International Journal of Engineering, Mathematical and Physical Sciences
ISSN: 2517-9934
Vol:8, No:1, 2014

A Problem in Microstretch Thermoelastic Diffusive Medium

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Abstract**—**The general solution of the equations for a homogeneous isotropic microstretch thermo elastic medium with mass diffusion for two dimensional problems is obtained due to normal and tangential forces. The Integral transform technique is used to obtain the components of displacements, microrotation, stress and mass concentration, temperature change and mass concentration. A particular case of interest is deduced from the present investigation.

Keywords**—**Normal and tangential force, Microstretch, thermoelastic, The Integral transform technique.

I. Introduction

ERINGEN [1] developed the theory of micropolar elastic solid with stretch. He derived the equations of motion, constitutive equations and boundary conditions for a class of micropolar solids which can stretch and contract. This model introduced and explained the motion of certain class of granular and composite materials in which grains and fibers are elastic along the direction of their major axis. This theory is a generalization of the theory of micropolar elasticity and is a special case of the micromorphic theory. Eringen [5] developed a theory of thermomicrostretch elastic solids in which he included microstructural expansions and contractions. The material points of microstretch solids can stretch and contract independently of their translations and rotations. Microstretch continuum is a model for Bravious Lattice with a basis on the atomic level and a two phase dipolar solid with a care on the macroscopic level. For example, composite materials reinforce with chopped elastic fibers, porous medium where pores are filled with gas or in viscid liquids, asphalt or other inclusions and 'solid-liquid' crystals etc., are characterized as microstretch solids. Thus, in these solids, the motion is characterized by seven degrees of freedom namely three for translation, three for rotation and one for microstretch. In the frame work of the theory of thermomicrostretch solids, Eringen [3] established a uniqueness theorem for the mixed initial boundary valued problem. This theory was illustrated with the solution of one-dimensional wave and compared with lattice dynamical results. The asymptotic behavior of solutions and an existence result were presented by Bofill and Quintanilla [1].Eringen [4] studied the effect of Micropolar elastic solids with stretch.

The transmission of the load across a differential element of the surface of a microstretch elastic solid is described by a force vector, a couple stress vector and a microstretch vector. The theory of microstretch elastic solid differs from the theory of micropolar elasticity in the sense that there is an additional degree of freedom called stretch and there is an additional kind of stress called microstretch vector. The materials like porous elastic materials filled with gas or in viscid fluid, asphalt, composite fibers etc. lie in the category of microstretch elastic solids. Diffusion is defined as the spontaneous movement of the particles from a high concentration region to the low concentration region, and it occurs in response to a concentration gradient expressed as the change in the concentration due to change in position. Thermal diffusion utilizes the transfer of heat across a thin liquid or gas to accomplish isotope separation. Today, thermal diffusion remains a practical process to separate isotopes of noble gases (e.g., Xenon) and other light isotopes (e.g., Carbon). Tomar and Garg [15] discussed the reflection and refraction of plane waves in microstretch elastic medium. Quintanilla [13] studied the spatial decay for the dynamical problem of thermo-microstretch elastic solids. Singh and Tomar [14] discussed Rayleigh-Lamb waves in a microstretch elastic plate cladded with liquid layers. Cicco [2] discussed the stress concentration effects in microstretch elastic bodies. A spherical inclusion in an infinite isotropic microstretch medium was discussed by Liu and Hu [12]. Kumar and Partap [7] analyzed free vibrations for Rayleigh-Lamb waves in a microstretch thermoelastic plate with two relaxation times. Kumar and Partap [6] discussed the dispersion of axisymmetric waves in thermo microstretch elastic plate. Othman [9] studied the effect of rotation on plane wave propagation in the context of Green-Naghdi (GN) theory type-II by using the normal mode analysis. Ezzat and Awad adopted the normal mode analysis technique to obtain the temperature gradient, displacement, stresses, couple stress, micro rotation etc. Othman [10] studied the effect of diffusion on 2-dimensional problem of generalized thermoelastic with Green-Naghdi theory and obtained the expressions for displacement components, stresses, temperature fields, concentration and chemical potential by using normal mode analysis. Othman [11] used normal mode analysis technique to obtain the expressions for displacement components, force, stresses, temperature, couple stress, and microstress distribution in a thermomicrostretch elastic medium with temperature dependent properties for different theories. Kumar et al. [8] investigated the disturbance due to force in normal and tangential direction and porosity effect by using

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normal mode analysis in fluid saturated porous medium. Kumar et al. [16] investigated the effect of viscosity on plane wave propagation in heat conducting transversely isotropic micropolar viscoelastic half space. Gravitational effect on plane waves in generalized thermo-microstretch elastic solid under green Naghdi theory was studied by Othman, Atwa, and Khan [12]. In the present paper general model of the equations of microstretch thermoelastic with mass diffusion for a homogeneous isotropic elastic solid is developed. The Normal mode analysis technique is used to obtain the expressions for the displacement components, couple stress, temperature, mass concentration and microstress distribution. Some special cases have been deduced from the present investigation.

