On the Representation of Actuator Faults Diagnosis and Systems Invertibility

Sallem F., Dahhou B., Kamoun A.

Abstract—In this work, the main problem considered is the detection and the isolation of the actuator fault. A new formulation of the linear system is generated to obtain the conditions of the actuator fault diagnosis. The proposed method is based on the representation of the actuator as a subsystem connected with the process system in cascade manner. The designed formulation is generated to obtain the conditions of the actuator fault detection and isolation. Detectability conditions are expressed in terms of the invertibility notions. An example and a comparative analysis with the classic formulation illustrate the performances of such approach for simple actuator fault diagnosis by using the linear model of nuclear reactor.

Keywords—Actuator fault, Fault detection, left invertibility, nuclear reactor, observability, parameter intervals, system inversion.

I. INTRODUCTION

FAULT Detection and Isolation [1], [2], abbreviated as FDI in the literature, plays a vital role in providing information about faults in the system to enable appropriate reconfiguration to take place. The main function of FDI is to detect a fault and to find its location so that corrective action can be made to eliminate or minimize the effect on the overall system performance. Our objective of this work is to provide actuator fault detection and isolation i.e. determination of the fault present in the actuator, the time of detection and the place where it occurs. A great deal of attention has formerly been devoted to actuator fault detection and isolation for linear systems [3]–[5].

However, the existent results consider the actuators as the constants in the input coefficient matrix of the process system and consider actuator faults as the changes of the input coefficient matrix elements, or the unknown additive terms of the process system control variable vectors. But, usually this is not the reality.

In this work, firstly, we formulate actuator fault detection problems for a class of linear systems. In a practical engineering control system, an actuator is a device with its interior structure and dynamic characteristic. In most cases, it cannot be simply approximated as constant coefficients. We think that this kind of actuator formulation is not effective to describe the nature of actuator faults. Different from this kind of formulation, we consider actuator as a subsystem connected with the process subsystem in cascade manner. Using this system formulation, actuator fault can be modeled as the parameter changes of the actuator subsystem. In this manner, the dynamic behavior of the faults can be perfectly described and logically explained. The objective is to obtain the condition of actuator fault detection and that is based on the invertibility concept. Moreover, the basic concept of that field, the fault detectability is defined in terms of the invertibility notions.

This work is organized as follows: Section II defines the problem settings and recalls the conventional description of the method used for actuator fault detection and isolation. Section III gives our new system formulation to obtain the condition of the actuator fault detection. In Section IV some notions about the concepts of invertibility and system inversion are studied. Actuator fault detectability conditions are investigated in Section V. The Section VI deals with the actuator fault isolation. A presentation of the nuclear reactor is given in Section VII. In Section VIII, simulation results and a brief comparison are provided to demonstrate the effectiveness of this new proposed approach. Conclusions and perspectives on the future works are given in the last section.

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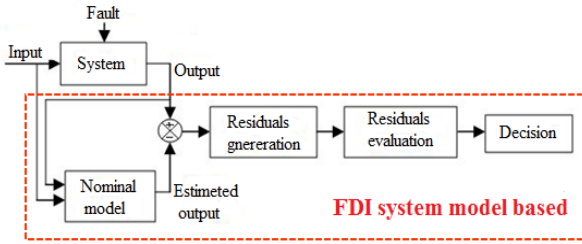
II. ACTUATOR FAULT DETECTION FOR LINEAR SYSTEMS

Different approaches for fault detection using mathematical models have been developed in the last 20 years, see for example [6]-[8], [11]-[14]. The task consists of the detection of faults in the processes, actuators or sensors by comparing the measurable outputs with the system's available mathematical model. Fig. 1 shows the general structure of the model based FDI.

A. General Linear System Structure

Consider the following linear system given by:

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) \\ y(t) = Cx(t) \end{cases} \qquad (1)$$

where: $x(t) \in X \subset \Re^n$ is the dynamic system state vector, $A \in \Re^{n \times n}$ is a constant state matrix of the dynamic system, $y(t) \in Y \subset \Re^l$ is the dynamic system output vector, $C \in \Re^{l \times n}$ is a constant output coefficient matrix of the dynamic system, $B \in \Re^{n \times m}$ is the control input coefficient matrix of the dynamic system, $u(t) \in U \subset \Re^m$ is the dynamic system input vector.

We assume that an actuator fault can occur and it is defined as a deviation or a change of B matrix parameters.

Fig. 1 General structure of the FDI based model

B. Fault Representation and Observer Structure

A fault is defined as an unpermitted deviation of at least one characteristic property of the system from an acceptable behavior. Therefore, the fault is a state that may lead to a malfunction or failure of the system. We assume that only actuator fault can occur, that is, $u_j^f(t) = \theta_j$ for $t \geq t_f$ and $\lim_{t \to \infty} |u_j(t) - \theta_j| \neq 0$, where θ_j is a constant and u_j^f is the actual output of the jth actuator when it is faulty, $j \in 1, 2, \cdots, m$ while $u_j(t)$ is the expected output when it is healthy.

Considering $B = (b_1, \ldots, b_m)$, if the lth actuator is faulty, the corresponding faulty [3] model is given by the state space model:

$$\begin{cases} \dfrac{dx}{dt} = Ax + \sum_{j \neq l} b_j u_j + b_l \theta_l \\ y = Cx \end{cases} \quad (2)$$

We desire construct an observer to detect the faulty actuator, so by using adaptation technique, the observers [3] for all possible faults are described below:

$$1 \leq i \leq m \begin{cases} \dfrac{d\hat{x}_i}{dt} = Ax + \sum_{j \neq i} b_j u_j + b_i \hat{\theta}_i + H(\hat{x}_i - x) \\ \dot{\hat{\theta}}_i = -2\gamma(\hat{x}_i - x)^T P b_i \\ \hat{y}_i = C\hat{x}_i \end{cases} \quad (3)$$

where H is a Hurwitz matrix that it can be chosen freely, γ is a scalar and P is a positive definite matrix. The two matrixes P and H are calculated by using the following Lyapunov equation:

$$H^T P + PH = -Q \quad (4)$$

where Q is any positive definite matrix that also can be chosen freely.

The basic idea is to estimate the output y_i and compare it to that generated via the model. The advantage of this proposed method, residual-based FDI, is that it can detect single fault actuator rapidly by using the FDI scheme. The residual r_i is given by:

$$r_i(t) = \|\hat{y}_i - y\| \quad (5)$$

Then, when we applied a constant fault in the one of these actuators, the correspondent residual reach zero after a short time. On the other hand the others residuals take a new constant value and remain there.

