International Journal of Engineering, Mathematical and Physical Sciences
ISSN: 2517-9934
Vol:8, No:4, 2014

Arbitrary Amplitude Ion-Acoustic Solitary Waves in Electron-Ion-Positron Plasma with Nonthermal Electrons

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***Abstract*—**Using pseudo potential method arbitrary amplitude ion-acoustic solitary waves have been theoretically studied in a collisionless plasma consisting of warm drifting positive ions, Boltzmann positrons and nonthermal electrons. Ion-acoustic solitary wave solutions have been obtained and the dependence of the solitary wave profile on different plasma parameters has been studied numerically. Lower and higher order compressive and rarefactive solitary waves are observed in presence of positrons, nonthermal electrons, ion drift velocity and finite ion temperature. Inclusion of higher order nonlinearity is shown to have significant correction to the solitary wave profile for the same values of plasma parameters.

***Keywords*—**Ion-acoustic waves, Nonthermal electrons, Sagdeev potential, Solitary waves.

I. Introduction

In the last two decades there has been a great deal of interest in the study of nonlinear wave phenomena in electron-positron-ion (e-p-i) plasmas [1]-[6]. This is due to the fact that e-p-i plasmas occur in many astro physical environments such as active galactic nuclei [7], pulsar magnetosphere [8], polar regions of neutron stars [9], centre of our galaxy [10], the early universe [11], [12], and solar atmosphere [13]. The e-p-i plasmas have also been produced in some laboratory environments [14]-[16]. Inclusion of positrons is found to drastically modify the ion-acoustic wave supported by simple electron-ion (e-i) plasma. Popel et al [2] have reported decrease in ion-acoustic soliton amplitude in presence of positrons. The presence of non-Maxwellian electrons is common in space and astrophysical plasmas [17]. The soliotary structures with density depression in the magnetosphere observed by Freja [18] and Viking [19] satellites have been explained by Cairns et al [20] by assuming a plasma with nonthermal electrons and cold ions. In fact the presence of the presence of nonthermal electrons in plasma gives rise to many interesting characteristics in nonlinear propagation of waves including the excitation of ion-acoustic solitons [10], [21]-[24] and double layers in plasma. Nonlinear ion-acoustic solitary waves in e-p-i plasma with nonthermal electrons have been considered by some authors [2], [21], [23] assuming ions to be cold. In practice ions have finite temperature and it can significantly modify the characteristics of nonlinear ion-acoustic structures [24]-[26]. The effect of ion temperature on ion-acoustic solitary waves in e-p-i plasma with nonthermal electrons has been studied only by a few authors [27], [28]. Pakzad [27] has shown that in warm plasma solitons speed is more but solitons amplitude is less than that in cold plasma. Mamun [28] has shown that the effects of ion temperature change the minimum value of the nonthermal parameter as well as the Mach number for which compressive and rarefactive solitons co-exist and also change the width and amplitude of the solitary waves. The purpose of the present paper is to make a detailed analysis of the nonlinear propagation of small as well as large amplitude ion-acoustic solitary structures in e-p-i plasma with nonthermal electrons, warm ions and Boltzmann positrons.

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II. Basic Equation

We consider an unmagnetized collisionless plasma consisting of warm positive ions, Boltzmann positrons and nonthermal electrons. The normalized basic equations governing ion dynamics for unidirectional propagation in such plasma in dimensionless form are the following [29]

$$\frac{\partial n_i}{\partial t}+\frac{\partial}{\partial x}(\mathrm{n}_i \mathrm{v}_i)=0 \tag{1}$$

$$\frac{\partial v_i}{\partial t}+v_i\frac{\partial v_i}{\partial x}+\frac{3\sigma_i}{(1-\chi)^2}n_i\frac{\partial n_i}{\partial x}=-\frac{\partial \phi}{\partial x} \tag{2}$$

$$\frac{\partial^2 \phi}{\partial x^2}=n_e-n_p-n_i \tag{3}$$

In the above equations, the parameters $n_i\, and v_i$ are respectively the concentration and velocity of the positive ions; n_e and n_p are respectively the concentration of electrons and positrons; ϕ denotes the electrostatic potential, other parameters have their usual meaning; the velocities are normalized by $\sqrt{\frac{k_B T_e}{m_i}}$; the densities are normalized by equilibrium electron density n_{e0}; all the length x by the electron Debye length $\lambda_{De}=\sqrt{\frac{k_B T_e}{4e^2 n_{e0}}}$; time by $\frac{\lambda_{De}}{C_s}$; ion temperature by T_i by T_e $\left(\sigma_i=\frac{T_i}{T_e}\right)$ and the potential ϕ by $\frac{k_B T_e}{e}$; where k_B is the Boltzmann's constant. The nonthermal electron density is given by;