II. Basic Equations

The basic equations for homogeneous, isotropic microstretch generalized thermoelastic diffusive solids in the absence of body force, body couple, stretch force and heat source are given by:

$$(\lambda+\mu)\nabla(\nabla.\boldsymbol{u})+(\mu+K)\nabla^2\boldsymbol{u}+K\nabla\times\boldsymbol{\phi}+\lambda_0\nabla\phi^*-\beta_1\left(1+\tau_1\frac{\partial}{\partial t}\right)\nabla T-\beta_2\left(1+\tau^1\frac{\partial}{\partial t}\right)\nabla C=\rho \quad (1)$$

$$(\gamma\nabla^2-2K)\boldsymbol{\phi}+(\alpha+\beta)\nabla(\nabla.\boldsymbol{\phi})+K\nabla\times\boldsymbol{u}=\rho j\ddot{\boldsymbol{\phi}} \quad (2)$$

$$(\alpha_0\nabla^2-\lambda_1)\phi^*-\lambda_0\nabla.\boldsymbol{u}+\nu_1\left(1+\tau_1\frac{\partial}{\partial t}\right)T+\nu_2\left(1+\tau^1\frac{\partial}{\partial t}\right)C=\frac{\rho j_0}{2}\ddot{\phi}^* \quad (3)$$

$$K^*\nabla^2T=\rho c^*\left(\frac{\partial}{\partial t}+\tau_0\frac{\partial^2}{\partial t^2}\right)T+\beta_1T_0\left(\frac{\partial}{\partial t}+\varepsilon\tau_0\frac{\partial^2}{\partial t^2}\right)\nabla.\boldsymbol{u}+\nu_1T_0\left(\frac{\partial}{\partial t}+\varepsilon\tau_0\frac{\partial^2}{\partial t^2}\right)\phi^*+aT_0\left(\frac{\partial}{\partial t}+\gamma_1\frac{\partial^2}{\partial t^2}\right)C \quad (4)$$

$$D\beta_2\nabla^2(\nabla.\boldsymbol{u})+Da\left(1+\tau_1\frac{\partial}{\partial t}\right)\nabla^2T+\left(\frac{\partial}{\partial t}+\varepsilon\tau^0\frac{\partial^2}{\partial t^2}\right)C-Db\left(1+\tau^1\frac{\partial}{\partial t}\right)\nabla^2C=0 \quad (5)$$

$$t_{ij}=(\lambda_0\phi^*+\lambda u_{r,r})\delta_{ij}+\mu(u_{i,j}+u_{j,i})+K(u_{j,i}-\epsilon_{ijk}\phi_k)-\beta_1\left(1+\tau_1\frac{\partial}{\partial t}\right)\delta_{ij}T-\beta_2\left(1+\tau^1\frac{\partial}{\partial t}\right)\delta_{ij}C \quad (6)$$

$$m_{ij}=\alpha\phi_{r,r}\delta_{ij}+\beta\phi_{i,j}+\gamma\phi_{j,i}+b_0\epsilon_{mji}\phi^*_{,m} \quad (7)$$