III. SYSTEM STRUCTURE WITH RESPECT TO THE ACTUATOR SUBSYSTEM

In a practical engineering control system, an actuator is a device with its interior structure and dynamic characteristic. In most cases, it cannot be simply approximated as constant coefficients. We think that this kind of actuator formulation is not effective to describe the nature of actuator faults. Different from this kind of formulation, we consider actuator as a subsystem connected with the process system by cascade manner. Using this system formulation, actuator faults can be modeled as the interior parameter changes of the actuator subsystem. The dynamic temperaments of the faults can be perfectly described and logically explained. On the contrary, in the conventional formulation the dynamic temperaments of the faults cannot be well formulated. They are usually brutally supposed as some time functions, and even constants.

B. Problem Formulation

Considering the system given by Fig. 2, it consists of two subsystems, i.e. the process subsystem and the actuator subsystem.
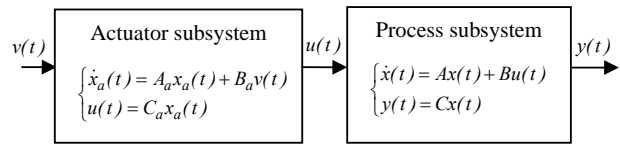

Fig. 2 System structure

We assume that the process subsystem is described by the same system (1), where $u(t) \in \Re^m$ is the process subsystem input vector, also it is the output vector of actuator subsystem. The actuator subsystem is given by:

$$\begin{cases} \dot{x}_a(t) = A_a x_a(t) + B_a v(t) \\ u(t) = C_a x_a(t) \end{cases} \quad (6)$$

where: $x_a(t) \in \Re^{n_a}$ is the state vector of the actuator subsystem. $A_a \in \Re^{n_a \times n_a}$ is a constant state matrix of the actuator subsystem, $B_a \in \Re^{n_a \times m_a}$ is a constant input coefficient matrix of the actuator subsystem, $v(t) \in \Re^{m_a}$ is the control input variable vector of the actuator subsystem, it is the output variable vector of the controller, $C_a \in \Re^{l_a \times n_a}$ is a constant output coefficient matrix of the actuator subsystem. As $u(t)$ is the process subsystem input vector and the output vector of actuator subsystem, we assume $l_a = m$.

In this new formulation, our objective is the fault detection in the actuator subsystem. We consider the fault in actuator subsystem as the changes of the actuator parameters, that is to say the changes of the elements of the matrix A_a, B_a and C_a.

We assume that the actuator output $u(t)$ cannot be measured, so we get information of actuator fault only by the output variable $y(t)$ of the process system. This consideration comes from the practice application in process control systems. Usually, the actuators are far from the central control room, we cannot set measuring cable from the central control room to each actuator to measure the actuator output signal because, on one hand, it is not necessary for the control, and on the other hand, it will increase the cost of the system. From the point of view of academic research, if we can get the measurement data of the output $u(t)$ of the actuator subsystem, the fault diagnosis problem of the actuator subsystem is degenerated as the general fault diagnosis problem when the actuator subsystem is considered as a dynamic system.

C. Faulty Actuator System Structure

In order to study the actuator fault detection problem, the actuator subsystem (6) is rewritten as:

$$\begin{cases} \dot{x}_a(t) = A_a x_a(t) + B_a v(t) + \sum_{i=1}^{k} L_i \Delta_i(t) \\ u(t) = C_a x_a(t) + \sum_{j=1}^{s} l_j \delta_j(t) \end{cases} \quad (7)$$

where, $\sum_{i=1}^{k} L_i \Delta_i(t)$ represents k possible faults caused by the parameter changes of the matrix A_a or the matrix B_a, i.e. the changes of the elements of the matrix A_a or the matrix B_a. The arbitrary function $\Delta_i(t) \in U_{F_i} \subset \Re^{m_{\Delta_i}}$ is the unknown fault mode. The matrix L_i represents the mapping $L_i : U_{L_i} \to X_a$, it is assumed monic, U_{L_i} and X_a are the sets of $\Delta_i(t)$ and of $x_a(t)$ respectively. When the fault $L_i \Delta_i(t)$ does not occur, $\Delta_i(t)$ is identically zero. Similarly $\sum_{j=1}^{s} l_j \delta_j(t)$ represents s possible faults caused by the parameter changes of the matrix C_a. The arbitrary function $\delta_i(t) \in U_{f_j} \subset \Re^{m_{\delta_j}}$ is the unknown fault mode. The matrix l_j represents the mapping $l_j : U_{f_j} \to U$, it is assumed monic, U_{f_j} and U are the sets of $\delta_j(t)$ and of $u(t)$ respectively. When the fault $l_j \delta_j(t)$ does not occur, $\delta_j(t)$ is identically zero.
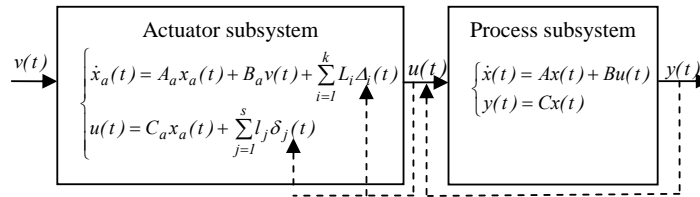

Fig. 3 System structure with respect to fault detection

We indicated that $L_i \Delta_i(t)$ can represent the fault of the matrix A_a or of the matrix B_a, and $l_j \delta_j(t)$ can represent the fault of the matrix C_a, this is the problem of fault modeling, we can see [7] and [9] for the detail of this problem. With (4), the system structure with respect to actuator fault detection is given in Fig. 3.

The effects of actuator faults may be propagated with the control variable $u(t)$. As $u(t)$ is inaccessible, we diagnose the faults using the data of the process subsystem output $y(t)$. In other words we observe $u(t)$ using $y(t)$, then observe the faults using $u(t)$, what is related to the concept of system invertibility. In simple terms, an invertible system is the system in which different input produces different output. We will see later, if the actuator subsystem is invertible with respect to a fault, and if the process subsystem is invertible
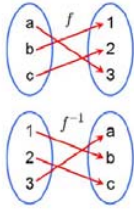
with respect to u, then this actuator fault can be detected using $y(t)$.

