Performance Evaluation of Cooperative Diversity in Flat Fading Channel with Error Control Coding

Oluseye Adeniyi Adeleke, Mohd Fadzli Salleh

Abstract—Cooperative communication provides transmit diversity, even when, due to size constraints, mobile units cannot accommodate multiple antennas. A versatile cooperation method called coded cooperation has been developed, in which cooperation is implemented through channel coding with a view to controlling the errors inherent in wireless communication. In this work we evaluate the performance of coded cooperation in flat Rayleigh fading environment using a concept known as the pair wise error probability (PEP). We derive the PEP for a flat fading scenario in coded cooperation and then compare with the signal-to-noise ratio of the users in the network. Results show that an increase in the SNR leads to a decrease in the PEP. We also carried out simulations to validate the result.

Keywords—Channel state information, coded cooperation, cooperative systems, pairwise-error-probability, Reed-Solomon codes.

I. Introduction

UPLINK transmit diversity is a powerful technique for mitigating fading in wireless systems. However this technique is actually not practicably applicable to mobile units due to their small size. The concept of user cooperation or cooperative diversity has been proposed as a means of creating transmit diversity in the uplink of a wireless system, without the need for multiple antennas on the mobile devices. Cooperative diversity is a concept used in describing a process whereby single-antenna mobiles share their antennas in order to achieve transmit diversity [1], [2].

Previously, methods proposed include, users detecting and repeating estimates of the symbols transmitted by the partner, or amplifying and forwarding the partner's data [3], [4]. These two aforementioned methods have been shown to improve the capacity and the signal-to-noise ratio, in spite of noise on the channel between the cooperating users. However, these earlier schemes are not without some limitations; some of which are highlighted as follows:

1. These schemes include the propagation of the partner's noise during the cooperative process, or in other words, these schemes encourage the forwarding of erroneous estimates of the partner's symbols,
2. These schemes involve some form of repetition; and from the standpoint of channel coding, this is not a prudent use of the available bandwidth,
3. For optimal maximum likelihood (ML) detection at the receiver, these previous methods require that either the SNR or the BER be known [3], [5].

In order to address these aforementioned limitations, a new scheme was proposed [6], [7] and is called coded cooperation in which cooperation occurs as part of channel coding. Coded cooperation has become very versatile as a scheme in cooperative diversity because it is simple; it can be applied to various multiple access schemes; it removes error propagation and provides coding gain seamlessly [1]. Moreover, incorporating cooperation with channel coding makes it possible for the cooperative diversity scheme to vary the code rate for the inter-user and user-receiver transmissions to allow for adaptability to various channel conditions [8], [1].

The work of Almawgani and Salleh [10] used the outage probability to evaluate the performance of coded cooperation. In the work, numerical and simulation results of outage probability showed that coded cooperation provides significant diversity gains over the non-cooperative scheme. In [8], the end-to-end bit error probability was used as the metric for evaluating the performance of coded cooperation over different cooperation scenarios. Chang Li et al. [16] also made use of outage probability in evaluating the performance of coded diversity, but with multiple relays.

However, in this work an analytical methodology for performance evaluation of coded cooperation is carried out using the family of Reed-Solomon (R-S) codes, though other error correcting codes can be used, with particular attention to the pairwise error probability (PEP). The reason for using the R-S codes is that they are very effective in correcting random burst errors over fading channels. The code symbols for the two time slots or frames may be selected through puncturing using a mask vector that partitions the parity check into two parts, different from the rate-compatible punctured code (RCPC) method given in [6]. Hunter and Nostratinia in [2], [6], [7] worked on coded cooperative diversity scheme using convolutional codes. In their work, the RCPC codes were used to split the message into two codewords [9], whereas the R-S coded diversity scheme involves using the Reed-Solomon codes for this purpose.

The rest of this paper is organized as follows: Section II introduces the background of R-S coded cooperation scheme. Section III discusses the pairwise error probability for the flat fading scenario and derives the PEP. In Section IV, the results are discussed, while the conclusion is given in Section V.

