Prediction of Computer and Video Game Playing Population: An Age Structured Model

T. K. Sriram, Joydip Dhar

Abstract—Models based on stage structure have found varied applications in population models. This paper proposes a stage structured model to study the trends in the computer and video game playing population of US. The game paying population is divided into three compartments based on their age group. After simulating the mathematical model, a forecast of the number of game players in each stage as well as an approximation of the average age of game players in future has been made.

Keywords—Age structure, Forecasting, Mathematical modeling, Stage structure.

I. INTRODUCTION

STAGE structure models are based on the premises that during their lifetime, individuals pass through a variety of stages. In a particular stage, the characteristics of individuals in the same stage are similar to each other and different from those in other stages. Depending on the factors such as age, size and awareness, many models based on stage structure have been proposed. S. Liu and L. Chen [1] argued that although stage structure models have received a lot of attention but models such as non autonomous stage structured models and those based on impulsive differential equations have not been extensively studied.

Models based on stage-structure can help in understanding and simulating natural processes. Rogers [2] argued that adoption of an innovation is a multi-step process and consists of five stages namely; awareness, interest, evaluation, trial and adoption. These five stages help a non-adopter in deciding whether to adopt an innovation or not, are facilitated through a number of communication channels. Researchers have proposed many stage structure models having immature and mature stages of particular population [3]-[7]. In particular, W. G. Aiello et al. [5] has proposed another stage-structured model of population growth in which the time to maturity of the population was state dependent.

The age structure of the population may be used to determine the future growth and composition of the population. Since the traits and habits of individuals differ across age groups, it is prudent to incorporate age-structure in population models. Norhayati and G. C. Wake [6] developed a non linear model by taking the effect of population density on age distribution of the population into consideration. Abia et

al. [7] discussed some numerical approaches to solve problems on age-structured models. Motivated from few interesting compartmental models of population dynamics and epidemiology [8]-[10], in this paper, an age-structured compartmental model is developed by dividing the video and computer game players into three stages. Data required for estimating the parameters was collected from reports of Entertainment Software Association and United States Census Bureau [11], [12].

II. MATHEMATICAL MODEL

The entire video and computer game playing population of US was split up into three stages based on their age group. The first stage (N1) consisted of gamers between (2-17) years of age. The second stage (N2) consisted of gamers between (18-49) years of age and the third stage (N3) had gamers who are more than 49 years of age. We assume the existence of a frustration group which comprises of ex-gamers who may have got bored with the existing games and stopped playing. The rate at which gamers join this group from the three stages is taken as the frustration rate and the rate at which people in the frustration group rejoin the game playing population is taken as the rejoining rate. In this model, the frustration and rejoining rates is taken to be independent of the size of the game playing population, i.e., both the rates were considered to be fixed. Additions of new gamers to the three stages as well as the death rate of gamers have been taken into account. Shifting of gamers from one stage to the next has also been included. Based on the above conditions, the following model is proposed:

$$\frac{dN1}{dt} = a - \alpha N1 - d1 - f1 + r1, \qquad (1)$$

$$\frac{dN2}{dt} = b + \alpha N1 - \beta N2 - d2 - f2 + r2, \qquad (2)$$

$$\frac{dN3}{dt} = c + \beta N2 - d3 - f3 + r3, \qquad (3)$$

$$\frac{dN4}{dt} = f1 + f2 + f3 - r1 - r2 - r3, \qquad (4)$$

where, N4 is population density of frustrated group, a, b and c are new recruitment rates in stage I, II, III respectively, again d1, d2 and d3 are death rates of gamers in Stage I, II, III respectively. Let as assume that α and β are shifting rates of gamers from Stage I to Stage II and Stage II to Stage III respectively. Finally, assume that f1, f2, f3 are frustration rates of gamers from Stages I, II, III and r1, r2, r3 are rejoining rate of non-gamers from frustration group into Stages I, II, III

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respectively.

Data regarding the video and computer game players was compiled from the "Essential Facts about the Computer and Video Game Industry" reports published by ESA during the years 2006-2010 and using US Census Bureau population data [11], [12]. Here, only the relevant data is presented. Table I provides the game playing population data for the years 2006-2010.