IV. INVERTIBILITY OF DYNAMIC SYSTEM

A. Notion of Invertibility

An inverse function is a function that undoes another function: If an input x into the function f produces an output y, then putting y into the inverse function g produces the output x, and vice versa. i.e., $f(x) = y$, and $g(y) = x$. A function f that has an inverse is called invertible (Fig. 4). The inverse function is then uniquely determined by f and is denoted by f^{-1}.

Fig. 4 Invertible function f and its inverse f^{-1}

Definition 1. The function f with domain X and range Y is invertible if there exists a function g with domain Y and range X, such that:

$$f(x) = y \quad \text{if and only if} \quad g(y) = x \quad (8)$$

Not all functions have an inverse. For this rule to be applicable, each element $y \in Y$ must correspond to no more than one $x \in X$; a function f with this property is called injective.

From the view of set theory, let X and Y are two linear inner product spaces, σ and τ are two linear mappings.

Definition 2. Considering a linear mapping $\sigma : X \to Y$, for any two points $\alpha, \beta \in X$ and $\alpha \neq \beta$, if:

$$\sigma(\alpha) \neq \sigma(\beta) \quad (9)$$

it is said that the linear mapping σ is injective. If for any $\gamma \in Y$, we have $\alpha \in X$, such that:

$$\sigma(\alpha) = \gamma \quad (10)$$

It is said that σ is surjective, or a full mapping from X to Y.

Lemma 1. A linear mapping $\sigma : X \to Y$ is invertible, if and only if σ is injective and surjective. That is to say:

σ is invertible $\Leftrightarrow$ is injective and surjective.

Proof:

1) "=>". σ is invertible, therefore σ^{-1} exists. For any two points $\alpha, \beta \in X$, if $\sigma(\alpha) = \sigma(\beta)$, then:

$$\alpha = (\sigma^{-1}\sigma)(\alpha) = \sigma^{-1}(\sigma(\alpha)) = \sigma^{-1}(\sigma(\beta))$$
$$= (\sigma^{-1}\sigma)(\beta) = \beta$$

Therefore σ is injective. For $\forall \varsigma \in Y$, let $\alpha = \sigma^{-1}(\varsigma)$, then $\alpha \in X$, and $\sigma(\alpha) = \sigma(\sigma^{-1})(\varsigma) = \sigma\sigma^{-1}(\varsigma) = \varsigma$, therefore σ is surjective.

2) "<=", if σ is injective and surjective, therefore each point in Y corresponds an unique point in X, therefore the inverse mapping τ of σ exists. It is obvious that the inverse mapping τ of σ is also a linear mapping

$\tau : Y \to X$, and $\sigma\tau = \tau\sigma = E$, where, E is unite mapping, therefore σ is invertible, and $\tau = \sigma^{-1}$.

♠

It is well known that a mapping can be represented by a corresponding matrix, the study of mapping is usually transformed to the study of the corresponding matrix, we give following matrix version statements about invertibility to subdivided it as left invertibility and right invertibility [15].

Definition 3. For matrix $P \in \Re^{n \times m}$, if there exists a matrix $P^L \in \Re^{m \times n}$ such that $P^L P = I_m$, it is said that P is left invertible, while P is right invertible if there exists a matrix $P^R \in \Re^{m \times n}$ such that $PP^R = I_n$.

Lemma 2. Let $P \in \Re^{n \times m}$, then, the following statements are equivalent:
1) P is left invertible
2) P is injective
3) $rank(P) = m$
4) P has full column rank

The following statements are also equivalent:
1) P is right invertible
2) P is surjective
3) $rank(P) = n$
4) P has full row rank

Lemma 3. Let $P \in \Re^{n \times n}$ then, the following statements are equivalent:
1) P is nonsingular
2) P has a unique left inverse
3) P has a unique right inverse
4) P has a unique inverse
5) P is injective
6) P is surjective
7) P is left invertible
8) P is right invertible
9) P is invertible
10) $rank(P) = n$

Corollary 1: Let σ, ς two linear left invertible mapping, $P \in \Re^{n \times m}$, $Q \in \Re^{s \times n}$ be corresponding matrices, then the mapping $(\varsigma\sigma)$ is left invertible.

Proof: Let σ, ς are left invertible, it implies that $P \in \Re^{n \times m}$ and $Q \in \Re^{s \times n}$ are left invertible, therefore there exist two matrices $P^L \in \Re^{m \times n}$ and $Q^L \in \Re^{n \times s}$ such that: $(P^L Q^L)(QP) = (P^L Q^L QP) = (P^L I_n P) = (P^L P) = I_m$

According to Definition 3, (QP) is left invertible, consequently $(\varsigma\sigma)$ is left invertible. ♠

B. Invertibility and Input Observability of Linear System

We consider the linear dynamic system given as follows:

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) \\ y(t) = Cx(t) + Dw(t) \end{cases} \quad (11)$$

where, $u \in U \subset \Re^m$ and $w \in W \subset \Re^{m_w}$ are the input variable vectors, $x \in X \subset \Re^n$ is the state variable vector, and $y \in Y \subset \Re^l$ is output variable vector, $U \subset \Re^m$, $X \subset \Re^n$, $Y \subset \Re^l$ and $W \subset \Re^{m_w}$ are linear vector spaces. A, B, C and D are the matrices with appropriate dimensions.

For the faults affecting on the system (11), after appropriate modeling, can be considered as the additive terms in the input and output of the system equations, that is to say from the view of fault diagnosis can be considered as the system's inputs $u(t)$. We concern such a question that can we get the information of the faults by the measured data of the system output $y(t)$? If no, it implies that we cannot implement fault diagnosis procedure. This property of the system is related to the concept of invertibility [16].

The needed system invertibility in our study is essentially the "left invertibility", or "input observability". In [18], the input observability is defined as the naturally dual property of output controllability. Output controllability is a pointwise property, i.e. it is concerned with the existence of an input function that drives the output to a specified point in output space at a specified time. The concept of functional reproducibility, or output functional controllability, is a functional property concerned with the existence of an input which produces a specified output function. It is obvious that left invertibility is a functional property naturally dual to output functional controllability. The term input functional observability is thus an appropriate alternative to left invertibility. Left invertibility or input observability is shown as Fig. 5, from the right side of the system equation, i.e. the input $u(t)$, the information propagates along with left direction until output $y(t)$, then at the left side, the measured data $y(t)$ is used to reconstruct the input $u(t)$.
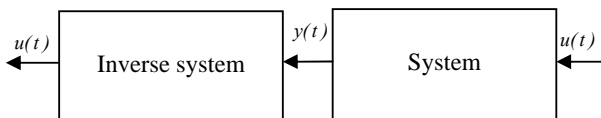

Fig. 5 Left invertible system

As the considered system is a linear system, it follows the superposition principle, therefore we can study the invertibility of $U \to Y$ and of $W \to Y$ respectively.

