Fuzzy Logic Based Active Vibration Control of Piezoelectric Stewart Platform

Arian Bahrami, Mojtaba Tafaoli-Masoule, Mansour Nikkhah Bahrami

Abstract—This paper demonstrates the potential of applying PD-like fuzzy logic controller for active vibration control of piezoelectric Stewart platforms. Through simulation, the control authority of the piezo stack actuators for effectively damping the Stewart platform vibration can be evaluated for further implementation of the system. Each leg of the piezoelectric Stewart platform consists of a linear piezo stack actuator, a collocated velocity sensor, a collocated displacement sensor and flexible tips for the connections with the two end plates. The piezoelectric stack is modeled as a bar element and the electro-mechanical coupling property is simulated using Matlab/Simulink software. Then, the open loop and closed loop dynamic responses are performed for the system to characterize the effect of the control on the vibration of the piezoelectric Stewart platform. A significant improvement in the damping of the structure can be observed by using the PD-like fuzzy controller.

Keywords—Active vibration control, Fuzzy controller, Piezoelectric stewart platform.

I. INTRODUCTION

FUTURE space telescope will require much more improved angular resolution than the current space telescopes to explore space. This need can only be obtained by interferometric devices or much larger telescopes. In interferometric devices, the signals of several independent smaller telescopes are combined together to create the final global resolution. The cost of interferometric devices is less than a larger telescope solution. As a result, it is logical to use interferometric devices in the upcoming future. In the upcoming decades, future space interferometers consist of various independent telescopes mounted on a giant truss which will be exposed to static and dynamic disturbances such as thermal loads, gravity loads, attitude control, etc. These types of disturbances in the form of vibrations can decrease the global resolution of the system. Vibration isolation of the system can be an essential solution to the problem. One method of vibration isolation is passive vibration isolation which can be used to isolate high frequency vibration but, it has been proved to be inappropriate to isolate low frequency vibration. Furthermore, it can hardly deal with the uncertainties of the system.

Arian Bahrami is with the Department of Mechanical Engineering, University of Tehran, Tehran, Iran (corresponding author to provide phone: +98 21 88793257; fax: +98 218807 9656; e-mail: arianbahrami@ut.ac.ir).

Mojtaba Tafaoli-Masoule is with the Department of Mechanical Engineering, Babol University of Technology, Babol, Iran (e-mail: mojtaba.tafaoli@yahoo.com).

Mansour Nikkhah Bahrami is with the Department of Mechanical Engineering, University of Tehran, Tehran, Iran (e-mail: mbahrami@ut.ac.ir).

The other method of vibration isolation is active vibration isolation techniques with the feedback control which can overcome the drawbacks of passive vibration isolation. Active vibration isolation can improve the vibration isolation performance in the low frequency vibration ranges but it needs actuators, sensors, and processors. With the development of smart actuators and sensors, active vibration isolation is becoming an attractive solution to vibration isolation problems. As mentioned before, it is crucial to set up a vibration-free environment for the space structures. This is where parallel robots such as Stewart platform appear as a perfect candidate for vibration isolation problems. A Stewart platform manipulator is a six DOF parallel mechanism consisting of a fixed plate and a moving plate, joined together by six legs [1]. Such a mechanism has high positioning accuracy and high force-to-weight ratio compared with conventional serial mechanisms. It can be applied as active mount for quiet components, isolation mount for a disturbance source, and active structural element of trusses for vibration control. According to the stiffness of the legs, two main categories exist for Stewart platforms; stiff platform and soft platform [2]. In the stiff platform, piezoelectric or magnetostrictive actuators are used, while in the soft platform, each leg consists of a voice coil actuator which can provide far more actuation stroke than stiff design ($1000\mu m$ or more). The focus of the paper is on the stiff piezoelectric Stewart platform. The piezoelectric Stewart platform can be used as a precision pointing device, a vibration isolator, and an active damping interface. Several researches have reported the design and manufacturing of the piezoelectric Stewart platforms [2]-[6]. Although a great number of control approaches have been proposed in recent years for the Stewart platforms such as robust PD controller [7], sliding mode control [8], adaptive control [9], etc., only the integral force feedback controller [2]-[5], and adaptive controller [6], have been applied for the active vibration control of the piezoelectric Stewart platform.

