A Comprehensive Approach in Calculating the Impact of the Ground on Radiated Electromagnetic Fields Due to Lightning

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Abstract—The influence of finite ground conductivity is of great importance in calculating the induced voltages from the radiated electromagnetic fields due to lightning. In this paper, we try to give a comprehensive approach to calculate the impact of the ground on the radiated electromagnetic fields to lightning. The vertical component of lightning electric field is calculated with a reasonable approximation assuming a perfectly conducting ground in case the observation point does not exceed a few kilometers from the lightning channel. However, for distant observation points the radiated vertical component of lightning electric field is attenuated due finitely conducting ground. The attenuation is calculated using the expression elaborated for both low and high frequencies. The horizontal component of the electric field, however, is more affected by a finite conductivity of a ground. Besides, the contribution of the horizontal component of the electric field, to induced voltages on an overhead transmission line, is greater than that of the vertical component. Therefore, the calculation of the horizontal electric field is great concern for the simulation of lightning-induced voltages. For field to transmission lines coupling the ground impedance is calculated for early time behavior and for low frequency range.

Keywords—Ground impedance, horizontal electric field, lightning, transient propagation, vertical electric field.

I. INTRODUCTION

PROPAGATION of lightning electromagnetic pulses on overhead lines can damage electric components due to induced overvoltages. These transient overvoltages are the response to either a direct strike or to a nearby lightning strike. The induced overvoltages to such external disturbances depend considerably on the ground impedances. Several approaches have been done to include the effect of the ground in the line impedances.

In this paper, from comparative studies of many works, accurate formulations are selected to calculate the impact of the transient ground impedance on the electromagnetic radiations. As we know, electromagnetic fields generated by lightning change their form while propagating over a finitely conducting ground. Depending on the distance, the electromagnetic fields measured far away from the source may deviate from their ideal values depending on both the distance of the propagation and the ground conductivity.

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II. PROBLEM FORMULATION

A. General Formulation

Reference [1] suggested for a single conductor above a lossy ground the use of the following relation:

$$\int_0^\infty \frac{e^{-2h\lambda}}{\lambda + \sqrt{\lambda^2 + \eta^2}} d\lambda \tag{1}$$

where: $\eta = \sqrt{\frac{j\omega\mu}{\rho} + \omega^2(\mu_0\varepsilon_0 - \mu_g\varepsilon_{g)}}$ with ω is the angular frequency, μ the permeability and ρ is the ground resistivity. h is the height of the conductors. The subscripts 0 and g refer to the air and the ground respectively.

Relation (1) is a generalized formulation for the different earth model expression available in the literature. Indeed, if the displacement currents in the air and in the earth are disregarded, we obtain Carson's formula. Sunde's relation can be obtained by only neglecting the displacement current in the air [2].

B. Low Frequency Approximation

References [3], [4] found an analytical inverse Fourier transform for (1) in case of low frequencies when $\rho \ll \omega\mu_0\varepsilon_g$.

$$\xi(t) = \frac{\mu_0}{\pi\tau_g}\left[\frac{1}{2\sqrt{\pi}}\sqrt{\frac{\tau_g}{t}} + \frac{1}{4}\exp\left(\frac{\tau_g}{t}\right).erfc\left(\frac{\tau_g}{t} - \frac{1}{4}\right)\right] \tag{2}$$

where $\tau_g = h^2\mu_0/\rho$ and $erfc(*)$ is the complementary error function.

$$erfc(t) = \frac{2}{\pi}\int_t^\infty e^{-\lambda} d\lambda \tag{3}$$

C. High Frequency Approximation (Early Time Behavior)

Relation (2) has singular behavior at $t = 0$ which leads to:

$$\xi(t) = \frac{\mu_0}{2\pi}\frac{1}{\sqrt{\pi\tau_g t}} \qquad \text{for } t \to 0 \tag{4}$$

Besides, due to the fundamental property of the Fourier transform, (2) is not valid for early time (high frequencies).

The authors, as in [3], [4], found a relation which gives the transient ground impedance as follows:

$$\xi(t) = \frac{1}{2\pi h}\sqrt{\frac{\mu_0}{\varepsilon_g}}\exp\left(-\frac{t}{2t_{min}}\right)I_0\left(\frac{t}{2t_{min}}\right) - \frac{\rho}{4\pi h^2}(1 \exp\left(-\frac{t}{t_{min}}\right)) \tag{5}$$

where I_0 is the modified Bessel function of the first kind and $t_{min} = \rho \varepsilon_g$

Equation (5) can be used to estimate the ground transient resistance for $0 < t < t_{min}$. Relations (2) and (5), when used together, can cover the entire range with satisfactory accuracy.

