electrical conductivity of the base fluid, α_1 is the material fluid parameter, T is the fluid temperature, α_m is the thermal diffusivity, $\tau\left(=(\rho C)_p / (\rho C)_f\right)$ is the ratio of effective heat capacity of the nanoparticle material to heat capacity of the fluid, C is the

nanoparticle volume fraction, D_B and D_T are the Brownian diffusion coefficient and the thermophoresis diffusion coefficient, T_∞ is the free stream temperature, C_p is the specific heat at constant pressure, and g, k are the acceleration due to gravity, the thermal conductivity of the fluid respectively.

To transform the governing equations into a set of similarity equations, the following dimensionless parameters are introduced

$$\eta = \sqrt{\frac{c}{\upsilon}}y, u = cxf'(\eta), v = -\sqrt{c\upsilon}f(\eta),$$
$$\theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \phi(\eta) = \frac{C - C_\infty}{C_w - C_\infty} \qquad (6)$$

The transformed momentum, energy and concentration equations together with the boundary conditions given by (1)-(4), (5a), (5b) can be written as

$$f''' + ff'' - f'^2 - Mf' - \alpha\left(f''^2 - 2ff''' + ff^{iv}\right) = 0 \qquad (7)$$

$$\frac{1}{Pr}\theta'' + f\theta' + Nb\theta'\phi' + Nt\theta'^2 + Q\theta = 0 \qquad (8)$$

$$\phi'' + Lef\phi' + \frac{Nt}{Nb}\theta'' = 0 \qquad (9)$$

The transformed boundary conditions are

$$f(0) = s, f'(0) = 1 + Kf''(0), \theta(0) = 1, \phi(0) = 1 \ at \ \eta = 0 \quad (10a)$$

$$f'(\infty) \to 0, \theta(\infty) \to 0, \phi(\infty) \to 0 \ as \ \eta \to \infty \qquad (10b)$$

where primes denote differentiation with respect to η and the seven parameters appearing in (7)-(9) are defined as follows

$$Pr = \frac{\upsilon}{\alpha_m}, Le = \frac{\upsilon}{D_B}, M = \frac{\sigma}{\rho_f}\frac{B_o^2}{b}, \alpha = \frac{\alpha_1 b}{\mu}, Q = \frac{Q_o}{b(\rho C)_f},$$
$$Nb = \frac{(\rho C)_p D_B (C_w - C_\infty)}{(\rho C)_f \upsilon}, Nt = \frac{(\rho C)_p D_T (T_w - T_\infty)}{(\rho C)_f T_\infty \upsilon} \qquad (11)$$

In (11), Pr, Le, M, α, Q, Nb and Nt denote the Prandtl number, the Lewis number, the magnetic field strength parameter, the viscoelastic parameter, the heat source/sink parameter, the Brownian motion parameter, and the thermophoresis parameter, respectively.

The physical quantities of interest are the local heat flux Nu and the local mass diffusion flux Sh from the vertical moving plate, which are defined as

$$Nu = \frac{xq_w}{k(T_w - T_\infty)}, Sh = \frac{xh_w}{D_B(C_w - C_\infty)}, \qquad (12)$$

where τ_w is the wall skin friction, q_w is the surface heat flux and h_w is the wall mass flux given by

$$q_w = -k\left(\frac{\partial T}{\partial y}\right)_{y=0}, h_w = -D_B\left(\frac{\partial C}{\partial y}\right)_{y=0} \qquad (13)$$

Using (6) in (12), one can obtain

$$\mathrm{Re}_x^{-1/2} Nu_x = -\theta'(0), \mathrm{Re}_x^{-1/2} Sh_x = -\phi'(0) \qquad (14)$$

where $\mathrm{Re}_x = u_w(x)x/\upsilon$ is the local Reynolds number based on the stretching velocity $u_w(x)$. Kuznetsov and Nield [6] referred $\mathrm{Re}_x^{-1/2} Nu_x$ and $\mathrm{Re}_x^{-1/2} Sh_x$ as the reduced Nusselt number $Nur = -\theta'(0)$ and reduced Sherwood number $Shr = -\phi'(0)$, respectively.