In the past decade, fuzzy theory has been used in many engineering applications. A rapid growth in the use of fuzzy logic in a wide variety of consumer products and industrial systems can be seen. Fuzzy logic was first introduced by Zadehto provide a possible mathematical representation of vagueness and approximation in a continuous fashion [10]. As an alternative to the classical control theory, fuzzy control deals with the uncertainty factors and does not need a precise mathematical model. Moreover, it has been applied in the field of active vibration control. Takawa et al. [11] developed a fuzzy controller for vibration suppression of a composite beam

which was based on the fuzzy model by using modern control theory. Casciati et al. [12] employed a fuzzy chip for nonlinear vibration control to solve the problem of slow reaction time for a software implemented controller. Also, Yoshimura et al. [13] proposed an active suspension system for passenger cars by using linear and fuzzy logic controllers. Furthermore, a fuzzy PID controller for vibration control of flexible structures was developed by Shen et al. [14].

Fuzzy control can also be used to suppress the vibrations of systems with uncertainties. For example, a fuzzy model reference learning controller was proposed by Mayhan and Washington to dampen the fundamental vibration mode of the cantilever beam system with piezoceramic actuators and sensors [15]. Also, Zeinoun and Khorrami [16] applied a fuzzy adaptive controller for active vibration suppression on a clamp-free beam instrumented with piezoceramic sensors and actuators.

To the best knowledge of the authors, there has been only one attempt to apply fuzzy controller for the active vibration control of the piezoelectric Stewart platform. In 2013, Bahrami et al. [17] applied the fuzzy force feedback controller for active vibration control of the piezoelectric Stewart platform. They demonstrated that the fuzzy integral force feedback controller can be used to dampen the vibration of piezoelectric Stewart platform.

In this paper, a PD-like fuzzy controller will be used to introduce much more damping in the mechanical system compared with the work of Bahrami et al. [17]. The active interface consists of a six-degree of freedom Stewart platform, a standard hexapod with a cubic architecture. Each leg of the active interface includes a linear piezoelectric actuator, a collocated velocity sensor, a collocated displacement sensor, and flexible tips for the connection with the two end plates. The proposed control architecture is based on six local/decentralized PD-like fuzzy logic controllers.

II. MODELING

The proposed piezoelectric Stewart platform is based on the cubic configuration which was invented by the Intelligent Automation Inc (IAI) [9]. The cubic configuration has several characteristics such as: uniform stiffness, uniform control capability in all directions and concise kinematics and dynamic analysis. The nominal cubic configuration can be obtained by cutting a cube by two planes as shown in Fig. 1. The two triangular planes are the base and the mobile platforms of the Stewart platform. The six legs of the hexapod are the edges of the cube connecting the two plates. Vibration isolation using the cubic configuration of Stewart platform has been studied by Geng and Haynes [9], Spanos et al. [18], and Thayer et al. [19]. The active legs of the Stewart platform consist of a piezo stack actuator, a collocated velocity sensor, and a collocated displacement sensor. A voltage is generated which is based on the measured velocity of the velocity sensor and the measured displacement of the displacement sensor. Then, this signal is applied to produce the control signal according to an appropriate control algorithm. Finally, the control signal is fed to a high voltage amplifier which drives

the actuator. The mentioned algorithm will be implemented in Matlab/Simulink in the next section. When a voltage V is applied to a linear piezoelectric actuator, it creates an expansion as δ:

