Verification and Application of Finite Element Model Developed for Flood Routing in Rivers

A. L. Qureshi, A. A. Mahessar, A. Baloch

***Abstract*—**Flood wave propagation in river channel flow can be enunciated by nonlinear equations of motion for unsteady flow. It is difficult to find analytical solution of these non-linear equations. Hence, in this paper verification of the finite element model has been carried out against available numerical predictions and field data. The results of the model indicate a good matching with both Preissmann scheme and HEC-RAS model for a river reach of 29km at both sites (15km from upstream and at downstream end) for discharge hydrographs. It also has an agreeable comparison with the Preissemann scheme for the flow depth (stage) hydrographs. The proposed model has also been applying to forecast daily discharges at 400km downstream in the Indus River from Sukkur barrage of Sindh, Pakistan, which demonstrates accurate model predictions with observed the daily discharges. Hence, this model may be utilized for flood warnings in advance.

***Keywords*—**Finite Element Method, Flood Forecasting, HEC-RAS, Indus river.

I. Introduction

FLOOD is the abnormal weather-related hazard, if flood can be predicted in advance then suitable warning and preparation can be adopted to mitigate the damages. For this purpose, many river basins have worked out to build up the flood forecasting system for flood mitigations [1].

One-dimensional unsteady open channel flow modeling is important in flood routing and prediction, stream flow modeling, river regulation and in the analysis of estuarine flows. Flood routing is the activity of mathematically modeling which is the most important activity in predicting flood stages and discharges as the functions of time and space along a river reach [2].

The movement of a flood wave in a river channel is a highly complicated phenomenon of unsteady flow. Not only the flow vary with time as the wave progressed downstream, but also the channel properties and amounts of lateral inflow/outflow can vary. Thus, the analytical solution to the problem becomes quite complicated [3].

The first numerical model of a river system was developed by [4] for the Ohio and Mississippi systems. It was followed by important developments in the late 1950s in particularly by [5]-[7].

Hydraulic routing is known as distributed routing based on conversation of mass and simplified Saint-Venant momentum equations. The most accurate theoretical approach to flood routing is the system of the Saint-Venant hydrodynamic equations. This is highly non-linear partial differential equations which cannot be solved analytically. Numerical schemes such as finite differences, finite volume and finite elements along digital computer must be utilized for solving non-linear problems [8]-[10].

The finite difference scheme has been used in most of numerical models i.e. is due to the fact of requiring small time and space steps in such schemes. Hence, the computing efficiency is decreasing when large space and time steps are being used. However, the finite element method (FEM) is not only efficient for such large space and time steps but also flexible in handling general shapes of domain and boundary conditions, which is basic required for fluid dynamic problems and computation of complex flows [11], [12].

II. Governing Equation and Numerical Model

The diffusive wave equation has terms of convection and diffusion which depends on two parameters, celerity and diffusion coefficient that is also functions of the discharge. The resolution of this equation depends on initial conditions, inflow hydrograph, lateral inflow/outflow and geometric characteristics of channel [13].

The diffusive wave model, simplified form of the Saint-Venant Equation, could give a good accuracy in the results from flood routing in the rivers [14]. The governing equation for computing flood discharge (Q) is described as follows:

$$\frac{\partial Q}{\partial t} = -C\,\frac{\partial Q}{\partial x} + D\,\frac{\partial^2 Q}{\partial x^2} \tag{1}$$

where, C is wave celerity, D is diffusion coefficient, t is time and x is flow direction.