III. METHOD OF SOLUTION

The Finite Element Method (FEM) is a numerical and computer-based technique of solving a variety of practical engineering problems that arise in different fields. It has been applied to a number of physical problems, where the governing differential equations are available. The method essentially consists of assuming the piecewise continuous function for the solution and obtaining the parameters of the functions in a manner that reduces the error in the solution. The steps involved in the finite element analysis are as follows:

- Discretization of the domain into set of finite elements.
- Weighted integral formulation of the differential equation.
- Defining an approximate solution over the element.
- Substitution of the approximate solution & the generation of the element equations.
- Assembly of the stiffness matrices for each element.
- Imposition of the boundary conditions.
- Solution of assembled equations.

The entire flow domain is divided into 10000 quadratic elements of equal size. Each element is three-noded and therefore the whole domain contains 20001 nodes. A system of equations has been obtained which is solved by the Gauss elimination method. The code of the algorithm has been executed in MATLAB running on a PC. Excellent convergence was achieved for all the results.

IV. RESULT AND DISCUSSION

The nonlinear ordinary differential equations (7)-(9) together with the boundary conditions (10a) & (10b) are solved numerically using FEM. The numerical computations have been carried out for different values of the parameters involved. The aim of the present study is to examine the variations of different quantities of parameters in which $0 \le K \le 10, 0 \le \alpha \le 10,\ 0 \le \mathrm{Pr} \le 70, 0.1 \le Nt \le 0.5,$

$0.1 \le Nb \le 0.5$, and $5 \le Le \le 30$. The computational work is carried out by taking size of the element $\nabla\eta = 0.0001$. It is observed that, if the number of elements is increased or the size of the element is decreased in the same domain, even then the accuracy is not affected.

Figs. 1-3 illustrate the velocity, temperature and concentration profiles for different values of the slip parameter K. Fig. 1 demonstrates that the effect of increasing value of slip parameter K is to shift the streamlines toward stretching boundary and thereby reduce thickness of the momentum boundary layer. Therefore, the effect of slip parameter K is seen to decrease the boundary layer velocity while the temperature and concentration are increased with the increase of the slip parameter. Figs. 4-6 show the effect of viscoelastic parameter α on the evolution of fluid motion and subsequent on the distribution of heat and mass across the sheet as time evolves. From this plot it is evident that increasing values of viscoelastic parameter α opposes the motion of the liquid close to the stretching sheet and assists the motion of the liquid far away from the stretching sheet. Increasing values of second-grade parameter enables the liquid to flow at a faster rate due to which there is decline in the heat transfer. This is responsible for the increase in momentum boundary layer whereas the thermal and concentration boundary layers reduce when the viscoelastic effects intensify.

The variations in velocity field, temperature distribution and nanoparticle concentration profile for various values of M are presented in Figs. 7-9. It is clear from these figures that the velocity decreases, whereas the temperature and concentration increase with the increase of the magnetic field parameter. The hydromagnetic force in (7) is a linear Lorentzian body force which acts transverse to the direction of application i.e. in the negative x-direction, parallel to the plate surface. It is directly proportional to the applied magnetic field, B_0. This force inhibits momentum development and decelerates the flow. The supplementary work done in dragging the conducting nanofluid against the action of the magnetic field, B_0, is manifested as thermal energy. This heats the conducting nanofluid and elevates temperatures. The warming of the boundary layer therefore also aids in nanoparticle diffusion which causes a rise in nanoparticle volume fraction, ϕ.

Figs. 10, 11 depict the effects of suction parameter S on velocity and concentration profile. It is noticed that both momentum and concentration boundary layer thickness decrease with the increase in the suction parameter. In order to understand the influence of magnetic parameter M on heat and mass transfer the local Nusselt and Sherwood number are plotted in Figs. 12, 13 for different values of viscoelastic parameter α. From the earlier graphical results, we have noticed that the thickness of the thermal and concentration boundary layers reduce when the viscoelastic effects intensify. This reduction is compensated with the increase in the rate of heat and mass transfer at the stretching surface.

In the present study, the local rate of heat transfer (Nur) and local rate of mass transfer at the sheet (Shr), defined in (14), are the important characteristics. The numerical values of Nur and Shr are exhibited in Tables I, II. Table I shows that the excellent correlation between the current FEM computations and the earlier results of Wang [19] and Gorla and Sidawi [20], for reduced Nusselt number ($-\theta'(0)$) by neglecting slip effect (K), the viscoelastic parameter (α), Brownian effect (Nb) and thermophoresis (Nt) for various values of Prandtl number (Pr) with step size, $h = 0.0003$. Variations of reduced Nusselt number Nur and reduced Sherwood number Shr for various values of Nb, Nt and Le when $Q = 0.1, \alpha = 0.5, M = 1.0, s = 0.5, K = 1.0, Pr = 1.0,$ are depicted in Table II.