where λ, μ, α, β, γ, K, λ_0, λ_1, α_0, b_0, are material constants, ρ is mass density, $\boldsymbol{u}=(u_1,u_2,u_3)$ is the displacement vector and $\boldsymbol{\phi}=(\phi_1,\phi_2,\phi_3)$ is the microrotation vector, ϕ^* is the scalar microstretch function, T is temperature and T_0 is the reference temperature of the body chosen, C is the concentration of the diffusion material in the elastic body, K^* is the coefficient of the thermal conductivity, c^* is the specific heat at constant strain, D is the thermoelastic diffusion constant, a is the coefficient describing the measure of thermo diffusion and b is the coefficient describing the measure of mass diffusion effects, j is the microinertia, $\beta_1=(3\lambda+2\mu+K)\alpha_{t1}$, $\beta_2=(3\lambda+2\mu+K)\alpha_{c1}$, $\nu_1=(3\lambda+2\mu+K)\alpha_{t2}$, $\nu_2=(3\lambda+2\mu+K)\alpha_{c2}$, α_{t1}, α_{t2} are coefficients of linear thermal expansion and α_{c1}, α_{c2} are coefficients of linear diffusion expansion, j_0 is the microinertia for the microelements, t_{ij} are components of stress, m_{ij} are components of couple stress, λ_i^* is the microstress tensor, e_{ij} are components of strain, e_{kk} is the dilatation, δ_{ij} is Kroneker delta function, τ^0, τ^1 are the diffusion relaxation times and τ_0, τ_1 are thermal relaxation times with $\tau_0\geq\tau_1\geq 0$. Here $\tau^0=\tau^1=\tau_0=\tau_1=\gamma_1=0$ for Coupled Thermoelastic theory (CT) model. $\tau_1=\tau^1=0$, $\varepsilon=1$, $\gamma_1=\tau_0$ For Lord-Shulman (LS) model and $\varepsilon=0$, $\gamma_1=\tau^0$ where $\tau^0>0$ for Green-Lindsay (GL) model. In the above equations symbol (“,”) followed by a suffix denotes differentiation with respect to spatial coordinates and a superposed dot (“ . ”) denotes the derivative with respect to time respectively.

III. Formulation of the Problem

We consider a rectangular Cartesian coordinate system $OX_1X_2X_3$ with x_3-axis pointing vertically outward the medium. We consider a normal or tangential force to be acting at the free surface of microstretch thermoelastic medium with mass diffusion half space. For two dimensional problems the displacement and microrotation vectors are of the form:

$$\boldsymbol{u}=(u_1,0,u_3),\ \boldsymbol{\phi}=(0,\phi_2,0), \quad (8)$$

For further consideration it is convenient to introduce in (1)-(5) the dimensionless quantities defined as:

$$u_r'=\frac{\rho\omega^*c_1}{\beta_1T_0}u_r,\ x_r'=\frac{\omega^*}{c_1}x_r,\ t'=\omega^*t,\ \phi^{*\prime}=\frac{\rho c_1^2}{\beta_1T_0}\phi^*,\ T'=\frac{T}{T_0},\ \tau_1'=\omega^*\tau_1,\ \tau_0'=\omega^*\tau_0,\gamma_1'=\omega^*\gamma_1,t_{ij}'=\frac{1}{\beta_1T_0}t_{ij},\omega^*=\frac{\rho c^*c_1^2}{K^*},\ \phi_i'=\frac{\rho c_1^2}{\beta_1T_0}\phi_i,\ \tau^{1\prime}=\omega^*\tau^1,\ c_1^2=\frac{\lambda+2\mu+k}{\rho},\ c_2^2=\frac{\mu+k}{\rho},\ c_3^2=\frac{\gamma}{\rho j},\ c_4^2=\frac{2\alpha_0}{\rho j_0},\ =\frac{\gamma^2T_0}{\rho^2c^*c_1},\ m_{ij}^*=\frac{\omega^*}{c\beta_1T_0}m_{ij},\ C'=\frac{\beta_2}{\rho c_1^2}C. \quad (9)$$

with the aid of (9) the (1)-(5) reduce to:

$$a_1\frac{\partial e}{\partial x_1}+a_2\nabla^2u_1-a_3\frac{\partial\phi_2}{\partial x_3}+a_4\frac{\partial\phi^*}{\partial x_1}-\left(1+\tau_1\frac{\partial}{\partial t}\right)\frac{\partial T}{\partial x_1}-a_5\left(1+\tau^1\frac{\partial}{\partial t}\right)\frac{\partial C}{\partial x_1}=\ddot{u}_1 \quad (10)$$

$$a_1\frac{\partial e}{\partial x_3}+a_2\nabla^2u_3+a_3\frac{\partial\phi_2}{\partial x_1}+a_4\frac{\partial\phi^*}{\partial x_3}-\left(1+\tau_1\frac{\partial}{\partial t}\right)\frac{\partial T}{\partial x_3}-a_5\left(1+\tau^1\frac{\partial}{\partial t}\right)\frac{\partial C}{\partial x_3}=\ddot{u}_3 \quad (11)$$

$$\nabla^2\phi_2-2a_6\phi_2+a_6\left(\frac{\partial u_1}{\partial x_3}-\frac{\partial u_3}{\partial x_1}\right)=a_7\ddot{\phi}_2, \quad (12)$$

