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Integral Image-Based Differential Filters

Kohei Inoue, Kenji Hara, Kiichi Urahama

Abstract—We describe a relationship between integral images and differential images. First, we derive a simple difference filter from conventional integral image. In the derivation, we show that an integral image and the corresponding differential image are related to each other by simultaneous linear equations, where the numbers of unknowns and equations are the same, and therefore, we can execute the integration and differentiation by solving the simultaneous equations. We applied the relationship to an image fusion problem, and experimentally verified the effectiveness of the proposed method.

Keywords—Integral images, differential images, differential filters, image fusion.

I. Introduction

INTEGRATION and differentiation are fundamental tools in various fields of research including image processing and computer vision. Differentiation is used for detecting discontinuity in images or edges. A number of differential operators have been proposed such as Sobel and Prewitt operators [1], [2]. Viola and Jones [3] introduced integral image which allows the features for face detection to be computed very quickly. After that, Porikli [4] proposed integral histogram to compute the histograms of all possible target regions in a Cartesian data space.

However, the relationship between integral and differential images has not been discussed sufficiently. In this paper, we derive a differential filter from the conventional integral image, and relate the integral images to the differential images by a set of simultaneous linear equations which has the same number of unknowns. Therefore, we can execute the integration and differentiation by solving the simultaneous equations. We also propose a method for image fusion based on the relationship between integral and differential images. Experimental results confirm the validity of the proposed relationship and method for image fusion.

The rest of this paper is organized as follows: Section II describes a relationship between integral and differential images, and shows examples using Sobel and Prewitt operators [1], [2]. Section III proposes a method for image fusion based on the relationship described in Section II. Section IV shows experimental results. Finally, Section V concludes this paper.

II. A Relationship between Integral and Differential Images

Let $f = [f_{ij}]$ be a grayscale image where $f_{ij} \in \{0, 1, \ldots, L\}$ denotes the pixel value at the position (i, j) for $i = 1, \ldots, m$ and $j = 1, \ldots, n$, and L denotes the maximal pixel value. Then the integral image of f is defined by

$$F = [F_{ij}], \quad F_{ij} = \sum_{k=1}^{i} \sum_{l=1}^{j} f_{kl}. \tag{1}$$

The integral image F can be computed in one pass by using the following pair of recurrences:

$$\begin{aligned} s_{ij} &= s_{i,j-1} + f_{ij}, && (2)\\ F_{ij} &= F_{i-1,j} + s_{ij}, && (3) \end{aligned}$$

where s_{ij} is the cumulative row sum, and initialized as $s_{i0} = 0$ and $F_{0j} = 0$ [3].

We consider the following problem: Assume that an integral image F is given. Find f which is related to F by (1).

From (1), we have mn simultaneous equations as follows:

$$\begin{array}{cccc} F_{11} = f_{11}, & F_{12} = f_{11} + f_{12}, & \cdots & F_{1n} = \sum_{j=1}^{n} f_{1j}, \\ F_{21} = f_{11} + f_{21}, & F_{22} = \sum_{i=1}^{2}\sum_{j=1}^{2} f_{ij}, & \cdots & F_{2n} = \sum_{i=1}^{2}\sum_{j=1}^{n} f_{ij}, \\ \vdots & \vdots & \ddots & \vdots \\ F_{m1} = \sum_{i=1}^{m} f_{i1}, & F_{m2} = \sum_{i=1}^{m}\sum_{j=1}^{2} f_{ij}, & \cdots & F_{mn} = \sum_{i=1}^{m}\sum_{j=1}^{n} f_{ij}, \end{array} \tag{4}$$

where the number of unknowns $\{f_{ij}\}$ is equal to that of equations. Therefore, we can solve for them as follows. Substituting (2) for (3), we have

$$\begin{aligned} F_{ij} &= F_{i-1,j} + s_{i,j-1} + f_{ij}, \\ &= F_{i-1,j} + s_{i,j-1} + F_{i-1,j-1} - F_{i-1,j-1} + f_{ij}, \\ &= F_{i-1,j} + F_{i,j-1} - F_{i-1,j-1} + f_{ij}, \end{aligned} \tag{5}$$