A. Graphs
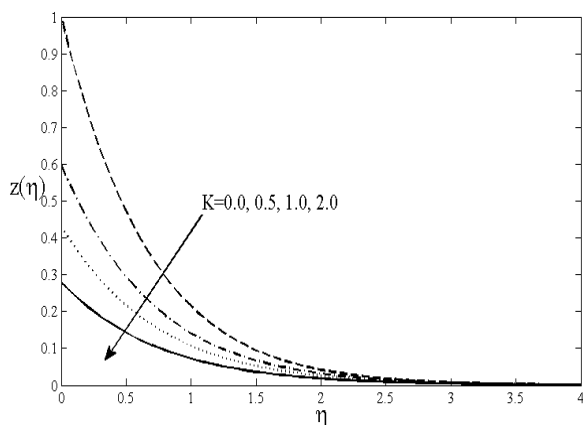
Fig. 1 Effect of K on velocity profile for
$Pr = Le = M = 1.0, Nb = Nt = 0.5, Q = 0.1, \alpha = 0.5, s = 0.5$
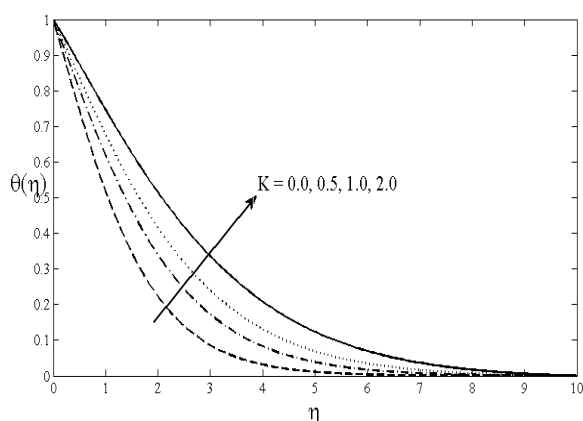
Fig. 2 Effect of K on temperature profile for $s = 0.5$
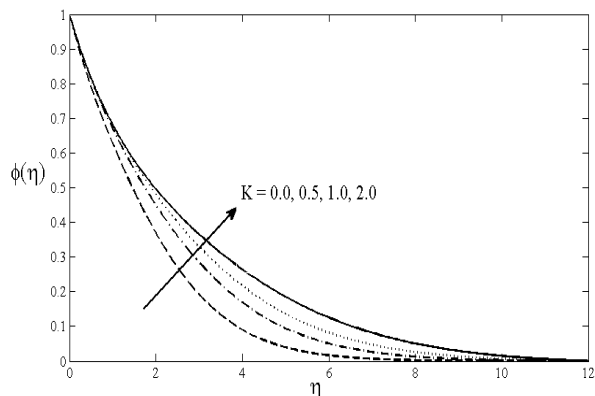$Pr = Le = M = 1.0, Nb = Nt = 0.5, Q = 0.1, \alpha = 0.5,$

Fig. 3 Effect of K on nanoparticle concentration profile for
$Pr = Le = M = 1.0, Nb = Nt = 0.5, s = 0.5, Q = 0.1, \alpha = 0.5,$
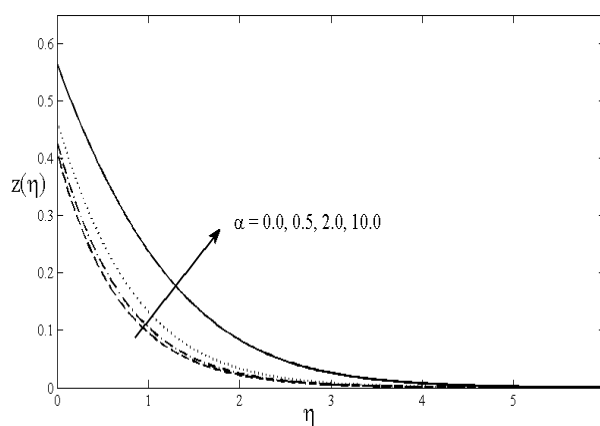