Firstly, we consider the invertibility of $U \to Y$ and we define linear mapping: $T_{u,y} : U \to Y$, and write for the state at $t = t_1$, when at time t_0, the system is in the zero state,

$$x(t_1) = \int_0^{t_1} e^{A(t_1-\tau)} Bu(\tau) d\tau \qquad (12)$$

and the system output,

$$y(t_1) = C \int_0^{t_1} e^{A(t_1-\tau)} Bu(\tau) d\tau \qquad (13)$$

The exponential may be represented in terms of its Taylor series; doing this we find,

$$y(t_1) = CB \int_0^{t_1} u(\tau) d\tau + CAB \int_0^{t_1} (t_1 - \tau) u(\tau) d\tau$$
$$+ CA^2 B \int_0^{t_1} \frac{1}{2} (t_1 - \tau) u(\tau) d\tau + \cdots \qquad (14)$$

We see that the terminal output variable is in the linear subspace spanned by the column vectors of the infinite sequence of matrices $CB, CAB, CA^2 B, \cdots, CA^n B, CA^{n+1} B, \cdots$.

According to Cayley-Hamilton theorem, for a n order matrix A, any order power of n can be expressed by the sum of its lower order powers with order $0, 1, \cdots, (n-1)$ respectively. So each term with type as $CA^i B, i > n$ can be expressed by the sum of $CB, CAB, CA^2 B, \cdots, CA^{n-1} B$. Consequently output variable $y(t_1)$ is in the linear subspace spanned by the column vectors of $CB, CAB, CA^2 B, \cdots, CA^{n-1} B$. That is to say the output space Y is spanned by these column vectors. The number of linear independent column vectors of $CB, CAB, CA^2 B, \cdots, CA^{n-1} B$ is the dimension of the spanned subspace. Recall that $T_{u,y} : U \to Y$ denotes the mapping from U to Y, obviously, the rank of the mapping $T_{u,y} : U \to Y$ is decided by this number, therefore:

$$rank(T_{u,y}) = rank\left(\begin{pmatrix} CB \\ CAB \\ CA^2 B \\ \vdots \\ CA^{n-1} B \end{pmatrix} \right) \qquad (15)$$

If we denote by $P_{u,y}$ the corresponding matrix of the mapping $T_{u,y} : U \to Y$, then,

$$rank(P_{u,y}) = rank\left(\begin{pmatrix} CB \\ CAB \\ CA^2 B \\ \vdots \\ CA^{n-1} B \end{pmatrix} \right) \qquad (16)$$

According to Lemma 2, if $P_{u,y}$ has full column rank, i.e. $rank(P_{u,y}) = m$, the matrix $P_{u,y}$ is left invertible, the input and output of the mapping is injective, the system is left

invertible. Therefore the condition of the dynamic system being left invertible is:

$$rank \left(\begin{array}{c} CB \\ CAB \\ CA^2B \\ \vdots \\ CA^{n-1}B \end{array} \right) = m \qquad (17)$$

Now, let's consider the invertibility of $W \rightarrow Y$.

1) $w(t) = u(t)$. In this case, we get the output $y(t_1)$ as:

$$y(t_1) = Du(t_1) + CB \int_0^{t_1} u(\tau)d\tau + CAB \int_0^{t_1} (t_1 - \tau)u(\tau)d\tau$$
$$+ CA^2B \int_0^{t_1} \frac{1}{2}(t_1 - \tau)u(\tau)d\tau + \cdots \qquad (18)$$

Along with the way above, we can get the condition of the dynamic system being left invertible is:

$$rank \left(\begin{array}{c} D \\ CB \\ CAB \\ \vdots \\ CA^{n-1}B \end{array} \right) = m \qquad (19)$$

This is the result in [1].

2) $w(t) \neq u(t)$ In this case, the output caused by $w(t)$ is:

$$y(t_1) = Dw(t_1) \qquad (20)$$

Similar to the previous cases, the condition of the dynamic system being left invertible is:

$$rank(D) = m_w \qquad (21)$$

That is to say the number of linear independent column vectors of D is m_w.

V. ACTUATOR FAULT DETECTABILITY

The fault detectability is the property which indicates if the fault in the system can be detected or in other word can be observed by using the system output data. Using the results of previous sections, it is easy to discuss the fault detectability of the system (7).

A. Actuator Subsystem Fault Detectability

Consider the fault $L_i \Delta_i(t)$ in the system (7), let L_i as the matrix B in the system (11), according to (17) the condition of fault detectable for the fault $L_i \Delta_i(t)$ is obtained:

$$rank \left(\begin{array}{c} C_a L_i \\ C_a A_a L_i \\ C_a A_a^2 L_i \\ \vdots \\ C_a A_a^{n_a-1} L_i \end{array} \right) = m_{\Delta_i} \qquad (22)$$

where m_{Δ_i} is the dimension of $\Delta_i(t)$.

Similarly, for the fault $l_j \delta_j(t)$ in the system (7), according to (21) the condition of fault detectable is:

$$rank(l_j) = m_{\delta_j} \qquad (23)$$

where m_{δ_j} is the dimension of $\delta_j(t)$.

B. Process Subsystem Invertibility

The invertibility of the process subsystem in our fault detection frame is the dynamic property which indicates whether all the information of $u(t)$ can pass through the process subsystem, therefore the fault information carried by $u(t)$ can completely pass through the process subsystem. From previous discussion it can be known that if the process subsystem is left invertible, then the information in the input of this subsystem can be "injective" mapped to its output. Therefore the fault information carried by $u(t)$ can completely passed through the process subsystem.

Comparing the system (1) and the system (11), and according to (17), the left invertible condition for the process subsystem is obtained as:

$$rank \left(\begin{array}{c} CB \\ CAB \\ CA^2B \\ \vdots \\ CA^{n-1}B \end{array} \right) = m \qquad (24)$$

where m is the dimension of the vector u.