$$n_e = (1-\beta\phi+\beta\phi^2)\exp(\phi) \tag{4}$$

where $\beta = \frac{4\delta}{1+3\delta}$ measures the deviation from the thermalized state and δ determines the presence of nonthermal electrons inside the plasma. The density of Boltzmann positrons is given by,

$$n_p = \chi\exp(-\sigma_p\phi) \tag{5}$$

where $\chi = \frac{n_{p0}}{n_{e0}}$ is the ratio between the unperturbed positron and electron number densities and $\sigma_p = \frac{T_e}{T_p}$ is the ratio between electron and positron temperatures. The equilibrium charge neutrality condition in normalized form is given by

$$\chi + n_{i0} = 1 \tag{6}$$

Using (4) and (5), (3) is re-written as

$$\frac{\partial^2\phi}{\partial x^2} = (1-\beta+\beta\phi^2)\exp(\phi) - \chi\exp(-\sigma_p\phi) - n_i \tag{7}$$

III. Sagdeev Potential

In order to derive the Sagdeev potential we use the following Galilean transformation

$$\eta = x - Vt \tag{8}$$

where V is the phase velocity of the solitary waves.

We also use the following boundary conditions:
As

$$x \to \infty,\ v \to v_{i0},\ n \to n_{i0} = (1-\chi),\ \phi \to 0 \tag{9}$$

Now using (8) in (1) and (2) we get,

$$n_i = \frac{(1-\chi)}{2\sqrt{3\sigma_i}}\left[\sqrt{(A-B-2\phi)} - \sqrt{(A+B-2\phi)}\right] \tag{10}$$

where,

$$A = 3\sigma_i + \left(V - v_{i0}\right)^2 \tag{11}$$

and

$$B = \sqrt{12\sigma_i\left(V - v_{i0}\right)^2} \tag{12}$$

Now from (7) we get,

$$-\frac{d^2\phi}{d\eta^2} = \left(1+\phi+\frac{\phi^2}{2}+\frac{\phi^3}{6}-\beta\phi-\beta\frac{\phi^3}{2}-\beta\frac{\phi^5}{24}+\beta\phi^3\right) - \left(\chi+\chi\sigma_p\phi-\chi\frac{\sigma_p^2\phi^2}{2}+\chi\frac{\sigma_p^3\phi^3}{2}\right) - \frac{(1-\chi)}{2\sqrt{3\sigma_i}}\left[\sqrt{(A-B-2\phi)} - \sqrt{(A+B-2\phi)}\right] \tag{13}$$

Equation (13) can be rewritten as:

$$\frac{d^2\phi}{d\eta^2} = -\frac{\partial\psi_s}{\partial\phi} = -\psi_s(\phi) \tag{14}$$

where, ψ_s is known as Sagdeev Potential.

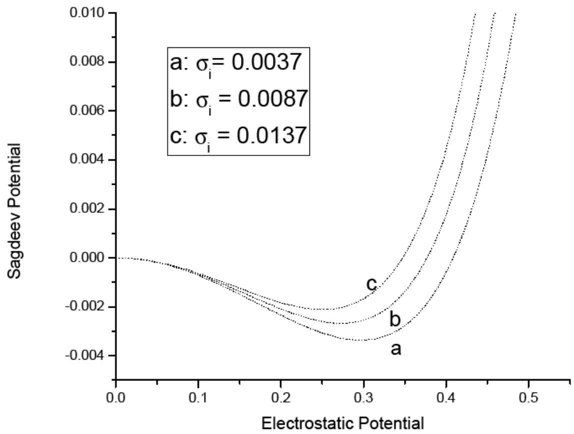

Fig. 1 Profile of Sagdeev potential by varying ion temperature (σ_i =0.0037, 0.0087 and 0.0137) where, nonthermal parameter β=0.15, positron temperature σ_p = 0.01, positron concentration χ=0.01, phase velocity V=1.705, initial velocity v_{i0}=0.5.

In Fig. 1, we plot Sagdeev potential for different values of ion temperatures. It shows that the depth of Sagdeev potential decreases as ion temperature increases.