Fig. 4 Effect of α on velocity profile for
$Pr = 2.0, M = 1.0, Le = 1.0, Nb = 0.1, Nt = 0.1, Q = 0.1, K = 1.0, s = 0.5$
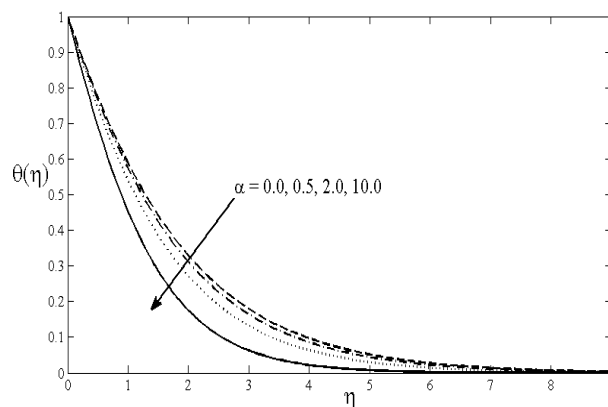

Fig. 5 Effect of α on temperature profile for
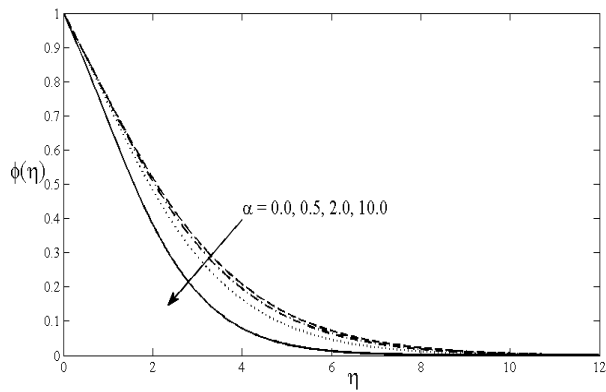$Pr = 2.0, M = 1.0, Le = 1.0, Nb = 0.1, Nt = 0.1, Q = 0.1, K = 1.0, s = 0.5$

Fig. 6 Effect of α on nanoparticle concentration profile for
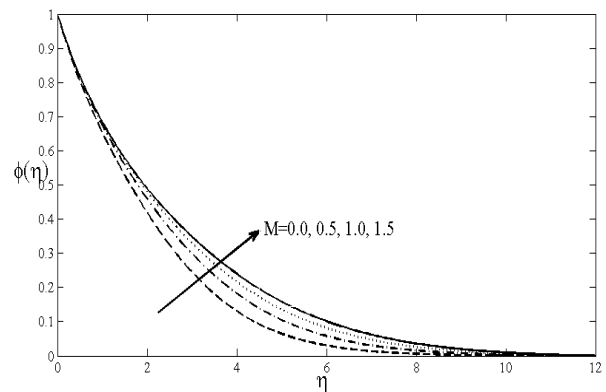$\Pr = 2.0, M = 1.0, Le = 1.0,\ Nb = 0.1, Nt = 0.1, Q = 0.1, K = 1.0, s = 0.5$

Fig. 9 Effect of M on nanoparticle concentration profile for
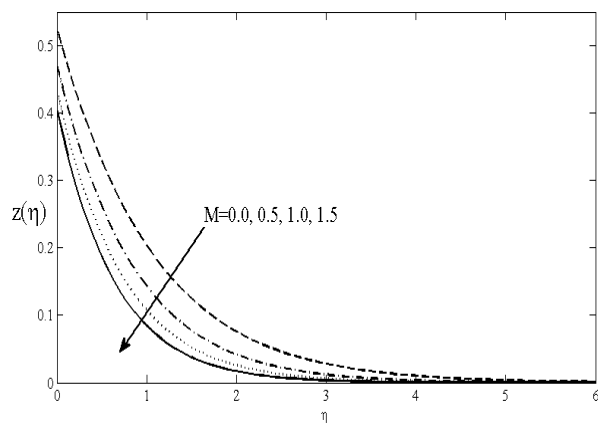$\Pr = Le = 1.0, Nb = Nt = 0.5, Q = 0.1,\ \alpha = 0.5,\ K = 1.0,\ s = 0.5$