$$\delta = d_{33}nV = g_aV \qquad (1)$$

where d_{33} is the piezoelectric coefficient, n is the number of piezoelectric ceramic layers in the actuator and g_a is the actuator gain. The effect of actuator on the Stewart platform can be represented by equivalent piezoelectric loads acting on the structure. The piezoelectric load which is applied axially to both ends of the active strut is equal to:

$$f = k\delta \qquad (2)$$

where k is the axial stiffness of the active strut and δ is the unconstrained piezoelectric expansion. Due to slight displacements of the piezo actuators, the nonlinear terms such as centripetal and coriolis forces can be neglected. Therefore, the motion equation of the Stewart platform without these terms and damping excited by six actuators is:

$$M\ddot{X} + KX = F + Bf = F + Bk\delta \qquad (3)$$

where M is the inertia matrix of the Stewart platform, K is the stiffness matrix and the vector F is the disturbance forces and moments acting on the payload platform $F = [F_x \quad F_y \quad F_z \quad M_x \quad M_y \quad M_z]^T$. $X=[x\ y\ z\ \psi\ \theta\ \varphi]^T$ is a vector containing the translational displacements x, y, z, and rotational displacements ψ, θ, φ of the end-effector center about the fixed axes of x, y, z respectively, B is the influence matrix of the active struts in the fixed coordinate system known as force Jacobian matrix, $f =[f_1 ,…,f_6]^T$ is the vector of actuator forces defined by (2), and $\delta = (\delta_1, …, \delta_6)^T$ is the vector of the six unconstraint displacements of the piezoelectric actuators. The inertia matrix of the Stewart platform can be expressed by:

$$M = \begin{bmatrix} m\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} & 0_{3\times3} \\ 0_{3\times3} & P^T\begin{bmatrix} I_x & 0 & 0 \\ 0 & I_y & 0 \\ 0 & 0 & I_z \end{bmatrix}P \end{bmatrix}$$

$$P = \begin{bmatrix} 1 & 0 & -s\theta \\ 0 & c\psi & s\psi c\theta \\ 0 & -s\psi & c\psi c\theta \end{bmatrix} \qquad (4)$$

where m is the mass of the end-effector; I_x, I_y, I_z are the moments of inertia of the moving platform expressed in the moving coordinate $\{P\}$. The end-effector is deflected away from its desired position in the presence of the external forces.

The overall stiffness matrix K_u of the Stewart platform can be defined as:

$$K_u = B \times diag(k_1, k_2, k_3, k_4, k_5, k_6) \times B^T \qquad (5)$$

where

$$k_i = \frac{A_i E_i}{l_i} \quad i = 1, 2, \dots, 6 \qquad (6)$$

In which E_i is the elasticity module of piezo, A_i is the cross section area of piezo stack and l_i is the nominal length of each leg. This matrix is always positive semi-definite and symmetric and because of its dependence on the Jacobian, it is also dependent on the posture of the end-effector. For obtaining the Jacobian matrix J and the stiffness matrix K_u, let us consider the vectorial representation of the hexapod as shown in Fig. 2. The cubic configuration of Stewart platform has a fixed platform, a moving platform andsix legseach connected to a connection point on the end-effector A_i and one of the connection points on the fixed triangular platform B_i, as shown in Fig. 2. {B} is considered as the inertial reference frame of the fixed platform which coincides with the mass center of the base platform and {P} is considered as the reference frame at the mass center C of the moving platform. r_{base} is the radius distance from the origin of {B} to the connection points on the base platform B_i and r_{end} is the radius distance from the origin of {P} to the connection points on the moving platform A_i. This expression can be obtained from Fig. 2:

$$q_i = x_0 + [R]^M p_i - r_i \quad i = 1, 2, 3, \dots, 6 \qquad (7)$$

where r_i is the position vector of the connection points on the base platform B_i expressed in {B}, p_i is the position vector of the connection points on the moving platform A_i expressed in {P}, x_0 is the position vector of point C, q_i is the cable length vector from B_i to A_i and R is the rotation matrix of the moving platform with respect to base platform with three rotation angles ψ, θ, φ about the fixed axes of {B} respectively and can be defined as:

$$[R] = \begin{bmatrix} c\phi c\theta & -s\phi c\psi + c\phi s\theta s\psi & s\phi s\psi + c\phi s\theta c\psi \\ s\phi c\theta & c\phi c\psi + s\phi s\theta s\psi & -c\phi s\psi + s\phi s\theta c\psi \\ -s\theta & c\theta s\psi & c\theta c\psi \end{bmatrix} \qquad (8)$$