Equation (1) is non-dimensionalized using non-dimensional variables: Q*, t* and x* in the following form (deleting * for brevity, for details [14]).

$$\frac{\partial Q}{\partial t} = -\frac{\partial Q}{\partial x} + \frac{\partial^2 Q}{\partial x^2} \tag{2}$$

In the development of finite element model, two-steps Lax-Wendroff predictor-corrector technique has been employed. In this model, semi-implicit Taylor-Galerkin scheme has been

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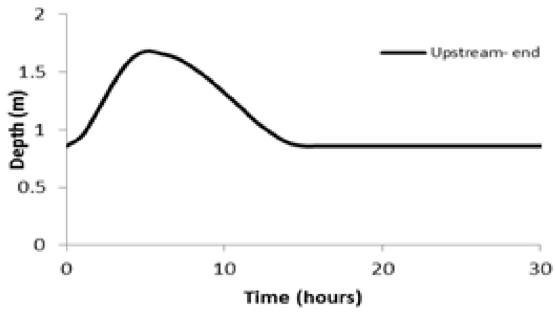
used. The developed model was validated with predicted hydrographs with and without lateral inflow and outflow of [15] which shows accuracy of the model in all three cases.

III. Results and Discussions

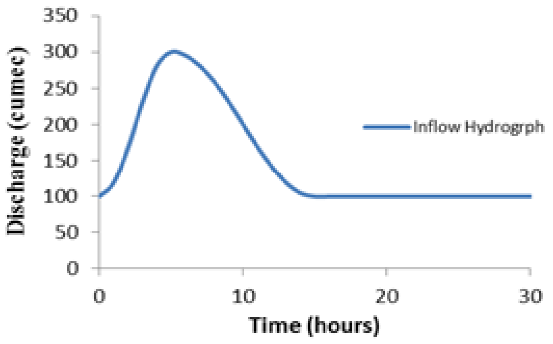
A wide rectangular river reach of 29km having average channel width (B) of 120m was selected for study to verify accuracy of the developed model. At upstream, an initial base flow 100 m^3/sec was recorded with the river reach with a bed slope (So) of 0.00061 and Manning's roughness co-efficient (n) of 0.023. The discharges and flow depths for 30 hours duration (shown in Fig. 1) have been given as time variable boundary condition at upstream of the river to the developed finite element model.

The discharge and stage hydrogrphs have been computed using the finite element model at both sites (i.e. 15km from upstream and at 29km, the downstream end) for the river reach; these hydrographs are shown in Figs. 2 and 3.

The predicted peak flow discharges (q = Q/B) obtained from the developed model at both sites (15km and 29km downstream) have been tabulated in Table I for comparison with peak discharges of the Preissmann and HEC-RAS models.



(a) Depth Inflow Hydrograph at upstream end



(b) Discharge Inflow Hydrograph at upstream end

Fig. 1 Stage & discharge hydrographs at upstream of river reach [5]

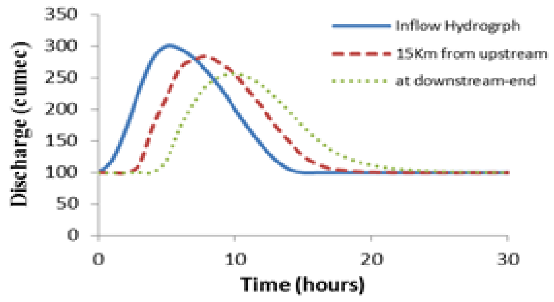


Fig. 2 Simulated discharge hydrogrphs using FEM model

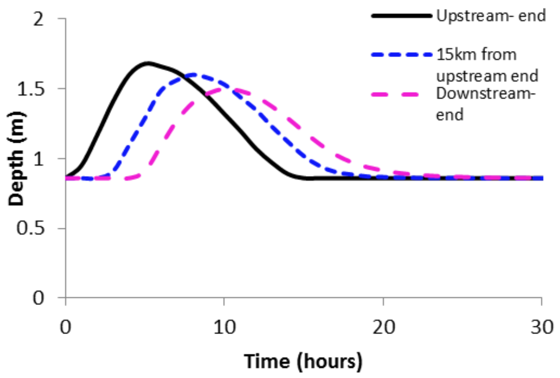


Fig. 3 Stage hydrographs predicted using FEM model

TABLE I
Comparison of Predicted Peak Discharges (M^3/sec per m width) Using Finite Element Model with Preissmann Finite Difference Scheme and HEC-RAS Model

Discharge (q = Q/B)	Preissmann finite difference scheme	HEC-RAS model	Presented finite element model
5 km from upstream	2.47	2.03	2.36
29 km from upstream	2.44	1.88	2.13

The above Table I demonstrates that the discharge predicted by the presented model have close agreement with those of Preissmann at both sites 15 and 29 km from upstream (see Fig. 4). The stages (flow depth hydrograph) of FEM show a small variation in peaks between the presented model and that of Preissmann technique/scheme.