Fig. 7 Effect of M on velocity profile for
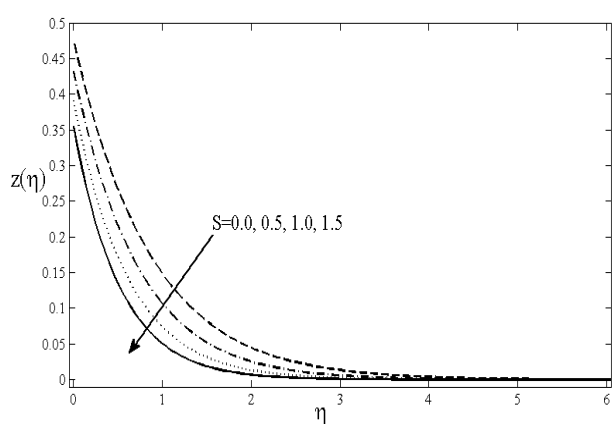$\Pr = Le = 1.0, Nb = Nt = 0.5, Q = 0.1,\ \alpha = 0.5,\ K = 1.0,\ s = 0.5$

Fig. 10 Effect of S on velocity profile for
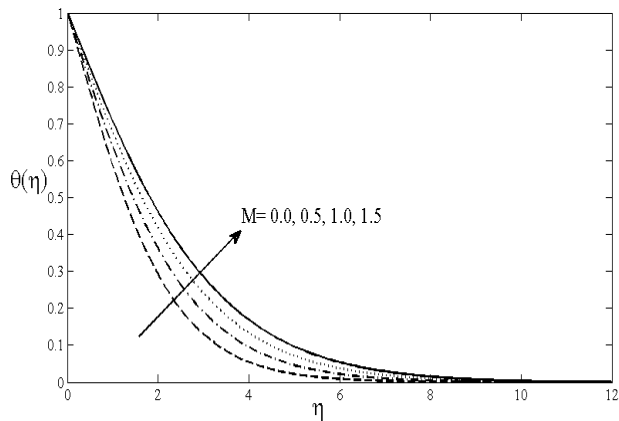$\Pr = M = 1.0,\ Le = 1.0,\ Nb = 0.3, Nt = 0.1, Q = 0.1,\ K = 1.0,$

Fig. 8 Effect of M on temperature profile for
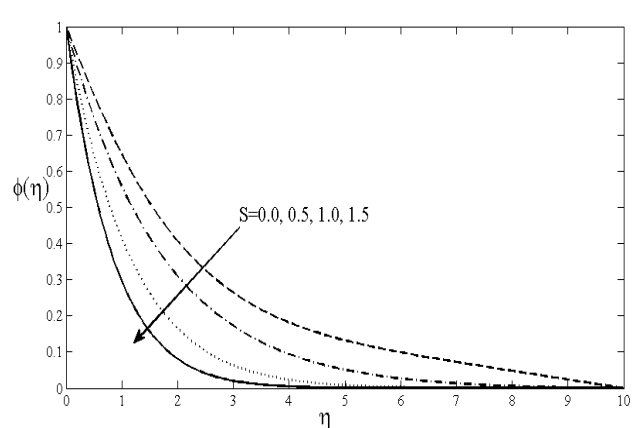$\Pr = Le = 1.0, Nb = Nt = 0.5, Q = 0.1,\ \alpha = 0.5, K = 1.0,\ s = 0.5$

Fig. 11 Effect of S on temperature profile for $\Pr = M = 1.0,$
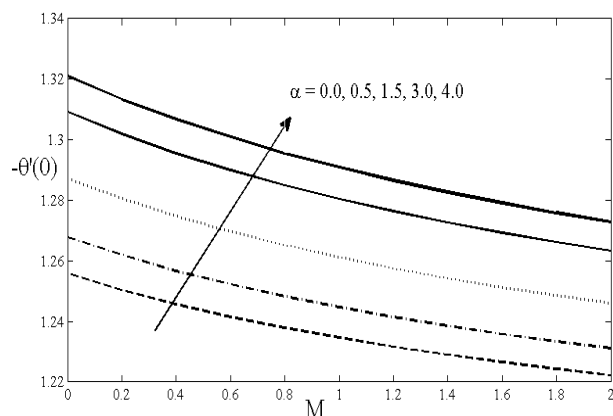$Le = 1.0,\ Nb = 0.3, Nt = 0.1, Q = 0.1,\ K = 1.0,$