The cable length of the leg can be defined as:

$$l_i = |q_i| = (q_i^T q_i)^{1/2} \qquad (9)$$

The influence matrix B known as the force Jacobian matrix can be obtained from the virtual work and is written in such way that its column is:

$$B_i = \begin{bmatrix} \dfrac{q_i}{|q_i|} \\ {}^B a_i \times \dfrac{q_i}{|q_i|} \end{bmatrix}, i = 1, 2, \dots, 6 \qquad (10)$$

where:

$$^B a_i = [R] p_i \qquad (11)$$

As mentioned before, in each leg of the hexapod, there is a velocity sensor and a displacement sensor collocated with an actuator. We know that the relationship between the leg extension velocity $\dot{y}$ and the payload frame velocity $\dot{X}$ can be expressed as $\dot{y} = J\dot{X} = B^T \dot{X}$ where J and B are the velocity and force Jacobian matrices respectively, defined by (10). Therefore, the velocity sensor output equation is:

$$\dot{y} = J\dot{X} = B^T \dot{X} \qquad (12)$$

where $\dot{y} = (\dot{y}_1, \dots, \dot{y}_6)^T$ is the six velocity sensor outputs, $\dot{X} = (\dot{x}, \dot{y}, \dot{z}, \dot{\psi}, \dot{\theta}, \dot{\varphi})^T$ is the velocity vector of payload frame. Also, the displacement sensor output equation can be written as:
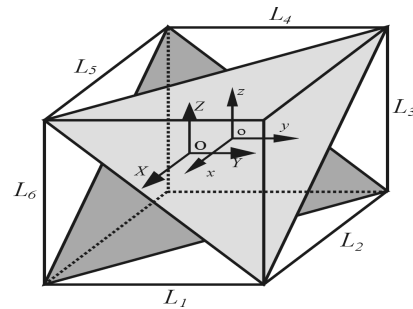
$$y = JX = B^T X \qquad (13)$$

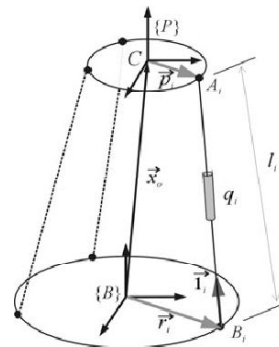Fig. 1 Cubic configuration of Stewart platform

