# Best Coapproximation in Fuzzy Anti-n-Normed **Spaces**

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Abstract—The main purpose of this paper is to consider the new kind of approximation which is called as t-best coapproximation in fuzzy n-normed spaces. The set of all t-best coapproximation define the t-coproximinal, t-co-Chebyshev and F-best coapproximation and then prove several theorems pertaining to this sets.

Keywords—Fuzzy-n-normed space, best coapproximation, co-proximinal, co-Chebyshev, F-best coapproximation, orthogonality

#### I. Introduction

THE concept of best coapproximation was introduced by Franchetti and Furi [2], in order to study some characteristic properties of real Hilbert spaces, and such problems were considered further by Papini and Singer, [12] and Rao and Saravanan [13]. The concept of n-norm on a linear space has been introduced and developed by Gähler in [3], [4]. Following Misiak [10], Malčeski [9] and Gunawan and Mashadi [5] developed the theory of n-normed space. The concept of fuzzy norm was initiated by Katsaras in [7] and further, Narayanan and Vijayabalaji [11] introduced the concept of fuzzy n-normed linear space. Moreover, Vijayabalaji and Thillaigovindan [17] introduced the notion of convergent sequence and Cauchy sequence in fuzzy n-normed linear space. In [6]Iqbal H. Jebril and Samanta introduced fuzzy anti-norm on a linear space depending on the idea of fuzzy anti-norm was introduced by Bag and Samanta [1] and investigated their important properties. In [8] Kavikumar et. al. introduced the notion of fuzzy anti-n-normed linear space. Further, Surender Reddy [15] introduced the notion of convergent sequence and Cauchy sequence in fuzzy anti-n-normed linear space. The set of all t-best approximations on fuzzy normed linear spaces was initiated and studied by Vaezpour and Karimi [16]. The set of all t-best approximations on fuzzy anti-n-normed linear space was introduced in [14]. In this paper we consider the set of all t-best coapproximation in fuzzy anti-n-normed spaces and then prove several theorems pertaining to this set.

#### II. PRELIMINARIES

Definition 1: [17]. Let  $n \in \mathbb{N}$  (natural numbers) and X be a real linear space of dimension  $d \geq n$ . (Here we allow d to be infinite). A real valued function  $\| \bullet, \bullet, \cdots, \bullet \|$  on  $X \times X \times ... \times X$  (n times)= $X^n$  satisfying the following four

•  $\parallel x_1, x_2, ..., x_n \parallel = 0$  if and only if  $x_1, x_2, ..., x_n$  are linearly dependent.

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- $||x_1, x_2, ..., x_n||$  is invariant under any permutation of
- $||x_1, x_2, ..., cx_n|| = |c| ||x_1, x_2, ..., x_n||$ , for any real
- $\| x_1, x_2, ..., x_{n-1}, y + z \| \le \| x_1, x_2, ..., x_{n-1}, y \| + \|$  $x_1, x_2, ..., x_{n-1}, z \parallel$

is called an *n*-norm on X and the pair  $(X, \| \bullet, ..., \bullet \|)$  is called an *n*-normed linear space.

Definition 2: [17]. Let X be a linear space over a real field  $\mathbb{F}$ . A fuzzy subset N of  $X^n \times [0, \infty)$  is called a fuzzy n-norm on X if and only if:

- $N(x_1, x_2, ..., x_n, t) > 0.$
- $N(x_1,x_2,...,x_n,t)=1 \Leftrightarrow x_1,x_2,...,x_n$  are linearly
- $N(x_1, x_2, ..., x_n, t)$  is invariant under any permutation of  $x_1, x_2, ..., x_n$ .
- $N(x_1, x_2, ..., cx_n, t) = N(x_1, x_2, ..., x_n, \frac{t}{|c|})$  if  $c \neq$
- $N(x_1,x_2,...,x_n+x_n',s+t) \ge N(x_1,x_2,...,x_n,t) * N(x_1,x_2,...,x_n',t)$  for all  $s,t\in\mathbb{R}$
- $N(x_1, x_2, ..., x_n, \cdot)$  is left continuous and non-decreasing function of  $\mathbb{R}$  such that  $\lim_{t\to\infty} N(x_1, x_2, ..., x_n, t) = 1$ .

Then (X, N) is called a fuzzy n-normed linear space.