C. Fault Detectability of the System

The fault detectability of the system in Fig. 2 represents the system property that if the faults in the actuator subsystem can be detected? According to Corollary 1, if the actuator subsystem is left invertible for the specified fault, and the process subsystem is left invertible, then the entire system is left invertible for this fault, which therefore can be detected. Consequently we get the conditions of fault detectable as follows:

Theorem 1. Consider the system in Fig. 2 which consists of the process subsystem (1) and the actuator subsystem (7), then the conditions of faults detectable are:

1) For the fault $L_i \Delta_i(t)$:

$$rank\left(\begin{array}{c} C_a L_i \\ C_a A_a L_i \\ C_a A_a^2 L_i \\ \vdots \\ C_a A_a^{n_a-1} L_i \end{array}\right) = m_{\Delta_i} \quad (25)$$

and,

$$rank\left(\begin{array}{c} CB \\ CAB \\ CA^2 B \\ \vdots \\ CA^{n-1} B \end{array}\right) = m \quad (26)$$

2) For the fault $l_j \delta_j(t)$:

$$rank(l_j) = m_{\delta_j} \quad (27)$$

and,

$$rank\left(\begin{array}{c} CB \\ CAB \\ CA^2 B \\ \vdots \\ CA^{n-1} B \end{array}\right) = m \quad (28)$$

VI. ACTUATOR FAULT ISOLABILITY

We defined the actuator fault isolability term as the property of the system indicating the possibility to locate the fault in the system, more precisely, to isolate the actuator causing the error.

A. Generality of the Fault Isolation

Fault isolation has been investigated using many techniques such as the adaptive observer approach [3], the geometric approach [9], and the interval based approach [10]. The choice of a method depends basically on the given situation such as the kind and the number of faults to be detected, the demands for the fault isolation, the robustness and the available measurements.

In our considered system, the actuator fault can be isolated. Of course, the fault information should be well transferred to the system output. Therefore the two subsystems should be left invertible and they verify the theorem 1 as mentioned in the previous section.

In this work, we will use the interval based approach in which a partition notion of each actuator parameter and the technique of monotonicity are used in order to locate and to determine the values of the faulty parameters in the actuator subsystem. The proposed method proves to be effective in the actuator fault isolation.

B. Actuator Fault Isolability in Parameter Intervals Based Approach

In parameter intervals based fault isolation approach, the isolation principle is not based on fault decoupling on space direction, while it based on the consistency test of the dynamic model function. That is to say, even with a same mapping matrix L_i, two faults $L_i \Delta_{i,j}(t)$ and $L_i \Delta_{i,k}(t)$ can be separated each other if the fault mode $\Delta_{i,j}(t)$ and $\Delta_{i,k}(t)$ are different functions of the time. If the information of these two faults can be "injective" transferred through the process subsystem to the output $y(t)$ of the system, so that it can be separated. As we have studied that if the process subsystem is left invertibility, the information of these two faults can be "injective" transferred to the process subsystem output.

We recall quickly the main points of the parameter interval based fault isolation method, for more details, the reader is refereed to [10].

In the parameter interval based method, the domain of each parameter θ is partitioned into a certain number of intervals and a parameter filter is built for each interval. The parameter filter consists of two isolation observers which correspond to two bounds of the interval $[\theta_j^\alpha \ \theta_j^\beta]$ of the jth parameter.

After the fault occurrence, the faulty parameter value must be in one of the parameter intervals.

Considering the model represented by:

$$\begin{cases} \dot{x}_a(t) = A_a(\theta_A)x_a(t) + B_a(\theta_B)v(t) \\ u(t) = C_a(\theta_C)x_a(t) \end{cases} \quad (29)$$

where θ_A, θ_B and θ_C are the parameter vectors used to study the faults caused by the changes of the elements of the matrices A_a, B_a or C_a respectively.

We consider, for example, the case of the dynamic fault in the actuator subsystem i.e. the fault caused by the changes of the elements of the matrix A_a.

For (29) the parameter filter with respect to the dynamic fault can be built with the two correspondent isolation observers that constitute the actuator parameter filter for i^{th} interval of jth actuator parameter:

$$\begin{cases} \dot{\hat{x}}_a^{\alpha(ij)} = A_a(\theta_A^{ob\alpha(ij)})\hat{x}_a^{\alpha(ij)} + B_a(\theta_B^0)v(t) \\ \qquad + KC_a(\theta_C^0)(x_a - \hat{x}_a^{\alpha(ij)}) \\ e^{\alpha(ij)} = x_a - \hat{x}_a^{\alpha(ij)} \\ \varepsilon^{\alpha(ij)} = C_a e^{\alpha(ij)} \end{cases} \quad (30)$$

$$\begin{cases} \dot{\hat{x}}_a^{\beta(ij)} = A_a(\theta_A^{ob\beta(ij)})\hat{x}_a^{\beta(ij)} + B_a(\theta_B^0)v(t) \\ \qquad + KC_a(\theta_C^0)(x_a - \hat{x}_a^{\beta(ij)}) \\ e^{\beta(ij)} = x_a - \hat{x}_a^{\beta(ij)} \\ \varepsilon^{\beta(ij)} = C_a e^{\beta(ij)} \end{cases} \quad (31)$$

where $\theta_A^{ob\alpha(ij)}$ and $\theta_A^{ob\beta(ij)}$ are the parameters vectors of the observers corresponding the parameter vector θ_A. We also assume that the fault is caused by the change of a single

parameter in the vector θ_A, so the parameters vectors θ_B and θ_C maintain their nominal values when the fault occurs. $\varepsilon^{\alpha(ij)}$ and $\varepsilon^{\beta(ij)}$ are the estimation errors of the observers.