Fig. 2 Vectorial representation of the Stewart platform

III. Fuzzy Controller Design

In a fuzzy-logic controller, the dynamic behavior of a fuzzy system is determined by a set of linguistic description rules based on expert knowledge usually of the form: IF (a set of conditions are satisfied) THEN (a set of consequence can be inferred). In order to simulate the PD-like fuzzy controller for the system, the state space of the dynamic equation of the structure (3), the sensor equations (12), (13) and the fuzzy control law (Table I) are employed in Matlab/Simulink to obtain the closed loop system for piezoelectric Stewart platform. The block diagram of the closed loop system is shown in Fig. 3. For implementation, the fuzzy controller is made of six independent sub-fuzzy controllers. The sub-fuzzy controller of each leg contains two inputs which are the actuator position error $e_i = y_d - y_i$ and the change of the position error $\dot{e}_i = \dot{y}_d - \dot{y}_i$ and one output which is the actuator force f_i. Where y_d and $\dot{y}_d$ are the desired error and desired change of the error respectively, supposed to be zero if the vibration of the system needs to be suppressed. The inputs to the controller system e_i and $\dot{e}_i$ are actual values in form of "crisp" numbers. Fuzzification converts the numerical value into a linguistic variable which can be understood by the fuzzy control system. Singleton fuzzification, which transforms a crisp value into a fuzzy singleton value, is selected in this fuzzy control scheme. For all the twelve inputs $\dot{e}_i$, e_i and six outputs f_i, seven triangular membership functions are defined over the range of input and output space as shown in Figs. 4, 5, and 6, due to their ease in real-time hardware implementation [20]. If the number of membership functions is chosen too big it will cause unnecessary computation, and if the number of membership functions is chosen too small it will result in inaccuracy. In order to cover the range of input and output variables with the proper overlap, membership functions are defined to be symmetric, equi-spaced with an equal area defined as negative big (NB), negative medium (NM), negative small (NS), zero (ZO), positive small (PS), positive medium (PM) and positive big (PB). If the overlap between different membership functions is too large, some values in the input domain may not have an effect on the output. If the overlap between different membership functions is too small, the corresponding linguistic variables will be difficult to differentiate. Also, 49 control rules are constructed for each sub-fuzzy controller as shown in Table I. Therefore, the PD-like fuzzy controller has a total of 294 control rules. An example of a rule thinking process of Table I can be explained as follows

"If $\dot{e}_i$ is negative big (NB) and e_i is negative big (NB), then fi is negative big (NB)."

To obtain the best possible conclusion, the max–min (Mamdani type) inference is employed. This type of inference is computationally easy and effective; thus it is appropriate for real-time control applications. The crisp control command fi is computed here using the center-of-gravity (COG) defuzzification [21]. The model of the piezoelectric Stewart platform is shown in Fig. 7.
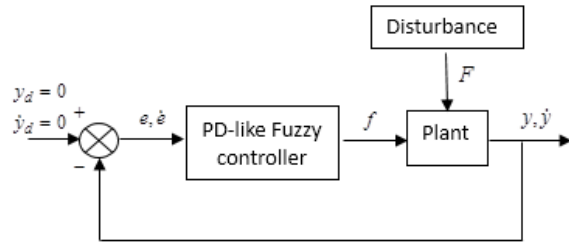

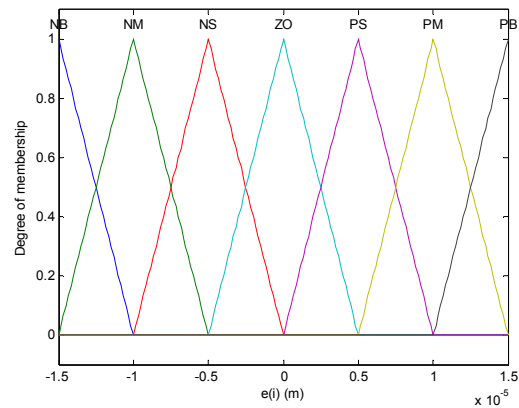Fig. 3 Block diagram of the closed loop system

Fig. 4 Membership functions for input error of six sub-fuzzy controllers
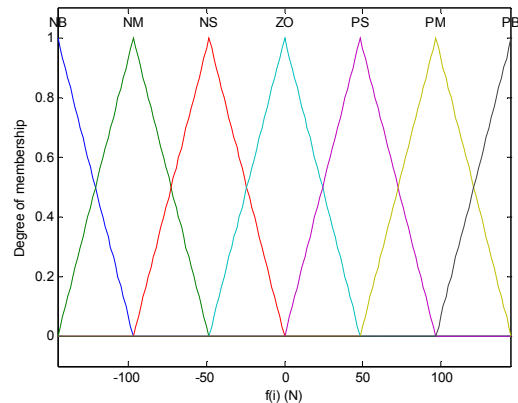