II. R-S Coded Cooperation Scheme

The R-S code scheme being considered in this work was

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first proposed by Almawgani and Salleh [9], [10] and Shakeel [11] in which each user's encoded word is partitioned into two frames and are then transmitted in two time slots. During the first time slot, the first frames (N_1) are directly transmitted to the destination, while later; the N_2 frames are forwarded to the destination via the relaying partners.

The users act independently in the second frame, not being sure whether their data is decoded successfully by their partner or not; and in view of this, four possible scenarios exist for the second frame transmission:

Case 1: Both users decode each other's data successfully; they both send parity bits for each other, and this results in full cooperation

Case 2: None of the users successfully decodes the other's data. In this case, the entire system reverts to the non-cooperative mode

Cases 3 and 4: If one user decodes the partner's data but not the other way round, then both users transmit one of the users' parity bits in the second frame

A detailed explanation of the R-S coded cooperation is given in [9] and Fig. 1 shows a coded cooperation scheme in which either user 1 or user 2 can perform the transmission or the relaying of data.
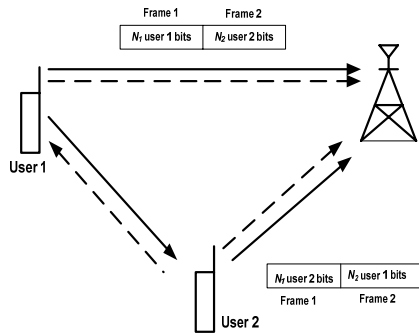

Fig. 1 Coded Cooperation scheme

III. PAIRWISE ERROR PROBABILITY (PEP)

Talking of coded cooperation, the pairwise error probability for a coded system can well be defined as favorably opting for the codeword $e = (e_1, e_2, ...e_N)$ when actually codeword c was sent or transmitted. In this analysis, the tools and techniques from Simon and Alouini [12], Craig [13] and Malkamaki and Leib [14] would be very helpful. In general, assuming a BPSK modulation scheme, coherent detection and maximum-likelihood decoding scheme, the pairwise error probability (PEP) conditioned on the set of fading coefficients $\alpha = (\alpha_1, \alpha_2, ...\alpha_N)$ is written as in [15]

$$P(c \rightarrow e | \alpha) = Q\left(\sqrt{2\sum_{n \in \eta} \frac{\alpha_n^2 E_n}{N_{o,n}}}\right) \quad (1)$$

where $Q(x)$ is the Gaussian Q-function and the expression $\frac{\alpha_n^2 E_n}{N_{o,n}}$ is the instantaneous signal-to-noise ratio (SNR) of bit n received, denoted by Γ_n. η denotes the set of all n for which $c_n \neq e_n$, and the cardinality of η equals the Hamming distance d between the codewords c and e. For all linear codes, of which Reed-Solomon code is one, the PEP only depends on the Hamming distance d and not on the codewords c and e in particular, hence the conditional PEP is represented by $P(d|\alpha)$ or in another form, $P(d|\Gamma)$.

A. Flat Fading Scenario

In flat fading, the fading coefficients for the user uplink channels are always constant over the entire codeword. And in that case, using the notations mentioned above, it means that $\alpha_n^{(i)} = \alpha^{(i)}$ and $\Gamma_n^{(i)} = \Gamma^{(i)}$ are constant for all n, where i denotes the uplink channel for user i. For cooperation involving two users, that is, a two-user cooperation, and assuming Case 1 as mentioned above, when both users successfully decode each other's data, each user's data bits are divided between their user channels. (1) then becomes

$$P\left(d|\Gamma^{(1)}, \Gamma^{(2)}\right) = Q\left(\sqrt{2d_1\Gamma^{(1)} + 2d_2\Gamma^{(2)}}\right) \quad (2)$$

where d_1 is the number of bits in the Hamming weight transmitted through the User 1's channel while d_2 is the number of bits transmitted through the User 2's channel, such that $d_1 + d_2 = d$.

