

# Cantor Interpolating Spline to Design Electronic Mail Boxes

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**Abstract**—Electronic mail is very important in present time. Many researchers work for designing, improving, securing, fastening, goodness and others fields in electronic mail. This paper introduced new algorithm to use Cantor sets and cubic spline interpolating function in the electronic mail design. Cantor sets used as the area (or domain) of the mail, while spline function used for designing formula. The roots of spline function versus Cantor sets used as the controller admin. The roots calculated by the numerical Newton – Raphson's method. The result of this algorithm was promised.

**Keywords**—Cantor sets, spline, electronic mail design, Newton – Raphson's method.

## I. INTRODUCTION

ELECTRONIC mail is very important in present time. Many researchers work for designing, benefit, improving, securing, fastening, good usage and others fields in electronic mail. For instance, Cheung addresses the impact of electronic direct mail on the design of the messages using Chi square [1]. Bothma and Hoffmann studied paper war and electronic mail abuse [2]. Bahreman proposed two families of protocols to certify electronic mail with enabling to exchange a receipt [3]. Hui examines the research and development of a prototype object-based multimedia electronic mail system [4]. This paper was concerned for designing new proposed electronic mail. Three ideas were composed in the design, Cantor sets, spline, and Newton – Raphson's method. Cantor sets have good topological properties represented in bounded, closure, compactness, measurable, infiniteness, and countable [5]. So it used as the area (or domain) of the design. For the smoothness of numerical spline method [6], it is used as a functioning or controlling the design. For fast time and less error, Newton – Raphson's method is used for calculating the roots of spline function. The initial point used as user name while the root used as password. Two procedures were introduced, the one, named as (send message) used for putting the message in box mail while the second, named as (open mail) used for owner mail.

## II. CUBIC SPLINE INTERPOLATING FUNCTIONS

When a function  $f$  defined on interval  $[x_0, x_N]$  and a set of nodes  $\{x_0, x_1, \dots, x_N\}$  such that  $a = x_0 < x_1 < \dots < x_N = b$ .

A cubic spline interpolating  $S$  for  $f$  is a function that satisfies the following conditions:

1.  $s(x)$  is a cubic polynomial, denoted  $s_j(x)$ , on the subinterval  $[x_j, x_{j+1}]$  for each  $j=0, 1, 2, \dots, N-1$ .
2.  $s(x_j) = f(x_j)$  for each  $j=0, 1, N$ .
3.  $s_{j+1}(x_{j+1}) = s_j(x_{j+1})$  for each  $j=0, 1, N-2$ .
4.  $s'_{j+1}(x_{j+1}) = s'_j(x_{j+1})$  for each  $j=0, 1, N-2$ .
5.  $s''_{j+1}(x_{j+1}) = s''_j(x_{j+1})$  for each  $j=0, 1, N-2$ .
6. One of the following set of boundary conditions is satisfied

$s''(x_0) = s''(x_N) = 0$  for free or natural boundary and

$s'(x_0) = f'(x_0), s'(x_N) = f'(x_N)$  for clamped boundary

Remark 1: To construct the cubic spline interpolating  $S$  for the function  $f$  which defined on the values [5]:

$$a = x_0 < x_1 < \dots < x_N = b$$

Satisfying

$$s''(x_0) = s''(x_N) = 0 :$$

$$s(x) = s_j(x) = a_j + b_j(x - x_j) + c_j(x - x_j)^2 + d_j(x - x_j)^3$$

for  $x_j \leq x \leq x_{j+1}$  :

Step 1. for  $i=0, 1, n-1$  set  $h_i = x_{i+1} - x_i$

Step 2. for  $i=0, 1, n-1$  set  $\alpha_i = \frac{3}{h_i}(a_{i+1} - a_i) - \frac{3}{h_{i-1}}(a_i - a_{i-1})$

Step 3. Set  $L_0 = 1, \mu_0 = 0, z_0 = 0$

Step 4. FOR  $i=1, 2, N-1$ , set

$$L_i = 2(x_{i+1} - x_{i-1}) - h_{i-1}\mu_{i-1}$$

$$\mu_i = \frac{h_i}{L_i}$$

$$z_i = \frac{(\alpha_i - h_{i-1}z_{i-1})}{L_i}$$

Step 5. set  $L_n = 1, z_n = 0, c_n = 0$

Step 6. FOR  $j=N-1, N-2, 0$ . Set

$$c_j = z_j - \mu_j c_{j+1}$$

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$$b_j = \frac{a_{j+1} - a_j}{h_j} - \frac{h_j(c_{j+1} + 2c_j)}{3}$$

$$d_j = \frac{c_{j+1} - c_j}{3h_j}$$

Step 7. Output  $a_j, b_j, c_j, d_j$  for  $j=0, 1, n-1$ .

The error in cubic spline interpolation method is  $R_N(x) \leq c_i h^{4-i}$ ,  $i=0,1,2,3$  for arbitrary constants  $c_i$ .