Fig. 1 shows the graphical representation of the age-structured population densities in each stage.

TABLE I
GAME PLAYING POPULATION

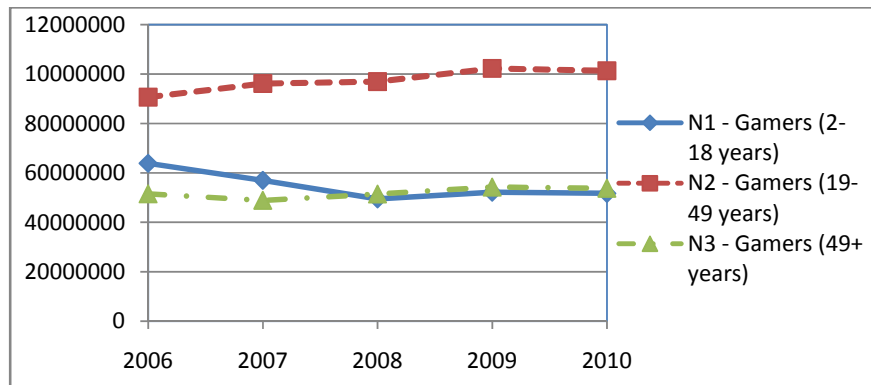|	2006	2007	2008	2009	2010
Total Game Playing Population	206029316	202058529	197843649	208764454	206859510
N1 - Gamers (2-17 years)	63869088	56980505	49460912	52191113	51714877
N2 - Gamers (18-49 years)	90652899	96179860	96943388	102294582	101361160
N3 - Gamers (49+ years)	51507329	48898164	51439348	54278758	53783472

Fig. 1 Age-structured Game Playing Population

III. PARAMETER ESTIMATION OF THE MODEL

Since there were no abrupt changes in the game playing population during the years 2006-2010, the shifting rate of gamers is assumed to be equal to the reciprocal of the number of years spent in a particular stage. Hence, $\alpha = 1/16$ and $\beta = 1/32$. Also, as the frustration and rejoining rates were not separately available for the three stages, same frustration and rejoining rates (i.e., $f = f1 = f2 = f3$ and $r = r1 = r2 = r3$) were assumed. The model can now be represented as:

$$\frac{dN1}{dt} = a - \frac{N1}{16} - d1 - f + r, \quad (5)$$

$$\frac{dN2}{dt} = b + \frac{N1}{16} - \frac{N2}{32} - d2 - f + r, \quad (6)$$

$$\frac{dN3}{dt} = c + \frac{N2}{32} - d3 - f + r, \quad (7)$$

$$\frac{dN4}{dt} = 3f - 3r. \quad (8)$$

Three levels of frustration group were considered so as to understand how changes in the frustration group may affect the population in the three stages. Frustration group was assumed to consist of a certain percentage (8.33%, 10% and 15%) of the total game playing population. To obtain the L.H.S. of the above system of equations, curve fitting was done using the Least-Square method. The equations of the curves are:

$$N1 = -1860655.8t^2 - 9837319.2t + 63869088 \quad (9)$$

$$N2 = -511496.7t^2 + 5103347.8t + 90652899 \quad (10)$$

$$N3 = 1604749.9t^2 - 3728702.6t + 51507329 \quad (11)$$

$$N4(8.3\%) = 246060.6t^2 - 704940.7t + 17162242 \quad (12)$$

$$N4(10\%) = 295391.1t^2 - 846267.9t + 20602931 \quad (13)$$

$$N4(15\%) = 443086.4t^2 - 1269401.1t + 3090439 \quad (14)$$

The values of dN/dt can be obtained by differentiating the above the equations with respect to t and putting t = 0 - 4 for the years 2006 - 2010 respectively.

In order to simplify the model, we substitute a - d1 = K1, b - d2 = K2, c – d3 = K3 and f - r = K4 in the model equations (5)-(8). K1, K2 and K3 denote the year wise intrinsic growth rates in stages N1, N2 and N3 respectively and K4 is the intrinsic frustration rate. The model finally reduces to:

$$\frac{dN1}{dt} = K1 - \frac{N1}{16} - K4 \quad (15)$$

$$\frac{dN2}{dt} = K2 + \frac{N1}{16} - \frac{N2}{32} - K4 \quad (16)$$

$$\frac{dN3}{dt} = K3 + \frac{N2}{32} - K4 \quad (17)$$

$$\frac{dN4}{dt} = 3K4 \qquad (18)$$

Ki's values obtained by solving the above system for different frustration levels are provided in Table II.