The equation in (2) is for a conditional PEP when Case 1 holds. However, to find the unconditional PEP, (2) must be averaged over the entire fading distributions, and this is given as

$$P(d) \int_0^\infty \int_0^\infty P\left(d|\Gamma^{(1)}, \Gamma^{(2)}\right) p\left(\Gamma^{(1)}\right) p\left(\Gamma^{(2)}\right) d\Gamma^{(1)} d\Gamma^{(2)} \quad (3)$$

where p(.) is the probability density function (pdf). How do we then obtain an exact solution to the equation in (3)? The techniques in [14] and [12] would come in handy here. Craig [12] originally derived an alternative notation for the Gaussian Q-function before being applied to fading channel performance analysis in [14].

The alternative representation of the Gaussian function is given as

$$Q(x) = \frac{1}{\pi} \int_0^{\frac{\pi}{2}} \exp\left(-\frac{x^2}{2\sin^2\theta}\right) d\theta, \ x \geq 0 \quad (4)$$

Using (4) in (2), we obtain

$$P\left(d|\Gamma^{(1)}, \Gamma^{(2)}\right) = \frac{1}{\pi} \int_0^{\pi/2} \exp\left(-\frac{1}{2\sin^2\theta}\right)\left(2d_1\Gamma^{(1)} + 2d_2\Gamma^{(2)}\right) d\theta \quad (5)$$

then putting (5) into (3),

$$P(d) = \int_0^\infty \int_0^\infty \frac{1}{\pi} \int_0^{\pi/2} \exp\left(-\frac{1}{2\sin^2\theta}\right)\left(2d_1\Gamma^{(1)} + 2d_2\Gamma^{(2)}\right)p(\Gamma^{(1)})\,p(\Gamma^{(2)})d\Gamma^{(1)}d\Gamma^{(2)}d\theta \qquad (6)$$

yielding

$$P(d) = \frac{1}{\pi}\int_0^{\frac{\pi}{2}}\left[\int_0^\infty \exp\left(-\frac{d_1\Gamma^{(1)}}{\sin^2\theta}\right)p(\Gamma^{(1)})d\Gamma^{(1)}\right] \times \left[\int_0^\infty \exp\left(-\frac{d_2\Gamma^{(2)}}{\sin^2\theta}\right)p(\Gamma^{(2)})d\Gamma^{(2)}\right]d\theta \qquad (7)$$

The two inner integrals are comparable to

$$\psi_x(s) = \int_0^\infty e^{sx}p(x)dx \qquad (8)$$

which is the moment-generating function of x. Equation (7) is then written as

$$p(d) = \frac{1}{\pi}\int_0^{\pi/2}\psi_{\Gamma_1}\left(-\frac{d_1}{\sin^2\theta}\right)\psi_{\Gamma_2}\left(-\frac{d_2}{\sin^2\theta}\right)d\theta \qquad (9)$$

Using Laplacian techniques, (9) becomes

$$\psi_\Gamma(-s) = \frac{1}{1+s\overline{\Gamma}} \quad s > 0 \qquad (10)$$

and by using (9) and (10), an exact expression for the unconditional PEP which can be evaluated by numerical integration is obtained, which is

$$P(d) = \frac{1}{\pi}\int_0^{\frac{\pi}{2}}\left(1+\frac{d_1\overline{\Gamma}^{(1)}}{\sin^2\theta}\right)^{-1}\left(1+\frac{d_2\overline{\Gamma}^{(2)}}{\sin^2\theta}\right)^{-1}d\theta \qquad (11)$$

where $\overline{\Gamma}$ is the average signal-to-noise ratio (SNR) of the uplink channel for a user. Thus (11) gives the expression for the unconditional PEP for cooperation in flat fading Rayleigh channel.