We assume that before t_f, the observer states $\hat{x}_a^{\alpha(ij)}(t)$ and $\hat{x}_a^{\beta(ij)}(t)$ converge on the actuator subsystem state $x_a(t)$ and:

$$e^{\alpha(ij)}(t_f) = \varepsilon^{\alpha(ij)}(t_f) = 0$$
$$e^{\beta(ij)}(t_f) = \varepsilon^{\beta(ij)}(t_f) = 0 \qquad (32)$$

At $t = t_f$ when the fault occurs, the sth parameter changes and the jth parameter of the observer changes in order to isolate the fault:

$$\forall t \geq t_f \begin{cases} (\theta_A)_s^f = (\theta_A)_s^0 + (\Delta_A)^f \\ (\theta_A)_l^f = (\theta_A)_l^0, \quad l \neq s \end{cases} \qquad (33)$$

$$(\theta_A)_j^{ob\alpha(ij)} = \begin{cases} (\theta_A)_j^0, \quad t < t_f \\ (\theta_A)_j^{\alpha(ij)}, \quad t \geq t_f \end{cases} \qquad (34)$$

$$(\theta_A)_l^{ob\alpha(ij)}(t) = (\theta_A)_l^0, \quad \forall t, l \neq j \qquad (35)$$

$$(\theta_A)_j^{ob\beta(ij)} = \begin{cases} (\theta_A)_j^0, \quad t < t_f \\ (\theta_A)_j^{\beta(ij)}, \quad t \geq t_f \end{cases} \qquad (36)$$

$$(\theta_A)_l^{ob\beta(ij)}(t) = (\theta_A)_l^0, \quad \forall t, l \neq j \qquad (37)$$

where $(\theta_A)_j^{\alpha(ij)}$ and $(\theta_A)_j^{\beta(ij)}$ are the bounds of the ith interval of the jth parameter of θ_A. The isolation index is:

$$v^{ij}(t) = sgn(\varepsilon^{\alpha(ij)}(t))\, sgn(\varepsilon^{\beta(ij)}(t)) \qquad (38)$$

It is assumed that the cascade observers which are built by using the actuator input v and the fault parameter as input data and, using process subsystem output y as output data, satisfy following assumption 1 and assumption 2.

Assumption 1

At any point (x_a, v), the linear function $f = A_a(\theta_A)x_a + B_a(\theta_B)v$ in (29) satisfies that any component f_i, $i \in \{1, ..., n\}$ which is an explicit function of the considered parameters $(\theta_A)_j$ or $(\theta_B)_j$ is a monotonous function of these two parameters.

Assumption 2

If the change of the actuator subsystem and the change of the observer are not at the same parameter i.e., $s \neq j$, whatever

the value of the change of the isolation observer parameter is, the dynamic difference between the isolation observer and the faulty actuator subsystem is great.

Using Assumption 1, it can be shown that for the case where $s = j$, the estimation error $\varepsilon^{\alpha(ij)}(t)$ of the observer is a monotonous function of the parameter difference:

$$\delta(\theta_A)_j^{\alpha(ij)} = (\theta_A)_j^{ob\alpha(ij)} - (\theta_A)_j^f \qquad (39)$$

and $\varepsilon^{\beta(ij)}(t)$ is a monotonous function of:

$$\delta(\theta_A)_j^{\beta(ij)} = (\theta_A)_j^{ob\beta(ij)} - (\theta_A)_j^f \qquad (40)$$

and no matter $s = j$ or not, the difference of the estimation error:

$$\varepsilon^{\beta\alpha(ij)}(t) = \varepsilon^{\beta(ij)}(t) - \varepsilon^{\alpha(ij)}(t) \qquad (41)$$

is a monotonous function of parameter difference $(\theta_A)_j^{\beta(ij)} - (\theta_A)_j^{\alpha(ij)}$ between the two interval bounds.

Using Assumption 2, the monotonicities of $\varepsilon^{\alpha(ij)}(t)$, of $\varepsilon^{\beta(ij)}(t)$ and of $\varepsilon^{\beta\alpha(ij)}(t)$, we can obtain the rule of the interval verification as follows:

Rule1 After the fault occurrence, if the ith interval of the jth parameter contains the faulty parameter value, then we have: $v^{ij}(t) = -1, \forall t$. Otherwise $v^{ij}(t) = 1$, then it can be concluded that this interval does not contain the faulty parameter value. If all the intervals of a parameter, after the fault occurrence, are excluded from containing the faulty parameter value, the fault is excluded from this parameter. If all the parameters except one are excluded from the fault, the fault is isolated. The parameter which is not excluded corresponds to the fault.

So we can prove that for the case where the interval contains the faulty parameter value, when $(\theta_A)_j^f \in [(\theta_A)_j^{\alpha(ij)} \; (\theta_A)_j^{\beta(ij)}]$, it will be:

$$v^{ij}(t) = -1 \qquad \forall t > t_f \qquad (42)$$

And for the case where the interval does not contain the faulty value, it exists $t_e \geq t_f$ that:

$$v^{ij}(t) = 1 \qquad (43)$$

VII. NUCLEAR REACTOR

To show the isolation ability of the parameter intervals approach in the new formulation, this method was applied to a one group reactor model and many simulations were carried out.

A. Reactor Model

As seen in Fig. 6 the closed-loop control action in a vertical tank-type moderator level control reactor is based on variation of the level of the heavy water in the reactor vessel by altering the outflow (by varying the port openings of a set of control valves), while maintaining the inflow constant (fixed pumping rate). The core reactivity is dependent on the moderator level (reactivity change is proportional to moderator level change).
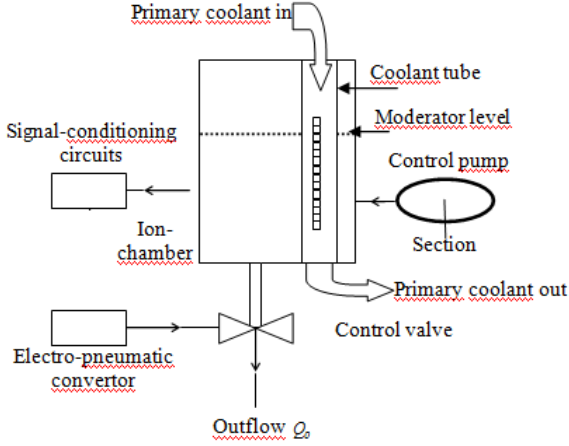

Fig. 6 Schematic of closed-loop control action in vertical tank-type reactor

The Reactor dynamics can be approximated by the point kinetic equations [17]:

$$\begin{cases} \dfrac{dn}{dt} = \dfrac{\rho - \overline{\beta}}{l} n(t) + \sum_{i=1}^{6} \lambda_i C_i(t) \\ \dfrac{dC_i}{dt} = \dfrac{\overline{\beta}_i}{l} n(t) - \lambda_i C_i(t) \qquad i = 1,2,\cdots \end{cases} \quad (44)$$

where, $n(t)$ is the density of the neutrons, ρ is the reactivity, $\overline{\beta}$ is the effective delayed neutron fraction. $\overline{\beta}_i$ is the effective fractional yield of the group of delayed neutrons, l is the prompt neutron lifetime, λ_i is the decay constant of the ith precursor group, $C_i(t)$ is the concentration of the ith precursor group.