On the contrary, the hydrograph computed through HEC-RAS at 15km from upstream is not matching with both the hydrographs (Fig. 4). However, the peak flow computed by FEM is closed to Preissmann finite difference scheme.

The maximum flow depth/stages values at 15 km and at downstream end for all three schemes are described in Table II, which shows a big variation in maximum stage readings of HEC-RAS model against both the finite difference scheme and the finite element model. To observe detail variation stage hydrograph of FE model at 15 km from upstream has been compared with Preissmann finite difference and HEC-RAS models (Fig. 5).

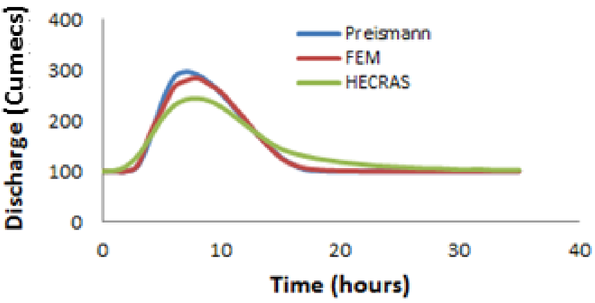


Fig. 4 Comparison of discharge hydrographs at 15 km from u/s

TABLE II
COMPARISON OF PREDICTED STAGES (M) USING FINITE ELEMENT MODEL WITH PREISSMANN FINITE DIFFERENCE SCHEME AND HEC-RAS MODEL

Stages	Preissmann finite difference scheme	HEC-RAS Model [16]	Presented finite element model
5 km from upstream	1.67	3.06	1.6
29 km from upstream	1.66	1.41	1.499

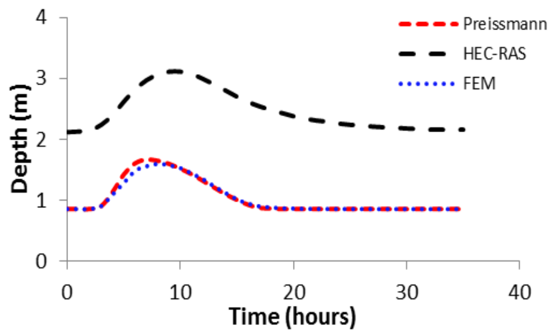


Fig. 5 Stage hydrographs of various schemes/models at 15 km

The above Fig. 5 demonstrates very close agreement of stage hydrographs of Preissmann and the presented FE model. However, HEC-RAS model shows very high depth at this section for all 35 hours durations. Hence, it is concluded that the presented finite element behaves very nice when comparing with Preissmann's finite difference scheme during verification process in a river reach.

Keeping in view above satisfactory results, the presented finite element model was tested against annual hydrograph of year 2009 of discharge data collected at downstream of Indus river reach. No lateral inflow or outflow was assumed in the reach of Sukkur barrage and Kotri barrage of the Indus River as shown in Fig. 6.

Inflow Hydrograph of 2009 (shown in Fig. 7) at downstream of Sukkur barrage of Indus River is used as inlet boundary condition. The model has computed discharge at a distance of 400 km downstream i.e. at upstream of Kotri barrage.