Definition 3: [8] Let X be a linear space over a real field  $\mathbb{F}$ . A fuzzy subset N of  $X^n \times [0, \infty)$  is called a fuzzy anti n-norm on X if and only if:

- for all  $t \in \mathbb{R}$  with  $t \leq 0$ ,  $N(x_1, x_2, ..., x_n, t) = 1$ .
- for all  $t \in \mathbb{R}$  with t > 0,  $N(x_1, x_2, ..., x_n, t) = 0 \Leftrightarrow$  $x_1, x_2, ..., x_n$  are linearly dependent.
- $N(x_1, x_2, ..., x_n, t)$  is invariant under any permutation of  $x_1, x_2, ..., x_n$ .
- for all  $t \in \mathbb{R}$  with t > 0,  $N(x_1, x_2, ..., cx_n, t) =$
- $\begin{array}{ll} N(x_1,x_2,...,x_n,\frac{t}{|c|}) \text{ if } c \neq 0, c \in \mathbb{F}(\text{field}) \\ \bullet \text{ for all } s,t \in \mathbb{R}, \ N(x_1,x_2,...,x_n+x_n',s+t) \leq \max\{N(x_1,x_2,...,x_n,s),N^*(x_1,x_2,...,x_n',t)\} \end{array}$
- $N(x_1, x_2, ..., x_n, \cdot)$  is right continuous and non-increasing function of  $\mathbb{R}$  such that

$$\lim_{t \to \infty} N(x_1, x_2, ..., x_n) = 0$$

Then (X, N) is called a fuzzy anti n-normed linear space. To strengthen the above definition, we present the following example.

Example 1: [8] Let  $(X, \| \bullet, \bullet, \dots, \bullet \|)$  be a *n*-normed linear space

Define,

$$\begin{split} N(x_1,x_2,...,x_n,t) &= \\ \left\{ \begin{array}{ll} 1 - \frac{t}{t+\parallel\ x_1,x_2,...,x_n\parallel} & \text{when } t(>0) \in \mathbb{R}, \forall x \in X \\ 1 & \text{when } t(\leq 0) \in \mathbb{R}, \forall x \in X \end{array} \right. \end{split}$$

Then (X, N) is a fuzzy anti n-normed linear space.

Definition 4: [15]A sequence  $\{x_k\}$  in a fuzzy anti-n-normed linear space (X, N) is said to be convergent to  $x \in X$  ig given t > 0, 0 < r < 1, there eixsts an integer  $n_0 \in \mathbb{N}$  such that

$$N(x_1, x_2, \dots, x_{n_1}, x_k - x, t) < r, \forall k \ge n_0.$$

Theorem 1: [15]In a fuzzy anti-n-normed linear space (X, N), a sequence  $\{x_k\}$  converges to  $x \in X$  if and only

$$\lim_{k \to \infty} N(x_1, x_2, \cdots, x_{n_1}, x_k - x, t) = 0, \forall t > 0.$$

Definition 5: [15] Let (X, N) be a fuzzy anti-n-normed linear space. Let  $\{x_k\}$  be a sequence in X then  $\{x_k\}$  is said to be a Cauchy sequence if

$$\lim_{k \to \infty} N(x_1, x_2, \dots, x_{n_1}, x_{k+p} - x_k, t) = 0, \forall t > 0$$

and  $p = 1, 2, 3, \cdots$ . A fuzzy anti-n-normed linear space (X, N) is said to be complete if every Cauchy sequence in X is convergent. A complete fuzzy anti-n-normed space (X, N)is called a fuzzy anti-n-Banach space. The open ball B(x, r, t)and the closed ball B[x, r, t] with the center  $x \in X$  and radius 0 < r < 1, t > 0 are defined as follows:

$$\begin{split} B(x,r,t) &= \{ y \in X : N(x_1,x_2,\cdots,x_{n_1},x-y,t) < r \}, \\ B[x,r,t] &= \{ y \in X : N(x_1,x_2,\cdots,x_{n_1},x-y,t) \leq r \}. \end{split}$$

A subset A of X is said to be open if there exists  $r \in (0,1)$ such that  $B(x,r,t) \subset A$  for all  $x \in A$  and t > 0. A subset A of X is said to be closed if for any sequence  $\{x_k\}$  in A converges to  $x \in A$ . i.e.,  $\lim_{k\to\infty} N(x_1, x_2, \cdots, x_{n_1}, x_k -$ (x,t) = 0, for all t > 0 implies that  $x \in A$ .