Then an upper bound can be obtained for (11). This is achieved by assuming that the maximum value of the integrand occurs when $\sin\theta = 1$, which also means $\sin^2\theta = 1$. And this yields

$$P(d) \le \frac{1}{2}\left(\frac{1}{1+d_1\overline{\Gamma}^{(1)}}\right)\left(\frac{1}{1+d_2\overline{\Gamma}^{(2)}}\right) \qquad (12)$$

When user 1 correctly decodes the data from user 2, but user 2 does not decode successfully the data from user 1, both users send the same parity bits for user 2 in the second frame, which will then be combined using the Maximal Ratio Combiner (MRC) at the receiver. For this situation, the conditional PEP for user 2 is obtained thus:

$$P(d|\Gamma^{(2)},\Gamma^{(1)}) = Q\left(\sqrt{2d_2\Gamma^{(2)} + 2d_1(\Gamma^{(2)} + \Gamma^{(1)})}\right) \quad (13)$$

$$= Q\left(\sqrt{2d_2\Gamma^{(2)} + 2d_1\Gamma^{(2)} + 2d_1\Gamma^{(1)}}\right)$$

$$= Q\left(\sqrt{2\Gamma^{(2)}(d_1+d_2) + 2d_1\Gamma^{(1)}}\right)$$

and since $d_1 = d_2$,

$$P(d|\Gamma^{(2)},\Gamma^{(1)}) = Q\left(\sqrt{2d\Gamma^{(2)} + 2d_1\Gamma^{(1)}}\right) \qquad (14)$$

Similarly, the conditional PEP for user 1 is given as

$$P(d|\Gamma^{(1)},\Gamma^{(2)}) = Q\left(\sqrt{2d\Gamma^{(1)} + 2d_2\Gamma^{(2)}}\right) \qquad (15)$$

In like manner, the unconditional PEP (as it was done in (11) and (12) for this scenario is obtained thus: For user 2,

$$P(d) = \frac{1}{\pi}\int_0^{\frac{\pi}{2}}\left(1+\frac{d\overline{\Gamma}^{(2)}}{\sin^2\theta}\right)^{-1}\left(1+\frac{d_1\overline{\Gamma}^{(1)}}{\sin^2\theta}\right)^{-1}d\theta$$

$$\le \frac{1}{2}\left(\frac{1}{1+d\overline{\Gamma}^{(2)}}\right)\left(\frac{1}{1+d_1\overline{\Gamma}^{(1)}}\right) \qquad (16)$$

For user 1,

$$P(d) = \frac{1}{\pi}\int_0^{\frac{\pi}{2}}\left(1+\frac{d\overline{\Gamma}^{(2)}}{\sin^2\theta}\right)^{-1}\left(1+\frac{d_2\overline{\Gamma}^{(2)}}{\sin^2\theta}\right)^{-1}d\theta$$

$$\le \frac{1}{2}\left(\frac{1}{1+d\overline{\Gamma}^{(1)}}\right)\left(\frac{1}{1+d_2\overline{\Gamma}^{(2)}}\right) \qquad (17)$$
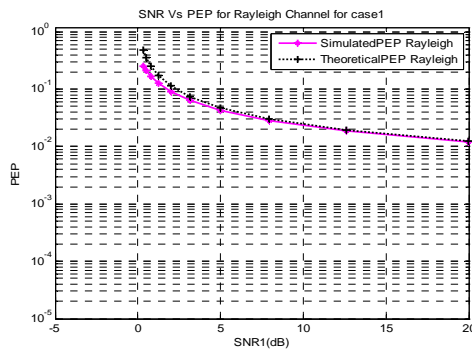
IV. RESULTS AND DISCUSSIONS

Fig. 2 Plots showing the theoretical vs simulated values of PEP

It can be observed from the expression in (16) that the PEP is inversely proportional to the product of the average SNR for the two user channels. It thus implies that provided $d_1 \neq 0$ and $d_2 \neq 0$, there is an achievement of a diversity order of 2 (two) when both user 1 and user 2 cooperate by successfully decoding their individual data, as in the Case 1 scenario.

Fig. 2 shows plots for the simulated against theoretical values of the PEP. The plot shows that there is good agreement between the theoretical and simulated. As the SNR increases, this agreement between the simulated and theoretical becomes better. It also shows that an increase in the SNR of the channel, leads to the lowering of the pairwise error probability (PEP).

V. CONCLUSION

In this paper, the pairwise error probability (PEP) for a coded cooperative system (with the use of R-S codes for error control) in a flat Rayleigh fading environment has been derived. We have also been able to show that the higher the signal-to-noise ratio (SNR) in a particular channel, the lower is the pairwise error probability.

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