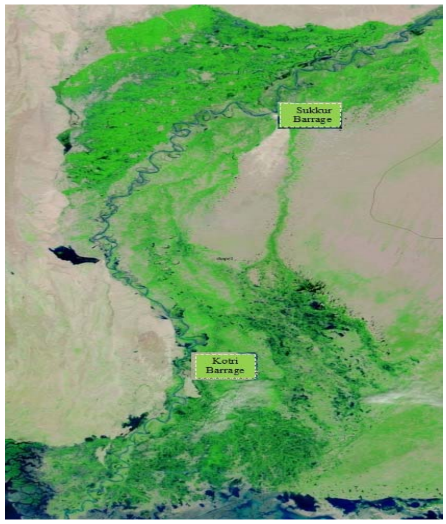


Fig. 6 Map of Indus River from Sukkur to Kotri barrage [17]

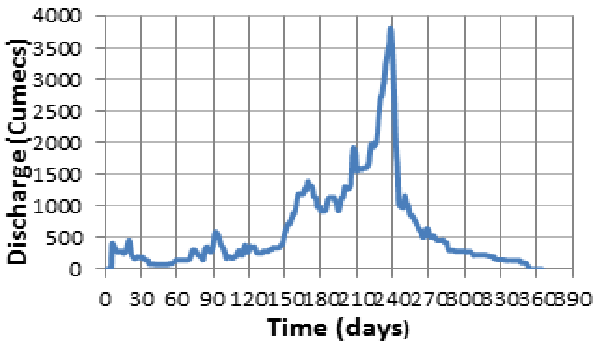


Fig. 7 Observed inflow hydrograph for 2009 at Sukkur barrage

The observed hydrograph of 2009 at Kotri barrage was collected from Sindh Irrigation and Drainage Authority (SIDA), Hyderabad and compared with computed discharge for the whole year (see Fig. 8); the comparison demonstrates a good agreement.

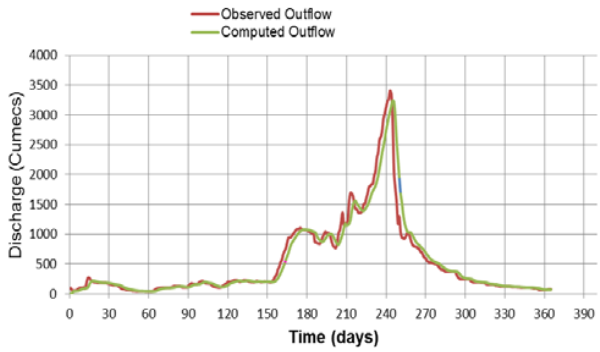


Fig. 8 Comparison between Observed and Computed discharges at upstream of Kotri Barrage)

There are four peaks; out of which the major peak which shows good matching of observed discharge (Q = 3405m^3/sec) with predicted one (Q= 3242 m^3/sec). The error between observed and simulated peak flow is about 4.77%. It was observed that the absolute relative error of observed and predicted for different months are varying; however, the average annual relative error is found about 13%.

The correlation coefficient R^2 of measured and predicted values of total volumes was calculated which comes 0.9; this reveals a good agreement between observed and simulated results. The observed and computed lag times are also calculated for the peak flows. The observed minimum lag time is three days. For the major peak flow, the observed lag time is 6 days, whereas the computed lag time comes 8 days. Meanwhile, the average lag time is calculated which is seven to eight days.

IV. Conclusions

The predicted discharge hydrographs using the proposed Finite Element model have been compared with those of Preissmann finite difference scheme and HEC-RAS model for both sites of river reach (15km from upstream and at 29km downstream end). These comparisons show a good matching among all three schemes. When comparing depth of flow computed from these schemes/models, stage hydrograph of the FE model having a good match with that of the Preissemann scheme; however, the output of HEC-RAS model does not show a fine matching with FE model and Preissemann scheme. This may be due to the fact that in HEC-RAS model, momentum equation is not used. Depth-values for 15 km site computed using HEC-RAS model are higher than those of present Finite element model and Preissmann scheme. Results also show that the arrival time of peak flow in the present model is 1 hour earlier than the HEC-RAS model at 15km after upstream, and it is 2 hours for the downstream-end.