where if $i - 1 < 1$ or $j - 1 < 1$, then $F_{i-1,j} = F_{i,j-1} = F_{i-1,j-1} = 0$. From (5), we have

$$f_{ij} = F_{ij} - F_{i-1,j} - F_{i,j-1} + F_{i-1,j-1}. \tag{6}$$

The subscripts i and j in the left side of (6) are not smaller than that of the right side. Hence, we can calculate f_{ij} in raster scan order. The coefficients of the differential filter defined by (6) is given by

$$W = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \end{bmatrix} [-1 \quad 1], \tag{7}$$

where $[-1 \quad 1]^T$ and $[-1 \quad 1]$ denote the coefficients of finite differences in the vertical and horizontal directions, respectively. Then we have

$$f = W * F, \tag{8}$$

where $*$ denotes convolution operator.

Kohei Inoue, Kenji Hara and Kiichi Urahama are with Kyushu University, Fukuoka, 815-8540 Japan (e-mail: k-inoue@design.kyushu-u.ac.jp).

Thus, we can derive two finite difference operators for row and column gradients from the definition of integral images. An integral image F and its differential image $f = \partial^2 F/\partial x \partial y$ are related to each other by the simultaneous equations like (4), where x and y denote the horizontal and vertical directions in image plane, respectively.

Let vec be the column vectorizing operator which stacks the columns of a matrix in a column vector, i.e., for an $m \times n$ matrix $F = [F_{ij}]$,

$$\text{vec}(F) = [F_{11}, F_{21}, \cdots, F_{m1}, \cdots, F_{1n}, F_{2n}, \cdots, F_{mn}]^T. \tag{9}$$

And let $\boldsymbol{v}_F = \text{vec}(F)$ and $\boldsymbol{v}_f = \text{vec}(f)$. Then we can write the transform from $\boldsymbol{v}_F$ into $\boldsymbol{v}_f$ as

$$\boldsymbol{v}_f = C\boldsymbol{v}_F, \tag{10}$$

where C is an $mn \times mn$ matrix of which the elements are the coefficients of the differential filter. The inverse transform is expressed as

$$\boldsymbol{v}_F = C^{-1}\boldsymbol{v}_f, \tag{11}$$

where C^{-1} is the inverse matrix of C.

Examples

Sobel operator [1], [2] has the coefficients of finite differences in vertical and horizontal directions as follows:

$$W_y^S = \begin{bmatrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ 1 & 2 & 1 \end{bmatrix}, \quad W_x^S = \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix}. \tag{12}$$

Similarly to (7), we have the coefficients of the differential filter for Sobel operator [2] as

$$W^S = W_y^S W_x^S = 6U, \quad U = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 0 & 0 \\ -1 & 0 & 1 \end{bmatrix}. \tag{13}$$

Prewitt operator [1], [2] has the coefficients of finite differences in vertical and horizontal directions as follows:

$$W_y^P = \begin{bmatrix} -1 & -1 & -1 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}, \quad W_x^P = \begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix}, \tag{14}$$

from which we have the coefficients of the differential filter for Prewitt operator [1], [2] as

$$W^P = W_y^P W_x^P = 3U. \tag{15}$$

Therefore, we can unify W^S and W^P in the form of

$$W^\alpha = \alpha U, \tag{16}$$

where $\alpha \in \{3, 6\}$. Then C in (10) for W^α becomes $C^\alpha = \alpha[C_{IJ}]$, where $I = m(j-1) + i$, $J = m(l-1) + k$ and

$$C_{IJ} = \begin{cases} 1, & \text{if} \quad k = i \pm 1 \text{ and } l = j \pm 1, \\ -1, & \text{else if} \quad k = i \pm 1 \text{ and } l = j \mp 1, \\ 0, & \text{otherwise.} \end{cases} \tag{17}$$

(a) Original image

(b) Differential image of (a)

(c) Integral image of (b)