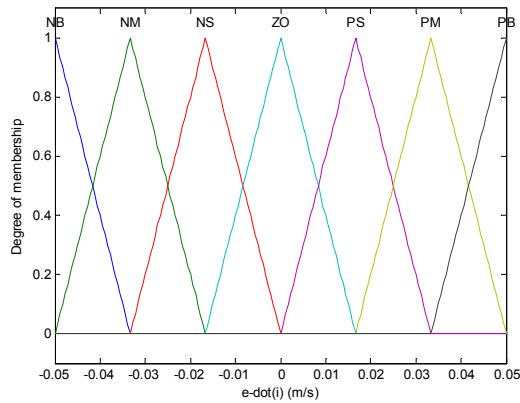Fig. 5 Membership functions for input changes of the errorof six sub-fuzzy controllers

Fig. 6 Membership functions for output of six sub-fuzzy controllers

TABLE I
RULES FOR SIX SUBFUZZY CONTROLLERS

$\dot{e}_i$ \ e_i	NB	NM	NS	ZO	PS	PM	PB
NB	NB	NB	NB	NB	NM	NS	ZO
NM	NB	NB	NB	NM	NS	ZO	PS
NS	NB	NB	NM	NS	ZO	PS	PM
ZO	NB	NM	NS	ZO	PS	PM	PB
PS	NM	NS	ZO	PS	PM	PB	PB
PM	NS	ZO	PS	PM	PB	PB	PB
PB	ZO	PS	PM	PB	PB	PB	PB

IV. SIMULATION

In order to simulate the piezoelectric Stewart platform, the state space of the dynamic equation of the structure (3), the sensor equations (12), (13) and the fuzzy control law (Table I) are applied in Matlab/Simulink to create the closed loop system. In order to compare the results of the proposed PD-like fuzzy controller with the fuzzy force feedback controller [17], the same condition is applied. For simulating the system, it is assumed that three forces and three moments are applied simultaneously at the center of the end-effector (top platform). The six white noise disturbance forces and moments are considered to be Gaussian distributed random signals with the mean value of zero and the variance value of 25 N^2 and 25 $(N.m)^2$ for the forces and moments respectively. By defining a constraint on the stack actuator voltage to be within 0-20 V, the system responses without and with control for white noise disturbances have been obtained. These assumptions are considered as shown in Tables II, III for the simulation. The model parameters of the piezoelectric Stewart platform are given in Table II. Table III shows the specification of the piezo stack actuators. As shown in Fig. 8, the PD-like fuzzy controller made a significant improvement in the damping of the structure $X = [x \ y \ z \ \psi \ \theta \ \varphi]^T$. By comparing the results of the proposed PD-like fuzzy controller (Fig. 8) with the results of the fuzzy force feedback controller [17] presented in Fig. 9, it is evident that the proposed PD-like fuzzy controller has introduced much more damping in the system which is in the interests of the designers to apply the proposed PD-like fuzzy

controller for active vibration control of piezoelectric Stewart platform.
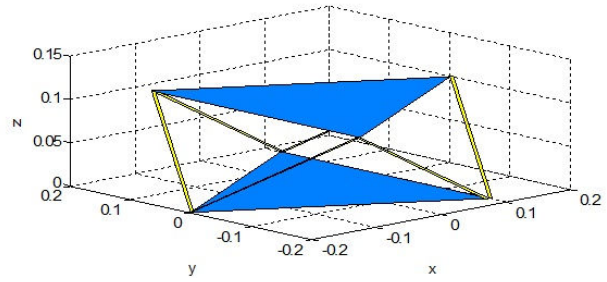