Corollary 1: [15] Let (X, N) be a fuzzy anti-n-normed linear space. Then N is a continuous function on

$$\underbrace{X\times X\times \ldots \times X}_{n}\times \mathbb{R}.$$

### III. T-BEST COAPPROXIMATION

Definition 6: Let A be a nonempty subset of fuzzy anti-n-normed space (X, N) and t > 0. For  $x \in X$ , an element  $y_0 \in A$  is said to be a t-best coapproximation of x from A if  $N(x_1, x_2, \dots, x_{n-1}, y_0 - y, t) \le N(x_1, x_2, \dots, x_{n-1}, x - y_0)$ y, t), for all  $y \in A$ . The set of all elements of t-best coapproximation of x from A is denoted by  $R_A^t(x)$ ; i.e.,  $R_A^t(x) = \{y_0 \in A : N(x_1, x_2, \dots, x_{n-1}, y_0 - y, t) \le$  $N(x_1, x_2, \cdots, x_{n-1}, x - y, t), \forall y \in A\}.$ 

For t > 0 putting

$$\check{A}_x^t = \{x \in X; N(x_1, x_2, \cdots, x_{n-1}, y, t) 
\leq N(x_1, x_2, \cdots, x_{n-1}, y - x, t) \forall y \in A\} 
= (R_A^t)^{-1}(\{0\}).$$

It is clear  $y_0 \in R_A^t(x)$  if and only if  $x - y_0 \in \mathring{A}_x^t$ .

Definition 7: Let A be a nonempty subset of a fuzzy anti-n-normed space (X, N). If for t > 0 and each  $x \in X$ has at least (respectively exactly) one t-best coapproximation in A, then A is called a t-best coproximinal (respectively t-co-Chebyshev) set. Also A is called t-quasi-co-Chebyshev set if  $R_A^t(x)$  is a compact set.

Theorem 2: Let (X, N) be a fuzzy anti-n-normed space and A be a subspace of X and t > 0. Then for each  $x \in X$ 

- (a) A is a t-coproximinal if and only if  $X = A + \mathring{A}_x^t$ .
- (b) A is a t-co-Chebyshev subspace if and only if  $X = A \oplus$  $A_x^t$ .

*Proof:* (a)( $\Rightarrow$ ) Assume that A is t-coproximinal,  $x \in X$ and  $y_0 \in R_A^t(x)$ . Then,  $x - y_0 \in A_x^t$ . Now,  $x = y_0 + (x - y_0) \in$  $A + \mathring{A}_x^t$ . Hence  $X = A + \mathring{A}_x^t$ .

 $(\Leftarrow)$  Let  $x \in X = A + \check{A}_x^t$ .  $x = y_0 + \overline{y}, y_0 \in A, \overline{y} \in \check{A}_x^t$  and so  $0 \in R_A^t(\bar{y}) = R_A^t(x - y_0)$ . Since,  $N(x_1, x_2, \dots, x_{n-1}, 0 - y_0)$  $(x-y_0),t) \leq \tilde{N}(x_1,x_2,\cdots,x_{n-1},y-(x-y_0),t),$  so  $N(x_1, x_2, \cdots, x_{n-1}y_0 - x, t) \le N(x_1, x_2, \cdots, x_{n-1}, (y + x_n))$  $(y_0) - x, t)$  where  $y + y_0 \in A$ ; hence  $y_0 \in R_A^t(x)$ . Therefore A is t-coproximinal.

(b)( $\Rightarrow$ ) Suppose that A is t-co-Chebyshev subspace,  $x \in X$ , and  $x = y_1 + \overline{y}_1 = y_2 + \overline{y}_2$ , where  $y_1, y_2 \in A$  and  $\overline{y}_1, \overline{y}_2 \in A_x^t$ . We show that  $y_1 = y_2$  and  $\overline{y}_1 = \overline{y}_2$ . Since  $x=y_1+\overline{y}_1=y_2+\overline{y}_2,$  then  $x-y_1=\overline{y}_1,x-y_2=\overline{y}_2,$  this implies that  $y_1, y_2 \in R_A^t(x)$ . Therefore  $y_1 = y_2$ , it follows that  $\overline{y}_1 = \overline{y}_2$ . Thus  $X = A \oplus \mathring{A}_x^t$ .