Fig. 12 Effect of M and α on reduced Nusselt number Nur for
$\Pr = Le = 1.0,\ Nb = 0.3, Nt = 0.1,\ Q = 0.1,\ K = 1.0,\ s = 1.5$

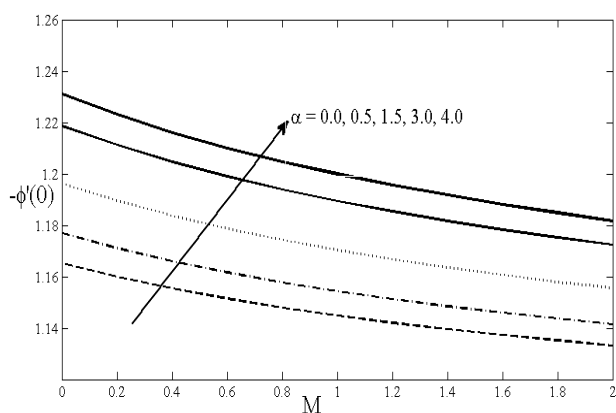
Fig. 13 Effect of M and α on reduced Sherwood number Shr for
$\Pr = Le = 1.0,\ Nb = 0.3, Nt = 0.1, Q = 0.1,\ K = 1.0,\ s = 1.5$

B. Tables

TABLE I

COMPARISON OF RESULT FOR REDUCED NUSSELT NUMBER $-\theta'(0)$ FOR

$K = \alpha = s = M = 0, Le = 10, Nb = Nt = 0,$

Pr	Wang [16]	Gorla and Sidawi [17]	Present result
0.07	0.0656	0.0656	0.0695
0.20	0.1691	0.1691	0.1694
0.70	0.4539	0.5349	0.4541
2.00	0.9114	0.9114	0.9120
7.00	1.8954	1.8905	1.8954
20.0	3.3539	3.3539	3.3539
70.0	6.4622	6.4622	6.4623

TABLE II

VARIATION OF Nur AND Shr WITH Nb, Nt AND Le WHEN

$s = 0.5, K = 1.0, \Pr = 1.0,\ Q = 0.1, M = 1.0, \alpha = 0.5,$

Nb	Nt	$Le = 1$		$Le = 10$	
		Nur	Shr	Nur	Shr
0.1	0.1	0.50278	0.25287	0.48513	4.6238
	0.2	0.48406	-0.15312	0.45966	4.3467
	0.3	0.46576	-0.52569	0.43552	4.1034
	0.4	0.44788	-0.86599	0.41262	3.8909
	0.5	0.43041	-1.1751	0.39089	3.7066
0.3	0.1	0.44173	0.44173	0.41204	4.8534
	0.2	0.42444	0.40848	0.39010	4.7790
	0.3	0.40757	0.34848	0.36947	4.7140
	0.4	0.39110	0.24442	0.34984	4.6575
	0.5	0.37502	0.15688	0.33120	4.6089
0.5	0.1	0.38546	0.62732	0.34938	4.8975
	0.2	0.36959	0.56696	0.33066	4.8621
	0.3	0.35412	0.51221	0.31292	4.8313
	0.4	0.33903	0.46290	0.29611	4.8048
	0.5	0.32430	0.41882	0.28015	4.7821

V. CONCLUSION

The problem of MHD boundary-layer flow of a viscoelastic nanofluid past a stretching sheet has been solved numerically to exhibit the effect of partial slip (i.e. Navier's condition) and heat source/sink on the fluid flow and heat transfer characteristics. The result can be summarized as follows:

1. With the increase in the second grade parameter α, the velocity and the momentum boundary layer thickness increases, however the temperature and nanoparticles concentration decrease.

2. There is a decrease in the velocity, but temperature and concentration are found to increase with an increase in velocity slip parameter K.

3. Magnetic field decelerates the flow and enhances temperatures and nano-particle volume fraction (concentration) distributions in the boundary layer.

4. With increase in the slip parameter K, heat transfer rate and mass transfer rate decrease.

5. By the increase of thermophoretic number Nt, the effect of velocity slip parameter K on reduced Nusselt number Nur and reduced Sherwood number Shr increase and decrease respectively.

6. The reduced Nusselt number and reduced Sherwood number both increases with the increase of viscoelastic parameter α.

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