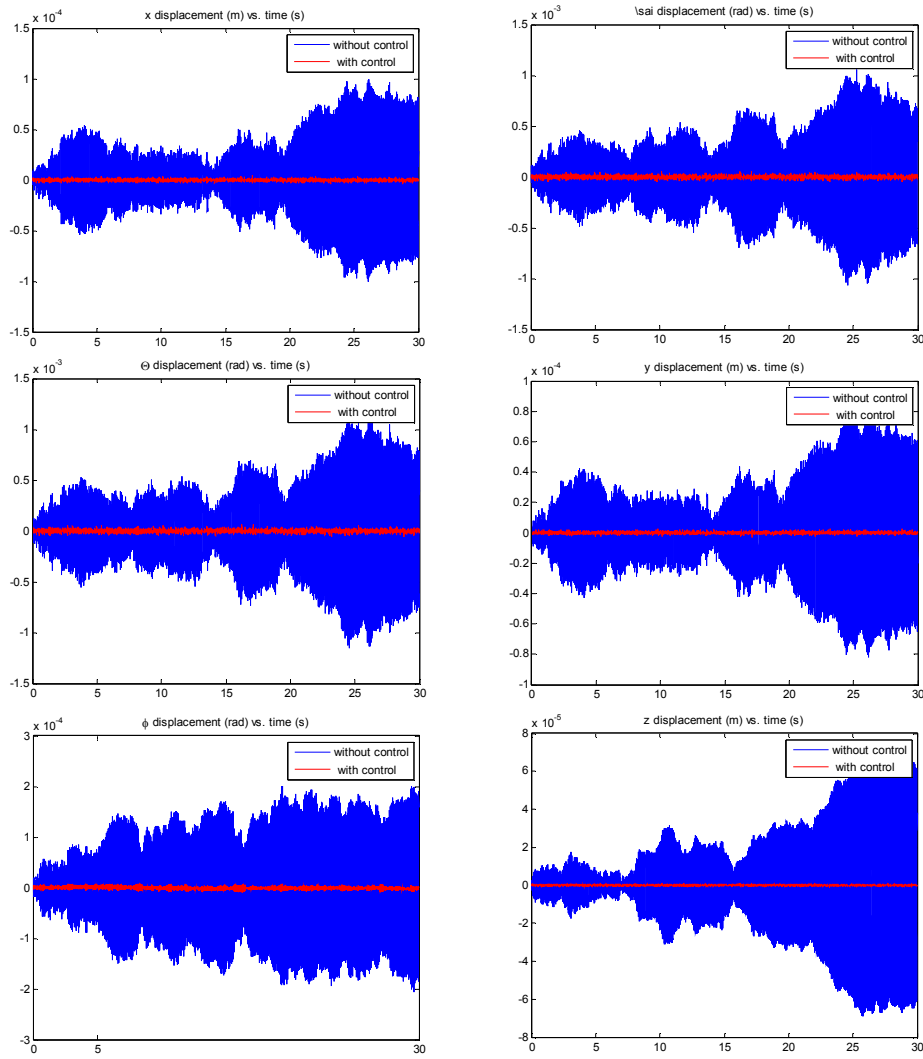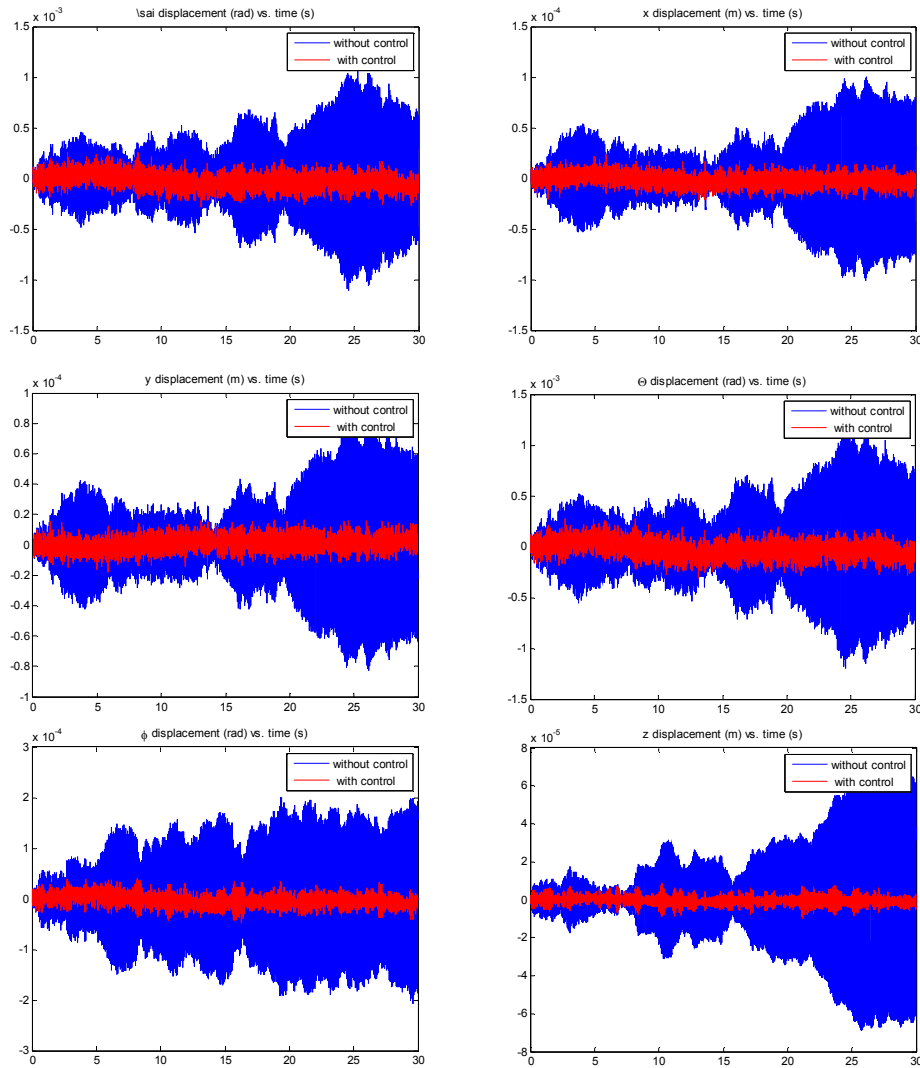Fig. 7 Model of piezoelectric Stewart Platform in Matlab/Simulink