 $(\Leftarrow)$  Let  $X = A \oplus \check{A}_x^t$ , and suppose for  $x \in X$ , there exist  $y_1, y_2 \in R_A^t(x)$ . Then  $x - y_1, x - y_2 \in A_x^t$  and therefore,  $x = y_1 + \overline{y}_1 = y_2 + \overline{y}_2$ , where  $\overline{y}_1 = x - y_1, \overline{y}_2 = x - y_2$ . It follows that  $y_1 = y_2$  and  $\overline{y}_1 = \overline{y}_2$ .

Theorem 3: Let A be a nonempty subset of a fuzzy anti-n-normed space (X, N). The for t > 0 and for each

- $\begin{array}{ll} \text{(a)} & R^t_{A+y}(x+y) = R^t_A(x) + y, \text{ for every } x,y \in X. \\ \text{(b)} & R^{|\alpha|t}_{\alpha A}(\alpha x) = \alpha R^t_A \text{ for every } x \in X \text{ and } \alpha \in \mathbb{R} \backslash \{0\}. \end{array}$
- (c) A is t-coproximinal (respectively t-co-Chebyshev) if and only if A + y is t-coproximinal (respectively *t*-co-Chebyshev), for any  $y \in X$ .
- (d) A is t-coproximinal (respectively t-co-Chebyshev) if and only if  $\alpha A$  is  $|\alpha|$  t-coproximinal (respectively  $|\alpha|$ t-co-Chebyshev), for any  $y \in X$ , for any given  $\alpha \in$

*Proof*: (i) For any  $x, y \in X$ , t > 0,  $y_0 \in R_{A+y}^t(x+y)$ if and only if

 $N(x_1, x_2, \cdots, x_{n-1}, y_0 - (a + y), t)$  $N(x_1, x_2, \dots, x_{n-1}, x+y-(a+y), t)$  for all  $(a+y) \in A+y$ if and only if,

only if,  $(y_0 - y) \in R_A^t(x)$ , i.e.,  $y_0 \in R_A^t(x) + y$ .

(ii) For any  $x \in X$ ,  $\alpha \in \mathbb{R} \setminus \{0\}$ , and t > 0,  $y_0 \in R_{\alpha A}^{|\alpha|t}(\alpha x)$ if and only if,

 $N(x_1, x_2, \cdots, x_{n-1}, (y_0 - \alpha a) = \alpha$  $N(x_1, x_2, \dots, x_{n-1}, \alpha x - \alpha a), \alpha t)$  for all  $a \in A$ if and only if  $N(x_1,x_2,\cdots,x_{n-1},(\frac{1}{\alpha}y_0-a,\mid\alpha\mid t)\leq N(x_1,x_2,\cdots,x_{n-1},x-a),t)$  for all  $a\in A$  if and only if,  $\frac{1}{\alpha}y_0\in R_A^t(x)$  if and only if,  $y_0\in\alpha R_A^t(x)$ . Therefore  $R_{\alpha A}^{|\alpha|t}(\alpha x)=\alpha R_A^t$ 

- (iii) Is an immediate consequence of (i)
- (iv) Is an immediate consequence of (ii).

Corollary 2: Let M be a nonempty subspace of a fuzzy anti-n-normed space X. Then for t > 0 and each  $x \in X$ .

- $\begin{array}{ll} \text{(a)} & R_M^t(x+y) = R_M^t(x) + y, \text{ for every } x,y \in X. \\ \text{(b)} & R_M^{|\alpha|t}(\alpha x) = \alpha R_M^t \text{ for every } x \in X \text{ and } \alpha \in \mathbb{R} \backslash \{0\}. \end{array}$

Proof: The proof is an immediate consequence of theorem 3 and this fact that M+y=M and  $\alpha M=M$  for all  $y\in M$ and  $\alpha \in \mathbb{R} \setminus \{0\}$ .