The linearization of the point kinetics equations around an equilibrium operation level using of the one-group approximation permits the model to be expressed in matrix form as:

$$\dot{x} = \begin{pmatrix} \delta \dot{n} \\ \delta \dot{C} \end{pmatrix} = \begin{pmatrix} -\dfrac{\overline{\beta}}{l} & \lambda_e \\ \dfrac{\overline{\beta}}{l} & -\lambda_e \end{pmatrix} \begin{pmatrix} \delta n \\ \delta C \end{pmatrix} + \begin{pmatrix} \dfrac{n_0}{l} \\ 0 \end{pmatrix} \delta\rho \quad (45)$$

We assume that $l = 10^{-4}\,s$ and the effective decay constant is $\lambda_e = 0.0768$. We use the delayed neutron a constant and we normalize the neutron and precursor density based on the

initial neutron density $n_0 = 10^8$. The process subsystem matrices are:

$$\begin{aligned} A &= \begin{pmatrix} -65.0 & 0.0768 \\ 65.0 & -0.0768 \end{pmatrix} \\ B &= \begin{pmatrix} 10^4 \\ 0 \end{pmatrix} \\ C &= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \end{aligned} \quad (46)$$

B. Actuator Model for the Nuclear Reactor

We consider the transfer function [17] given by, that is present the relationship between the reactivity and the valve opening:

$$\frac{\delta\rho(s)}{X(s)} = \frac{-\dfrac{\partial Q_o/\partial X}{\partial Q_o/\partial H}}{1 + \dfrac{A}{\partial Q_o/\partial H} s} \quad (47)$$

By differentiating the above equation, we obtain:

$$d\frac{d^2 \delta\rho}{dt^2} = -[\frac{\partial Q_o}{\partial H}\frac{d\delta\rho}{dt} + \frac{\partial Q_o}{\partial X}\frac{dX}{dt}] \quad (48)$$

where d is the diaphragm area of the tank, Q_o is the outflow, X is the valve opening and H is the moderator level.

Therefore the state representation is given by:

$$\dot{x}_a = \begin{pmatrix} \delta\dot{\rho} \\ \delta\ddot{\rho} \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 0 & -\dfrac{f_1}{A} \end{pmatrix} \begin{pmatrix} \delta\rho \\ \delta\dot{\rho} \end{pmatrix} + \begin{pmatrix} 0 \\ -\dfrac{f_2}{A} \end{pmatrix} v(t) \quad (49)$$

where $q_1 = \dfrac{\partial Q_o}{\partial H}$, $q_2 = \dfrac{\partial Q_o}{\partial X}$ and $v(t) = \dfrac{dX}{dt}$.

We assume that $\dfrac{q_1}{d} = 12$ and $\dfrac{q_2}{d} = 0.1$. The actuator subsystem matrices are:

$$\begin{aligned} A_a &= \begin{pmatrix} 0 & 1 \\ 0 & -12 \end{pmatrix} \\ B_a &= \begin{pmatrix} 0 \\ -0.1 \end{pmatrix} \end{aligned} \quad (50)$$

VIII. SIMULATION RESULTS

A. Classic Formulation of Actuator Fault

To show in detail the actuator fault isolation algorithm in the first formulation, we have chosen the example where the faulty actuator parameter is $(\dfrac{n_0}{l})^f = b_1^f = \theta_1^f = 2 \cdot 10^4$.

We have applied a fault at time $t_f = 20\,days$ in the first actuator parameter b_1. In Fig. 7, we presented the two residuals r_i associated to the two observers. We can see that

residuals leave zero at $t = 20\,days$ but after a very short period, $r_1(t)$ that corresponds to the 1st input return to its initial value. While we observe that the 2nd residual take new values for a many time and then converges to zero.
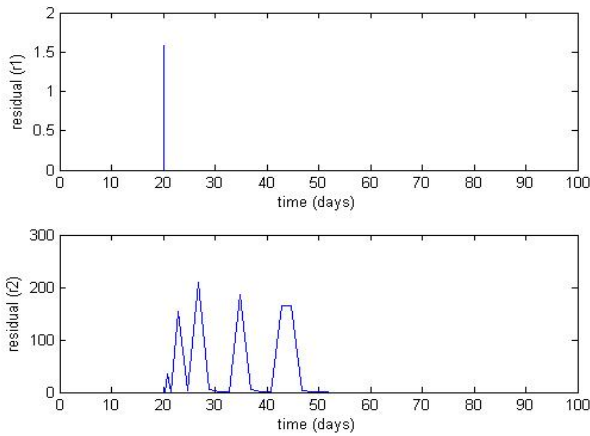

Fig. 7 Two residuals $r_1(t)$ and $r_2(t)$

Consequently, we have isolated the fault actuator correctly, and it is in the 1st actuator. In this case, the isolation time is $t_{iso} = 0.11\,days$, because the fault appears at $t_f = 20\,days$ and it has been isolated at $t_1 = 20.11\,days$.

B. New Formulation of Actuator Fault

We use $\theta_1 = -\dfrac{q_1}{d} = (\theta_A)_1^0$ and $\theta_2 = -\dfrac{q_2}{d} = (\theta_B)_2^0$ the nominal parameter values of the actuator subsystem to simulate the actuator subsystem fault. Their possible domain are assumed as $(\theta_A)_1 \in [-12 \quad -10]$ and $(\theta_B)_2 \in [-0.3 \quad -0.1]$.

The domain of each parameter θ_1 and θ_2 is partitioned into 4 intervals (Tables I and II):

TABLE I
VALUES OF PARAMETER FILTERS OF θ_1

N	1	2	3	4
θ_1^{α}	-12	-11.5	-11	-10.5
θ_1^{β}	-11.5	-11	-10.5	-10

It is assumed that the fault is caused at $t_f = 20\,days$ by the variation of θ_1, its value changes from -12 to -10.25, while the parameter θ_2 maintains its nominal value -0.1. We also note that this actuator subsystem fault has the same effect on the output of the process subsystem that affects the control variable in the first case.