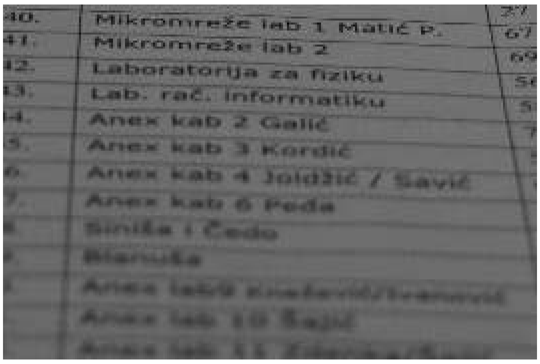
Fig. 1 Differential and integral images based on Sobel operator [1], [2]

III. Application to Image Fusion

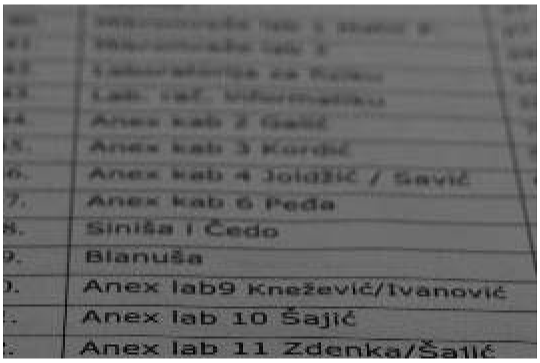
In this section, we propose a method for applying the above relationship between integral and differential images to multifocus image fusion.

Let $A = [A_{ij}]$ and $B = [B_{ij}]$ be two input images to be fused. Then we first calculate the row gradients

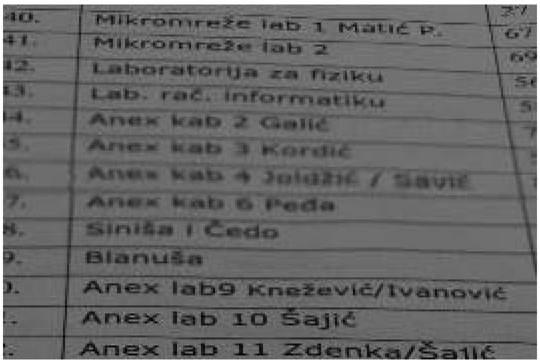
$$A_x = \frac{\partial A}{\partial x} = [A_{xij}], \quad A_{xij} = A_{ij} - A_{i,j-1}, \tag{18}$$