Definition 8: For  $x \in X$ ,  $a \in A$ , 0 < r < 1, and t > 0, define

$$e_a^t(x) = N(x_1, x_2, \cdots, x_{n-1}, x - a, t)$$

Theorem 4: Let (X, N) be a fuzzy anti n-normed space, A be a subset of X,  $x \in X \setminus \overline{A}$  and t > 0. Then we have

$$R_A^t(x) = \left[\bigcap_{a \in A} B[a, e_a^t(x), t]\right] \cap A.$$

*Proof:* By definition of  $R_A^t(x)$  for each  $a \in A$  we have

$$R_A^t(x) \subseteq [B[a, e_a^t(x), t]] \cap A$$

Therefore

$$R_A^t(x) \subseteq [\bigcap_{a \in A} B[a, e_a^t(x), t]] \cap A.$$

Conversely, let  $y\in [\bigcap_{a\in A}B[a,e^t_a(x),t]]\cap A$ , then we have  $y\in A$ , and for each  $a\in A$ ,  $N(x_1,x_2,\cdots,x_{n-1},a-y,t)\leq e^t_a=N(x_1,x_2,\cdots,x_{n-1},x-a,t)$ , which implies that  $y\in R^t_A(x)$ . So  $[\bigcap_{a\in A}B[a,e^t_a(x),t]]\cap A\subseteq R^t_A(x)$ , which completes the proof.

Corollary 3: Let (X,N) be a fuzzy anti-n-normed space, A be a subset of  $X, x \in X \setminus \overline{A}$  and t > 0. Then

- (a) The set  $R_A^t(x)$  is t-bounded.
- (b) If A is t-closed, then  $R_A^t(x)$  is t-closed.

Theorem 5: Let (X, N) be a fuzzy anti-n-normed space. For each  $x \in X$  and t > 0, if A is a convex subset of X, then  $R_A^t(x)$  is a convex subset of A (for  $R_A^t(x) \neq \emptyset$ ).

*Proof:* Let  $z_1, z_2 \in R_A^t$ , then for t>0 and each  $x\in X$ ,  $N(x_1, x_2, \cdots, x_{n-1}, y-z_1, t)\leq N(x_1, x_2, \cdots, x_{n-1}, x-y, t)$  and  $N(x_1, x_2, \cdots, x_{n-1}, y-z_2, t)\leq N(x_1, x_2, \cdots, x_{n-1}, x-y, t)$  for all  $y\in A$ . Now for each  $\lambda\in(0,1)$  we have

$$\begin{split} N(x_1, x_2, \cdots, x_{n-1}, y - (\lambda z_1 + (1 - \lambda) z_2, t) \\ &= N(x_1, x_2, \cdots, x_{n-1}, \lambda y - \lambda z_1 + y - \lambda y - z_2 + \lambda z_2, t) \\ &= N(x_1, x_2, \cdots, x_{n-1}, \lambda (y - z_1) + (1 - \lambda) (y - z_2), \\ &\qquad \qquad \lambda t + (1 - \lambda) t) \\ &\leq \max\{N(x - 1, x_2, \cdots, x_{n-1}, y - z_1, \frac{\lambda t}{\lambda}), \\ &\qquad \qquad N(x_1, x_2, \cdots, x_{n-1}, y - z_2, \frac{(1 - \lambda) t}{(1 - \lambda)})\} \end{split}$$

$$\leq \max\{N(x-1,x_2,\cdots,x_{n-1},x-y,\frac{\lambda t}{\lambda}),$$

$$N(x_1, x_2, \cdots, x_{n-1}, x-y, \frac{(1-\lambda)t}{(1-\lambda)})\}$$

$$\leq N(x_1, x_2, \cdots, x_{n-1}, x-y, t)$$

So  $\lambda z_1 + (1 - \lambda)z_2 \in R_A^t(x)$  and  $R_A^t(x)$  is convex.

Theorem 6: For t>0 and each  $x\in X$ . let A be a t-coproximinal subspace of a fuzzy anti-n-normed space (X,N). Then

- (a) If  $\check{A}^t_x$  is a t-compact set then A is t-quasi-co-Chebyshev.
- (b) If  $\check{A}^t_x$  is a *t*-closed set then  $R^t_A(x)$  is *t*-closed for every  $x \in X$

*Proof:* (i) Suppose  $x \in X$  and  $\{y_n\}$  is a sequence in  $R_A^t(x)$ . Since  $x-y_n \in \check{A}_x^t$  and  $\check{A}_x^t$  is a t-compact set, there exists a subsequence  $\{x-y_{n_k}\}$  that t-convergence to  $x-y_0 \in \check{A}_x^t$ . Consequently,  $\{y_n\}$  has a subsequence  $y_{n_k} \stackrel{t}{\to} y_0 \in R_A^t(x)$  and hence A is t-quasi-co-Chebyshev.