### III. CONSTRUCTION OF CANTOR SETS

The Cantor set is a subset of the real line.

Let  $C_0 = [0,1]$ . Remove the middle third, and get

$$C_1 = \left[0, \frac{1}{3}\right] \cup \left[\frac{2}{3}, 1\right] \quad (1)$$

It is a closed interval, it keeps endpoints. Continue in this the processing. Next is

$$C_2 = \left[0, \frac{1}{9}\right] \cup \left[\frac{2}{9}, \frac{1}{3}\right] \cup \left[\frac{2}{3}, \frac{7}{9}\right] \cup \left[\frac{8}{9}, 1\right] \quad (2)$$

In general,  $C_n$  is the union of  $2^n$  closed intervals, each of size  $3^{-n}$ . Clearly

$$C_0 \supset C_1 \supset C_2 \supset \dots \quad (3)$$

Definition 1, [6]: The Cantor set  $C$  is defined by

$$C = C_0 \cap C_1 \cap C_2 \cap \dots \quad (4)$$

### IV. NUMERICAL NEWTON-RAPHSON'S METHOD

Newton-Raphson's Method can be used for finding the roots  $p$  of given differentiable function

$$f(x): R \rightarrow R,$$

as follow:

Step 1. Set  $i=1$

Step 2. While  $i \leq N$  Do steps 3-6

Step 3. Set

$$p = p_0 - \frac{f(p_0)}{f'(p_0)}$$

Step 4. if  $|p - p_0| < \text{tolerance}$  then stop

Step 5. Set  $i = i + 1$

Step 6. Set  $p_0 = p$  Do step 3.

where  $p_0$  represent the initial value.

### V. PROPOSED DESIGN OF ELECTRONIC MAIL

The Electronic Mail Designer focuses on two terms, the (ID) and the password. It is clear the (ID) is public while the password should be top secret. So suitable mathematics must be used carefully for issue sub-domain for each mail which is not related with other mails and it is not possible to insert other domain in the sub-domains series.

Procedure 1 (Cantor spline design mail):

First: Take any stage of cantor set  $C_n$ .

Second: Choosing discrete mesh points set

$$M = \{(x, y) : 0 < x < 1, -1 < y < 1\}.$$

Third: using spline method for interpolation  $M$  such that each subset of  $C_n$  has an initial point  $p_0$ , and a unique root  $p$  which are both controlled by the spline function  $f$ .

Fourth: For calculating  $p$ , one can use the rule of Newton-Raphson

$$p = p_0 - \frac{f(p_0)}{f'(p_0)}$$

Example 1: For  $n=2$ , Fig. 1 is the result of procedure 1, where red and blue colors represent spline function and Cantor sets respectively. Clearly there are 8 boxes mail.

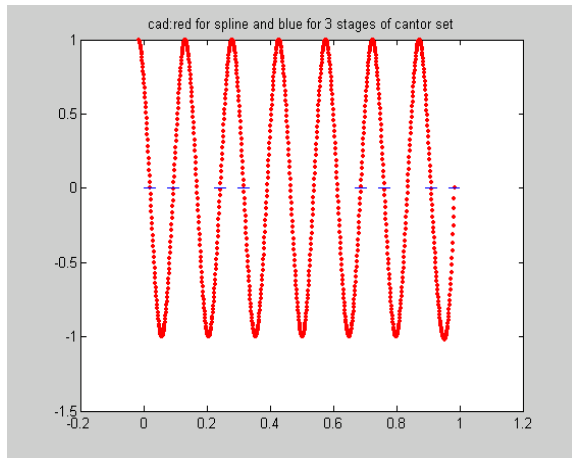


Fig. 1 (Cantor spline) red and blue colors represent spline function and Cantor sets respectively

(5) For next procedures, the user has the following mail choices:

User name: [ad@yahoo.com](mailto:ad@yahoo.com)

Password: ad1963uok

where [ad@yahoo.com](mailto:ad@yahoo.com) =  $p_0$ , ad1963uok =  $p$  and  $h = p_0 + p$ .

Procedure 2 (sending the message):

First: search for user name: [ad@yahoo.com](mailto:ad@yahoo.com).

Second:  $p_0 = \text{ad@yahoo.com}$ .

Third: calculate  $\bar{p}$  from  $\bar{p} = p_0 - \frac{f(p_0)}{f'(p_0)}$ .

Fourth: if  $|\bar{p} - p| < \alpha$  put message in box mail where  $\alpha = 10^{-6}$ .

Procedure 3 (read mail):

First: input User name ([ad@yahoo.com](mailto:ad@yahoo.com)) and password (ad1963uok).

Second: if  $p_0 + p = h$  open box mail.

## VI. CONCLUSION

A flexible procedure for electronic mail design was suggested. It is benefit and stable in computer tools and communication such as website net. For non interfacing among design points, Cantor sets were constructed. For faster calculation a mesh points were choose and interpolated by Spline function. Finally, the Newton–Raphson iterated method was used for finding the roots or design points of nonlinear equation.

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