(a) 1st input image

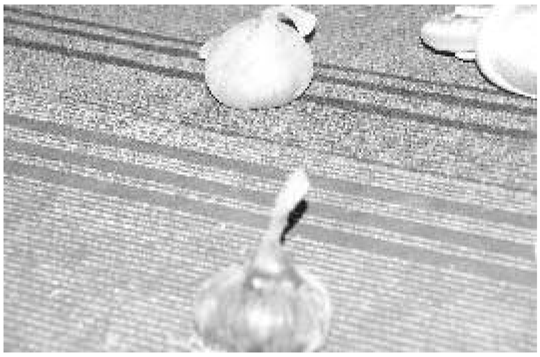


(b) 2nd input image

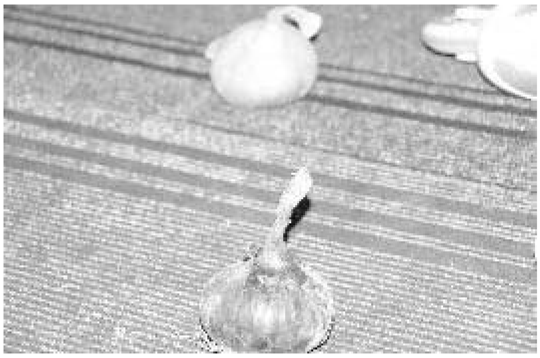


(c) Fused image

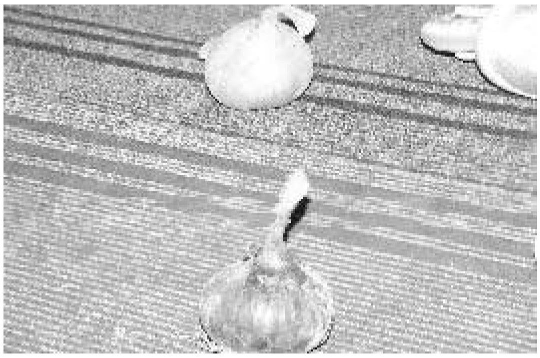
Fig. 2 Multifocus image fusion 1



(a) 1st input image



(b) 2nd input image



(c) Fused image

Fig. 3 Multifocus image fusion 2

$$B_x = \frac{\partial B}{\partial x} = [B_{xij}], \quad B_{xij} = B_{ij} - B_{i,j-1}, \quad (19)$$

where we use the coefficients of finite difference in the horizontal direction: $[-1 \;\; 1]$. Next, we fuse A_x and B_x as follows: Calculate a kind of bilateral distance [5] as follows:

$$d_{Ai} = \sum_{j=1}^{n-1} \sqrt{1 + (A_{xi,j+1} - A_{xij})^2}, \quad (20)$$

$$d_{Bi} = \sum_{j=1}^{n-1} \sqrt{1 + (B_{xi,j+1} - B_{xij})^2}. \quad (21)$$

Then we select the larger one, and have the following image:

$$S_x = [S_{xij}], \quad S_{xij} = \begin{cases} A_{xij}, & \text{if} \quad d_{Ai} \geq d_{Bi}, \\ B_{xij}, & \text{otherwise.} \end{cases} \quad (22)$$

In order to obtain a fused differential image, we calculate the column gradient of S_x as follows:

$$S_{xy} = \frac{\partial S_x}{\partial y} = [S_{xyij}], \quad S_{xyij} = S_{xij} - S_{x,i-1,j}, \quad (23)$$

where we use the coefficients of finite difference in the vertical direction: $[-1 \;\; 1]^T$. Finally, we integrate S_{xy} to obtain the fused image

$$T = [T_{ij}], \quad T_{ij} = \sum_{k=1}^{i} \sum_{l=1}^{j} S_{xykl}. \quad (24)$$

IV. Experimental Results

Fig. 1 shows the results of computation of differential and integral images based on Sobel operator [1], [2]. Fig. 1(a)

(a) 1st input image

(b) 2nd input image

(c) Fused image

Fig. 4 Multifocus image fusion 3

shows an original image of 512×512 pixels, i.e., $m = n = 512$. We vectorized it and transformed the vector into the vectorized differential image with (10). Then we reshaped the resultant vector into an $m \times n$ matrix to obtain the differential image shown in Fig. 1(b), where we linearly fitted the minimal and maximal values into 0 and L for the purpose of displaying the differential image. Finally, we integrated the differential image by (11) to obtain the integral image shown in Fig. 1(c), which coincides with Fig. 1(a). These results exemplifies the validity of (10) and (11).

Fig. 2 shows an example of multifocus image fusion. We fused two differently focused images [6] in Figs. 2(a) and (b) by the method described in Section III. The number of pixels in each image is 177×267. In Figs. 2(a) and (b), the bottom and top of the images are blurred, respectively. The fused image is shown in Fig. 2(c), where the bottom region is clearer than that of Fig. 2(a), and the top region is clearer than that of Fig. 2(b). We also show other examples of multifocus image fusion in Figs. 3 and 4.

We performed the above experiments using MATLAB R2011b on a desktop computer with Intel Core i5 CPU and 4.00GB RAM.

V. Conclusion

In this paper, we described a relationship between integral and differential images. The relationship is represented by simultaneous linear equations, where the number of unknowns is equal to that of equations, therefore, we can execute the integration and differentiation by solving the simultaneous equations. We also proposed a method for image fusion based on the relationship. We experimentally verified the effectiveness of the proposed method applied to multifocus image fusion.

Future work will include the application of the proposed method to high dynamic range imaging, non-photorealistic rendering and miniature faking.

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