(ii) The proof is similar to (i).

Definition 9: A subset A of a fuzzy anti-n-normed space (X, N) is called to be t-boundedly compact if every t-bounded

sequence in A has a subsequence t-converging to an element of X

Theorem 7: Suppose for some t > 0 and each  $x \in X$ , A is a t-boundedly compact and t-closed subset of a fuzzy anti-n-normed space (X, N), then, A is t-quasi-co-Chebyshev.

*Proof:* Let  $\{y_n\}$  be any sequence in  $R_A^t(x)$ . Then  $N(x_1,x_2,\cdots,x_{n-1},y_n-y,t)\leq N(x_1,x_2,\cdots,x_{n-1},x-y,t)$  for every  $y\in A$ . Since  $R_A^t(x)$  is t-bounded,  $\{y_n\}$  is a t-bounded sequence in A, and so  $\{y_n\}$  has a t-convergent subsequence  $\{y_{n_k}\}$ , let  $y_{n_k}\stackrel{t}{\to} y_0\in A$ , as A is t-closed. Consider

$$N(x-1, x_2, \dots, x_{n-1}, y_0 - y, t)$$

$$= \lim_{k} N(x_1, x_2, \dots, x_{n-1}, y_{n_k} - y, t)$$

$$\leq N(x_1, x_2, \dots, x_{n-1}, x - y, t)$$

for every  $y \in A$ . So  $y_0 \in R_A^t(x)$ , which implies that A is t-quasi-co-Chebyshev.

Definition 10: Let (X,N) be a fuzzy anti-n-normed space and A be a subset of X. For t>0 and an element  $x\in X$  is said to be t-orthogonal to an element  $y\in X$ , and we denote it by  $x\perp_x^t y$ , if  $N(x_1,x_2,\cdots,x_{n-1},x+\lambda y,t)\geq N(x_1,x_2,\cdots,x_{n-1},x,t)$  for all scalar  $\lambda\in\mathbb{R}$ ,  $\lambda\neq 0$ . We say  $A\perp_x^t y$  if  $x\perp_x^t y$  for every  $x\in A$ .

Theorem 8: For t>0 and each  $x\in X$  and  $y_0\in A$ , let (X,N) be a fuzzy anti-n-normed space and A be a subspace of X. If  $A\perp_x^t x-y_0$  then  $y_0\in R_A^t(x)$ .

# IV. F-BEST COAPPROXIMATION

Definition 11: Let A be a nonempty subset of a fuzzy anti-n-normed space (X, N). An element  $y_0 \in A$  is said to be an F-best coapproximation of  $x \in X$  from A if it is a t-best coapproximation of x from A, for every t > 0, i.e.,

$$y_0 \in \bigcap_{t \in (0,\infty)} R_A^t(x).$$

The set of all elements of F-best coapproximation of X from A is denoted by  $FR_A^t(x)$ , i.e.,

$$FR_A^t(x) = \bigcap_{t \in (0,\infty)} R_A^t(x).$$

If each  $x \in X$  has at least (respectively exactly) one F-best coapproximation in A, then A is called F-coproximinal (respectively F-co-Chebyshev) set.

*Example 2:* Let  $X=\mathbb{R}^3$ . Define  $N:X\times X\times X\times [0,\infty)\to [0,1]$  by

$$\begin{split} N(x_1,x_2,x_3,t) &= \frac{\|x_1,x_2,x_3\|}{t}, \quad if \quad t > 0, t \in \mathbb{R}, x_1,x_2,x_3 \in X \\ &= 1, \quad if \quad t \leq 0, t \in \mathbb{R}, x_1,x_2,x_3 \in X. \end{split}$$

where  $\|x_1,x_2,x_3\|=\min_{1\leq i\leq 3}\sum_{j=1}^3 |x_{ij}|$ . Then (X,N) is a fuzzy anti-3-normed linear space. Let  $A=\{(a,b,c)\in\mathbb{R}^3:a^2+b^2\leq 1,0\leq c\leq a^2+b^2\}$  and  $x_1=(1,0,0),\ x_2=(0,1,0),\ x_3=(0,0,4)$  are in X. Let  $a_0=(0,-1,1)$  and  $a_1=(0,1,1)$  are in A. Then  $(0,-1,1),(0,1,1)\in FR_A^t(0,0,4)$ . So A is not a F-co-Chebyshev set.