Fig. 8 Translational and rotational displacements (x, y, z, ψ, θ, φ)

Fig. 9 Translational and rotational displacements (x, y, z, ψ, θ, φ) [17]

TABLE II
SPECIFICATION OF STEWART PLATFORM

PROPERTY	SYMBOL	VALUE
Mass of end effector (kg)	m	1
R end-effector (m)	r_{end}	0.2
R base(m)	r_{base}	0.2
Moments of inertia (kg.m^2)	I_x	0.005
Moments of inertia (kg.m^2)	I_y	0.005
Moments of inertia (kg.m^2)	I_z	0.01
Length of each leg (m)	l	0.24

TABLE III
PIEZOCERAMIC PROPERTIES OF PIEZOSTACKACTUATOR

PROPERTY	SYMBOL	VALUE
Piezo Modulus (Gpa)	E_p	70
Piezo Density (kg/m^3)	ρ_p	7×10^3
Section area (m^2)	A_p	3.1×10^{-5}
Piezo stack length (m)	L_p	0.24
Piezo strain coefficient (m/V)	d_{33}	5×10^{-10}
Thickness of layers (m)	t	1×10^{-4}

V. CONCLUSION

The focus of the study is to evaluate the control authority of the piezo stack actuators for effectively damping the Stewart platform vibration. First, the dynamic equations of the piezoelectric Stewart platform and the six piezo stack actuators with their corresponding velocity and displacement sensors are modeled in Matlab/Simulink software. Then, six local PD-like fuzzy controllers have been used to demonstrate the effect of control on the overall response of the closed loop control to white noise disturbance forces, with the constraint on the stack actuator voltage to be within a specified bound. Using the proposed PD-like fuzzy controller shows much more improvement in the damping of Stewart platform vibration compared with the work of Bahrami et al. [17].

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