III. EFFECT OF THE GROUND ON THE RADIATED ELECTROMAGNETIC FIELDS

The ground conductivity plays an important role in the distortion of the radiated fields. The electromagnetic fields generated by lightning change their wave shapes and their amplitudes. Various studies have been done to describe the impact of the ground on both the horizontal and vertical components of radiated fields [5], [6].

A. Horizontal Electric Field

The presence of the finite conductivity of the ground can be introduced as a correction term in the total incident field on the line at a given height. The total field will be the sum of the horizontal electric field calculated under perfect ground and the electric field at the ground level. The electric at the ground level is the product of magnetic field with the surface impedance. Thus, the total horizontal electric field can be given by the following formula [7]:

$$E_r(r,z,j\omega) = E_{rp}(r,z,j\omega) - H_{\varphi p}(r,0,j\omega).\sqrt{\frac{\mu_0}{\varepsilon_{rg}+1/j\omega\rho}} \quad (6)$$

where r is the horizontal position, z is the vertical position, p refers to perfect ground and H_φ is the azimuthally component of the magnetic field.

The surface impedance can be expressed in time domain by the following expression:

$$Z_g = \beta e^{-at} \left\{ \begin{matrix} a[I_1(at) - I_0(at)] \\ \times u(t) + \delta(t) \end{matrix} \right\} \quad (7)$$

where $\beta = \sqrt{\frac{\mu_0}{\varepsilon_g}}$, $a = \frac{\iota\sigma}{2\varepsilon}$, ε_{rg} and I_0 and I_1 are the modified Bessel functions

For distances located beyond a few kilometers (6) is reduced to the Wavetilt formula:

$$W(j\omega) = \frac{E_r}{E_z} = \frac{1}{\sqrt{\varepsilon_{rg}+1/j\omega\rho\varepsilon_0}} \quad (8)$$

The wavetilt formula is not applicable for calculating the horizontal electric field for distances less than a few kilometers because the return stroke upward speed of about $1.10^8 \ m/s$ excites regions of few hundred meters in height in a few microseconds. This is because in the early time of the lightning return stroke, the electromagnetic fields are essentially radiation fields for distances larger than a few kilometers.

B. Vertical Electric Field

The vertical component of the electric field may be calculated with a reasonable approximation, assuming a perfectly conducting ground, in case the observation point does not exceed a few kilometers from the lightning stroke.

For far field radiations, neglecting the static and induction components of the electric field and ignoring the height of the lightning stroke due the long distance, the vertical electric field for an observation point at ground levels is:

$$E_z^{far}(r,t) = \frac{1}{2\pi\varepsilon_0 c^2 r} \int_0^h \frac{\partial i(z,t-\frac{r}{c})}{\partial t} dz \quad (9)$$

Reference [8] expressed (9) over a finitely conducting ground as follows:

$$E_{z\sigma}^{far}(r,t) = \frac{1}{2\pi\varepsilon_0 c^2 r} \int_0^t E_z^{far}(r,t-\tau)S(r,\tau)d\tau \quad (10)$$

ehere σ is the ground conductivity. $S(r,t)$ is an attenuation factor; for the case of a homogeneous ground, is given by:

$$S(r,t) = \frac{d}{dt}\left[1 - \exp\left(-\frac{t^2}{4\zeta^2}\right) + 2\delta(\varepsilon_{rg}+1)\frac{J(x)}{t}\right] \quad (11)$$

where $J(x) = x^2(1-x^2)\exp(-x^2)$, $x = t/2\zeta$, $\zeta^2 = r\rho/(2\mu_0 c^3)$ and $\delta = \rho/(2\mu_0 c^2)$ c is the speed of light.

IV. CONCLUSIONS

In this paper, exact solutions for the ground resistance are given. The effect of ground resistance, for low frequencies, can be calculated in an efficient way as in (2). However for early time transient resistance, the effect is calculated by the equation described as in (5). Although the comparison with other works is not mentioned in this paper, the validity of the relations given in this paper is done by the authors listed in the references. Components of the electric field can be calculated by the known methods (ground as a perfect conductor), added to it the attenuation due to the finitely conducting earth. The attenuation can be calculated for the nearby and for long distances from the return stroke.

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