IV. Solitary Waves

For small amplitude case $\left(|\phi| < 1\right)$ Sagdeev potential may be expanded as follows [30]:

$$\frac{d^2\phi}{d\eta^2} = C^{(1)}\phi + C^{(2)}\phi^2 + C^{(3)}\phi^3 \ldots \tag{15}$$

where,

$$C^{(1)} = 1-\beta+\chi\sigma_p+\frac{(1-\chi)}{2\sqrt{3\sigma_i}}\left(\frac{1}{\sqrt{A+B}}-\frac{1}{\sqrt{A-B}}\right) \tag{16}$$

$$C^{(2)} = -\frac{1}{2}+\frac{\chi\sigma^2{}_p}{2}-\frac{(1-\chi)}{4\sqrt{3\sigma_i}}\left\{\frac{1}{(A+B)^{3/2}}+\frac{1}{(A-B)^{3/2}}\right\} \tag{17}$$

$$C^{(3)} = \frac{1}{6} - \frac{\beta}{2} + \frac{\chi \sigma^3{}_p}{6} + \frac{(1-\chi)}{4\sqrt{3\sigma_i}} \left\{ \frac{1}{(A+B)^{5/2}} - \frac{1}{(A-B)^{5/2}} \right\} \quad (18)$$

Lowest order solitary wave solution is

$$\phi^{(1)} = \frac{3C^{(1)}}{2C^{(2)}} \sec h^2 \left(\frac{\eta}{\vartriangle} \right) \quad (19)$$

And higher order solitary wave solution is [30],

$$\phi^{(2)} = \frac{2\gamma^{(1)}}{\left\{ \left(\gamma^{(2)}\right)^2 - 4\gamma^{(1)}\gamma^{(3)} \right\}^{1/2} \left(2\cosh^2\theta - 1 \right) + \gamma^{(2)}} \quad (20)$$

where,

$$\vartriangle^{(1)} = \frac{2}{\sqrt{C^{(1)}}}, \quad \gamma^{(1)} = C^{(1)}, \quad \gamma^{(2)} = \frac{2C^{(2)}}{3}, \quad \gamma^{(3)} = -\frac{C^{(3)}}{2} \quad (21)$$

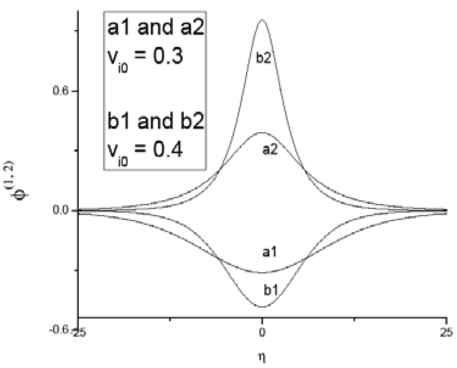

Fig. 2 Lower and higher order solitary structures by varying ion drift velocity (v_{i0}=0.3 and 0.4) where nonthermal parameter β=0.025, ion temperature σ_i = 0.015, positron temperature σ_p =0.015, positron concentration χ=0.028, phase velocity V=1.65.

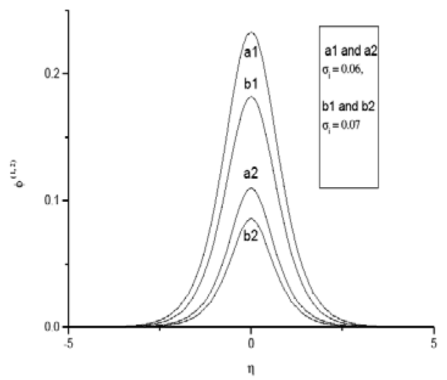

Fig. 3 Lower and higher order solitary structures by varying ion temperature (σ_i= 0.06 and 0.0.07), where nonthermal parameter β=0.055, positron temperature σ_p =0.015, positron concentration χ=0.028, phase velocity V=1.45, initial velocity v_{i0}=0.5.

Figs. 2 and 3 show the soliton profiles for lower and higher order solitary wave solutions. a1 and b1 represents solitary structures for lowest order and a2 and b2 represents higher order solitary structures. From the profiles we find that the nature of solitary structures depend significantly on various plasma parameters like nonthermal parameter, ion temperature, positron concentration and ion drift velocity.

V. Results

1. The depth of Sagdeev potential decreases as ion temperature increases.
2. The amplitude of compressive solitons decreases significantly as we include higher order nonlinearity.
3. In case of rarefactive solitons inclusion of higher order nonlinearity may change the nature of the soliton.

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