TABLE II
YEAR-WISE K VALUES

	Year	2006	2007	2008	2009	2010
	K4	-234980.22	-70939.82	93100.58	257140.99	421181.39
Frustration Group -8.3%	K3	-6796585.92	-3595763.25	-246083.29	2960232.10	6362941.74
	K2	3709452.67	3453753.64	3088635.46	2226269.71	1367912.03
	K1	-6080481.42	-2625665.86	789711.58	4845701.15	8701288.40
	K4	-282089	-85161.9	111765.5	308692.9	505620.3
Frustration Group -10%	K3	-6843695	-3609985.3	-227418	3011784	6447380.65
	K2	3662344	3439531.56	3107300	2277821.6	1452350.94
	K1	-6127591	-2639887.9	808376.5	4897253.1	8785727.31
	K4	-423134	-127742.77	167648.2	463039.1	758430.033
Frustration Group -15%	K3	-6984739	-3652566.2	-171536	3166130.2	6700190.38
	K2	3521299	3396950.7	3163183	2432167.8	1705160.67
	K1	-6268635	-2682468.8	864259.2	5051599.3	9038537.05

It is observed that larger the level of the frustration group, higher is the intrinsic frustration rate (K4). On taking a larger frustration group, higher intrinsic growth rate values (K1, K2 and K3) are obtained. Initial negative values of K4 indicate that during 2006-2007 more people were rejoining the game playing population. However, from 2008 onwards, the number of people joining the frustration group outnumbered the people rejoining the game playing population. It is evident from Table II that the intrinsic growth rates in Stages 1 and 3 were continuously increasing and changed from negative to positive over the years. Also, the intrinsic growth in Stage 2 showed a downward trend but remained positive.

Assuming the frustration group comprises of 8.33% of the game playing population for a given year, the model was simulated for the period (2006-2010) by substituting the Ki values in the model equations. Since the simulated trend lines as shown in Fig. 2 are very much similar to the actual trend lines in Fig. 1, this model is able to represent the trends in the game playing population correctly.

The model was further simulated till the year 2016 (Fig. 3). The results show that there would be a growth in the number of gamers in all the three stages with N3 stage witnessing the most significant increment in game players. Its population would become nearly equal to that of N2 by 2016.
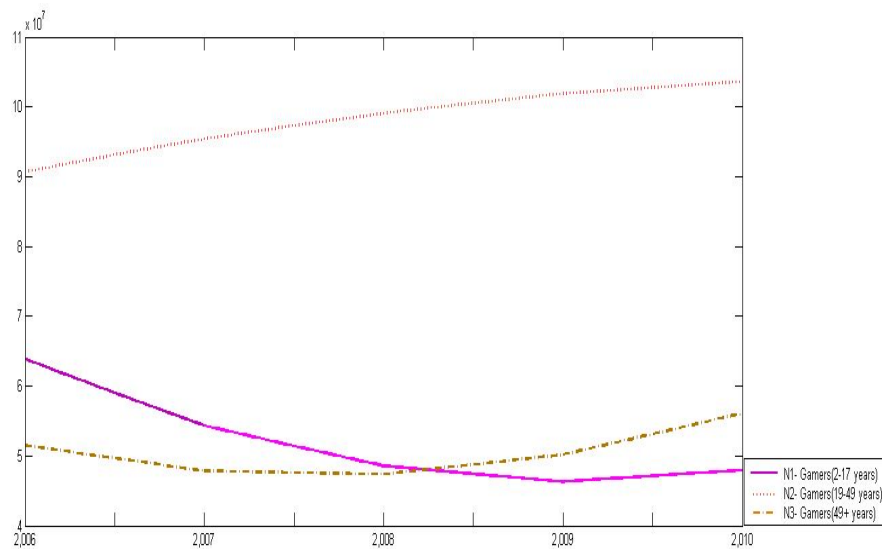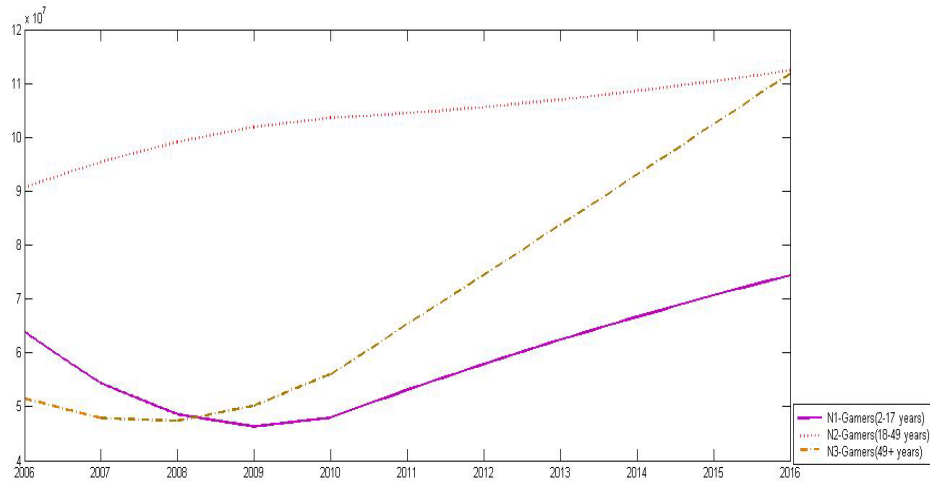