Theorem 9: Let  $\{\|.,.,\cdots,.\|_{\alpha}^* : \alpha \in (0,]\}$  be a descending family of  $\alpha$ -n-norm on X corresponding to the fuzzy anti-n-norm on X. Then  $y_0 \in A$  is a best coapproximation to  $x \in X$  in the descending family of  $\alpha$ -n-norm on X corresponding to the fuzzy anti-n-norm on X if and only if  $y_0$  is a F-best coapproximation to x in the fuzzy anti-n-normed space (X, N).

*Proof:* For each  $x\in X$ ,  $y_0$  is a best coapproximation to  $x\in X$  in the descending family of  $\alpha$ -n-norm on X corresponding to the fuzzy anti-n-norm on X. if and only if  $\|x_1,x_2,\cdots,y-y_0\|_{\alpha}^*\leq \|x_1,x_2,\cdots,x-y\|_{\alpha}^*$ , for every  $y\in A$ , if and only if  $\frac{t}{t+\|x_1,x_2,\cdots,y-y_0\|_{\alpha}^*}\geq \frac{t}{t+\|x_1,x_2,\cdots,x-y\|_{\alpha}^*}$  for every  $y\in A$  and  $t\in (0,\infty)$ , if and only if  $N(x_1,x_2,\cdots,x_{n-1},y-y_0,t)\leq N(x_1,x_2,\cdots,x_{n-1},x-y,t)$  for every  $y\in A$  and  $t\in (0,\infty)$ , if and only if  $y_0\in FR_A^t(x)$ .

Definition 12: Let (X,N) be a fuzzy anti-n-normed space and A be a subset of X. For each element  $x \in X$  is said to be F-orthogonal to an element  $y \in X$  and we denote it by  $x \perp_x^F y$ , if for every t > 0,  $x \perp_x^t y$ . We say  $A \perp_x^F y$  if  $x \perp_x^F y$  for every  $x \in A$ .

Theorem 10: Let  $\{\|.,.,\cdots,.\|_{\alpha}^* : \alpha \in (0,]\}$  be a descending family of  $\alpha$ -n-norm on X corresponding to the fuzzy anti-n-norm on X. Then  $x \in X$  is Brikhoff orthogonal to  $y \in X$  in the descending family of  $\alpha$ -n-norm on X corresponding to the fuzzy anti-n-norm on X if and only if x is a F-orthogonal to y in the fuzzy anti-n-normed space (X,N).

Proof: For each  $x\in X,\ x$  is a Brikhoff orthogonal to  $y\in X$  in the descending family of  $\alpha$ -n-norm on X corresponding to the fuzzy anti-n-norm on X. if and only if  $\|x_1,x_2,\cdots,x_{n-1},x\|_{\alpha}^*\leq \|x_1,x_2,\cdots,x_{n-1},x+\lambda y\|_{\alpha}^*$ , for every scalar  $\lambda\in\mathbb{R}\backslash\{0\}$ , if and only if  $\frac{t}{t+\|x_1,x_2,\cdots,x_{n-1},x\|_{\alpha}^*}\geq \frac{t}{t+\|x_1,x_2,\cdots,x_{n-1},x+\lambda y\|_{\alpha}^*}$  for every scalar  $\lambda\in\mathbb{R}\backslash\{0\}$  and t>0, if and only if  $N(x_1,x_2,\cdots,x_{n-1},x,t)\leq N(x_1,x_2,\cdots,x_{n-1},x+\lambda y,t)$  for every scalar  $\lambda\in\mathbb{R}\backslash\{0\}$  and t>0, if and only if  $x\perp_F^x y$ .

# V. CONCLUSION

In this paper we introduced the concept of t-best coapproximation in and F-best coapproximation in fuzzy anti-n-normed spaces and also introduced t-coproximinal and t-co-Chebyshev in fuzzy anti-n-normed spaces. Then prove several theorems pertaining to this sets illustrate with example.

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