The proposed developed model has been applied for the prediction of flood forecasting at Sukkur-Kotri barrage reach of the Indus River. The observed and computed outflow hydrographs at upstream of Kotri barrage provides satisfactory resolution of the model under field conditions. This model is capable of computing peak flow attenuation and time lag which is vital to be computed for flood forecasting. The analyzed results of model demonstrates that the model is accurate in computing and forecasting hourly to daily basis discharges which can be utilized in issuing flood warnings about flood hazardous in advance.

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References

[1] Yu, P.S. and Tseng, T.Y., "A model to forecast flow with uncertainty analysis", *Hydrological Sciences Journal,* vol. 41, no. 3, 1996, pp. 327–344.
[2] Habib Abida, Manel Ellouze and Med R. Mahjoub, "Flood routing of regulated flows in Medjerda River, Tunisia", *Journal Hydro-informatics,* vol. 7, 2005, pp. 209-216.
[3] Doiphode Sanjay, L. and Oak Ravindra, A., "Dynamic Flood Routing and Unsteady Flow Modeling: A Case Study of Upper Krishna River" *International Journal of Advance Engineering Technology IJAET,* vol. 3, no. 3, March, 2012, pp 55-59.
[4] Stoker, J. J. "Numerical solution of flood prediction and river regulation problems", Rep. IMM-200. Inst. Math. Sco., New York, 1953.
[5] Preissmann, A., "Propagation of translatory waves in channels and rivers." *Proc. 1st Congress of French Association for Computation, Grenoble, France,* 1961, pp. 433-442 (in French).
[6] Cunge J.A., Holly F.M. Jr, and Verwey A., *Practical Aspects of Computational River Hydraulics,* Pitman, London 1980.
[7] Mahmood, K., and Yevdjevich, V. *Unsteady flow in Open Channels*; Vols: I and II. Water Resource. Publ., Fort Collins, Colorado 1975.
[8] Vijay P. Singh, and Panagiotis D. Scarlatos, "Analysis of Nonlinear Muskingum Flood Routing", *Journal of Hydraulic Engineering,* vol. 113, no. 1, January 1987, pp. 61-79.
[9] Scarlatos, P.D., "Pure Finite Element Method for Saint-Venant Equations", J. *Coastal Engineering*, vol. 6, June 1982, pp. 27-45.
[10] Taylor, C., and Davis, J., "Finite Element Numerical Modeling of Flow and Dispersion in Estuaries", *IAHR Proceedings International Symposium on River Mechanics*, Paper C39, Thailand.
[11] Qureshi, A. L., and Baloch, A., "Finite Element Simulation of Sediment Transport: Development, Validation of Application of Model to Photo minor of Jamrao West Branch, Sindh, Pakistan", *In Advances in Sediment Transport Research*, edited by Fukuoka et al. CRC Press book, Taylor and Francis Group, London, UK, September, 2013.
[12] Qureshi, A. L., and Baloch, A., and Qureshi, Y. G., "Finite Element Modeling for the Prediction of Salt Concentration: Application to River Indus in the Vicinity of Upstream of Kotri Barrage", *Proceeding 4th International Bhurbhan Conference on Applied Science and Technology (IBCAST),* organized by National Centre of Physics, Quaid-i-Azam University, Islamabad, Pakistan, July, 2005.
[13] Moussa, R., and Bocquillon, C. (1996) "Criteria for the choice of flood routing methods in natural channels." *J. Hydrol.*, vol. 186, 1996, pp. 1-30.
[14] Mahessar A.A, Qureshi, A. L. and Baloch, A., "Numerical Study on Flood Routing Indus River", *International Water Technology Journal,* vol. 3, no. 1, March 2013, pp.3-12.
[15] Mahessar A. A., Baloch A. and Qureshi A. L., "Solution of Diffusive Wave Equation using FEM for Flood Forecasting", *Mehran University Research Journal of Engineering & Technology,* Jamshoro, Pakistan, submitted for publication.
[16] Moghaddam and Firoozi, "Development of Dynamic Flood Wave Routing in Natural Rivers through Implicit Numerical Method" *American Journal of Scientific Research*, vol. 14, 2011, pp.6-17.
[17] The Indus River http://en.wikipedia.org/wiki/Indus River.