Fig. 2 Model Simulation 2006-2010

Fig. 3 Model Forecast 2006-2016

The forecasted population densities of the three stages are shown in Table III.

TABLE III
PREDICTION OF GAME PLAYING POPULATION

Year	N1(10^8)	N2(10^8)	N3(10^8)
2011	0.5306	1.0445	0.6528
2012	0.5787	1.0559	0.7450
2013	0.6239	1.0697	0.8376
2014	0.6664	1.0859	0.9307
2015	0.7063	1.1040	1.0243
2016	0.7438	1.1240	1.1186

IV. AVERAGE AGE OF GAME PLAYERS IN FUTURE

Since, the population was assumed to be evenly distributed across all ages within a particular stage, the average of the game players can be calculated by taking the weighted average of the median age of gamers in the three stages. For Stage 3, we consider gamers up to 79 years of age. The average age of the game players can be calculated using:

$$Average\ age = \frac{10*N1+34*N2+64.5*N3}{N1+N2+N3} \qquad (19)$$

The predicted average age of gamers for the years 2011-2016 is shown in Table IV.

TABLE IV
AVERAGE AGE OF GAME PLAYERS

Year	Average age of Game Players
2011	37.22
2012	37.71
2013	38.18
2014	38.62
2015	39.04
2016	39.45

V. LIMITATIONS OF THE MODEL

1. Here, the proposed model is considered to be independent with respect to compartmental influence. Hence, the interaction among different stages is not well defined.
2. Since data corresponding to the game playing population was not available explicitly, some relevant assumptions have been made.
3. Exact frustration rates for the three stages were not available in literature. So, the frustration rate is assumed to be equal in all the three stages. However, it will be different in practice.
4. Impact of gender and work culture on the game playing population was not considered. It is possible that people who work more on computers would be drawn more towards playing computer and video games.

VI. CONCLUSIONS

In this paper, a compartmental model in the form of a system of differential equations to simulate the movements in the video and computer game playing population of US has been developed and analyzed. An increase in the intrinsic frustration rate along with positive intrinsic growth in the three stages indicate that although games are being developed to attract new gamers yet efforts are not being put to retain the existing game players as more people are getting bored with the existing games and are joining the frustration group. The model simulation results are similar to the actual trend in the three stages and it is expected that by 2016, the number of game players in N3 stage would rise rapidly and become nearly equal to those in N2 stage. Hence, special efforts need to be put to cater to the needs of these two segments.

In future, one can extend this model by considering population density dependent frustration and rejoining rates. Also, different values of frustration and rejoining rates for the three stages may be taken. The influence of gamers in one stage on other gamers may be studied in future researches.

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