TABLE II
VALUES OF PARAMETER FILTERS OF θ_2

N	1	2	3	4
θ_2^{α}	-0.30	-0.25	-0.20	-0.15
θ_2^{β}	-0.25	-0.20	-0.15	-0.1

Fig. 8 shows the result of the 1st parameter filter of θ_1, that is the case where $s = j$. After $t_f = 20\,days$ these two observers estimation errors have the same sign, so this interval does not contain the faulty parameter value.
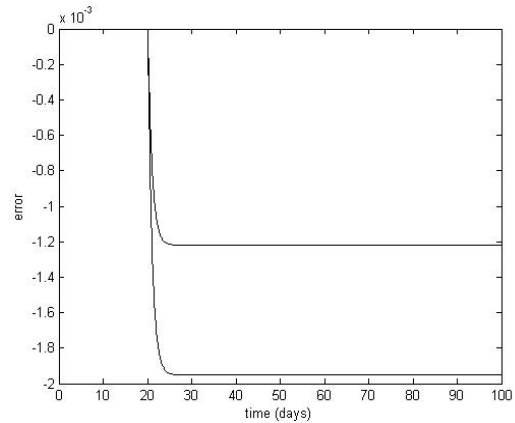

Fig. 8 1st parameter filter of θ_1

Fig. 9 shows the result of the 4^{tht} parameter filter of θ_1, that is also the case where $s = j$. After $t_f = 20\,days$ the sign of these two observers estimation errors is always different, so this interval contain the faulty parameter value and the parameter θ_1 cannot be excluded from the fault
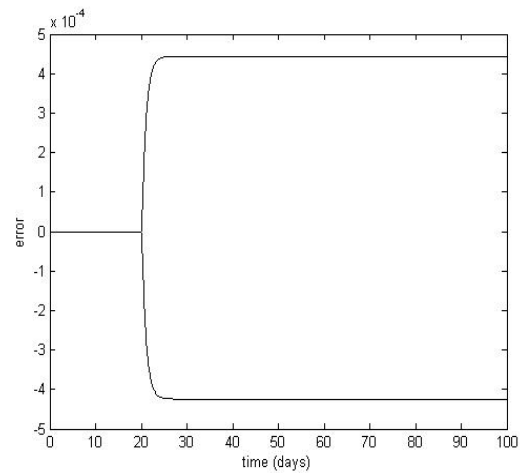

Fig. 9 4^{tht} parameter filter of θ_1

Fig. 10 shows the result of the 3rd parameter filter of θ_2, this is the case where $s \neq j$. After $t_f = 20days$ the signals of these two observers' estimation errors are the same, it means that this interval does not contain the faulty parameter value and that all the intervals of the parameter θ_2 cannot contain the faulty parameter value.
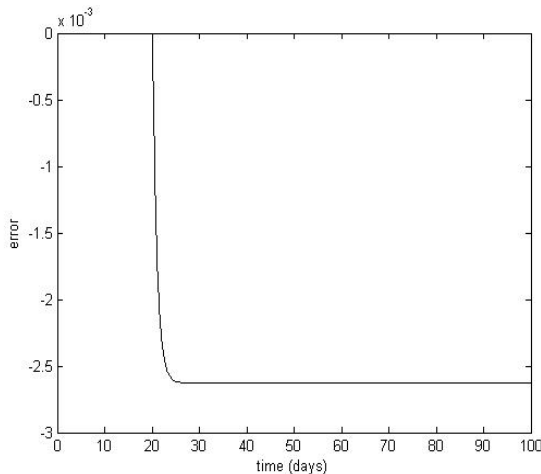

Fig. 10 3rd parameter filter of θ_2,

C. A Comparative Analysis

Simulation runs have been used to compare the actuator fault isolation using observers based method and which use parameter intervals based method. In [10] it is shown that the 2nd proposed method is faster than the 1st one, so will apply our new formulation with using the parameter intervals based.

Fig. 11 shows the isolation time of the actuator fault in these two previous different approaches.

We can conclude that these experimental results using the new formulation and based on parameter intervals are more realistic and faster than those based on classical adaptive observers.
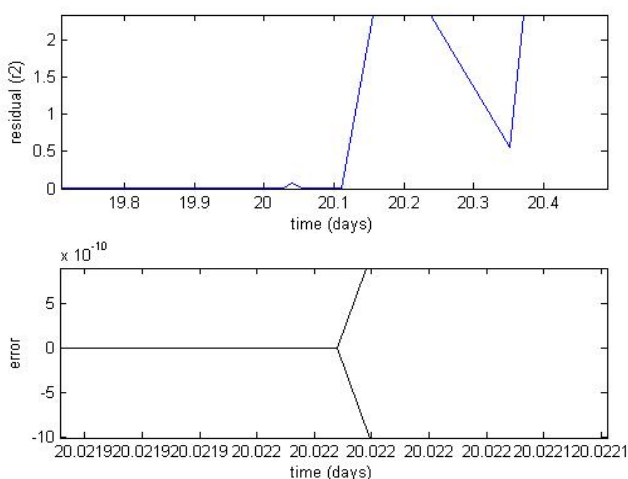

Fig. 11 3rd parameter filter

Though it is not so accurate as the detection and isolation results based on the 2nd method, but it requires less computation and it is effective for linear systems diagnosis.

The use of an interval notion contributes to the fault detection speed in a positive way and it is also fits large kind of linear dynamics systems. The only required conditions for the type of the linear system are that the dynamic of the system is a monotonous function [10] with respect to the considered parameter.

This method does not need any parameter identification procedure. It is proven that if the parameter intervals are small enough the isolation speed will be fast enough.

IX. CONCLUSION

In this paper, a new system formulation with respect to actuator fault detection is built. The new formulation is closer to practical system, can more clearly describe actuator faults. The new formulation is very useful for the further study of actuator fault diagnosis. The actuator fault detectability and isolability based on the new system formulation are studied, and the obtained conditions can be used to verify if the actuator faults can be detected and isolated. A comparative analysis of classical formulation of the actuator fault diagnosis and our contribution using two model based methods is given.

Experimental results show that the two detection and isolation methods are both effective and more accurate than others methods.

In the first method using adaptive observers the isolation can be carried out, but the isolation speed is not ideal due the parameter identification which lasts a long time. However the second one which is based on parameter intervals can solve this problem. Some simulation results illustrate these advantages.

In our work we only focus on the simple actuator fault, that is why one interesting future research direction is to extend this new formulation to multiple faults problem for linear dynamic systems.

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