$$\nabla^2\phi^*-a_8\phi^*-a_9e+a_{10}\left(1+\tau_1\frac{\partial}{\partial t}\right)T+a_{11}\left(1+\tau^1\frac{\partial}{\partial t}\right)C=a_{12}\ddot{\phi}^* \quad (13)$$

$$\nabla^2T=a_{13}\left(\frac{\partial}{\partial t}+\tau_0\frac{\partial^2}{\partial t^2}\right)T+a_{14}\left(\frac{\partial}{\partial t}+\varepsilon\tau_0\frac{\partial^2}{\partial t^2}\right)e+a_{15}\left(\frac{\partial}{\partial t}+\varepsilon\tau_0\frac{\partial^2}{\partial t^2}\right)\phi^*+a_{16}\left(\frac{\partial}{\partial t}+\gamma_1\frac{\partial^2}{\partial t^2}\right)C \quad (14)$$

$$\nabla^2e+a_{17}\left(1+\tau_1\frac{\partial}{\partial t}\right)\nabla^2T+a_{18}\left(\frac{\partial}{\partial t}+\varepsilon\tau^0\frac{\partial^2}{\partial t^2}\right)C-a_{19}\left(1+\tau^1\frac{\partial}{\partial t}\right)\nabla^2C=0. \quad (15)$$

here,

$$a_1=\frac{\lambda+\mu}{\beta_1T_0},a_2=\frac{\mu+K}{\beta_1T_0},a_3=\frac{K}{\rho c_1^2},a_4=\frac{\lambda_0}{\rho c_1^2},$$
$$a_5=\frac{\rho c_1^2}{\beta_1T_0},a_6=\frac{Kc_1^2}{\gamma\omega^{*2}},a_7=\frac{\rho jc_1^2}{\gamma},a_8=\frac{\lambda_1c_1^2}{\alpha_0\omega^{*2}},$$
$$a_9=\frac{\lambda_0\rho c_1^4}{\beta_1T_0\alpha_0\omega^{*2}},a_{10}=\frac{\upsilon_1\rho c_1^4}{\beta_1\alpha_0\omega^{*2}},a_{11}=\frac{\nu_2\rho^2c_1^6}{\beta_1\beta_2T_0\alpha_0\omega^{*2}},$$
$$a_{12}=\frac{\rho c_1^2j_0}{2\alpha_0},a_{13}=\frac{\rho c^*c_1^2}{\omega^*K^*},a_{14}=\frac{\beta_1c_1^2}{\omega^*K^*},a_{15}=\frac{\upsilon_{1\beta_1T_0}}{\omega^*\rho K^*},$$

$$a_{16}=\frac{a\rho c_1^4}{\omega^* \beta_2 K^*}, a_{17}=\frac{aT_0}{\beta_2}, a_{18}=\frac{\rho c_1^4}{\omega^* D\beta_2^2}, a_{19}=\frac{b\rho c_1^2}{\beta_2^2}.$$

The displacement components u_1 and u_3 are related to potential functions ϕ and ψ as:

$$u_1=\frac{\partial\phi}{\partial x_1}-\frac{\partial\psi}{\partial x_3}, \quad u_3=\frac{\partial\phi}{\partial x_3}+\frac{\partial\psi}{\partial x_1} \tag{16}$$

Using the relation (16), in the (10)-(15), we obtain:

$$(a_1+a_2)\nabla^2\phi-\ddot{\phi}+a_4\phi^*-\left(1+\tau_1\frac{\partial}{\partial t}\right)T-a_2\left(1+\tau^1\frac{\partial}{\partial t}\right)C=0 \tag{17}$$

$$\left(\nabla^2\text{-}a_8\text{-}a_{12}\frac{\partial^2}{\partial t^2}\right)\phi^*\text{-}a_9\nabla^2\phi+a_{10}\left(1+\tau_1\frac{\partial}{\partial t}\right)T+a_{11}\left(1+\tau^1\frac{\partial}{\partial t}\right)C=0 \tag{18}$$

$$a_{13}\left(1+\tau_0\frac{\partial}{\partial t}\right)\dot{T}+a_{14}\left(\frac{\partial}{\partial t}+\varepsilon\tau_0\frac{\partial^2}{\partial t^2}\right)\nabla^2\phi+a_{15}\left(1+\varepsilon\tau_0\frac{\partial}{\partial t}\right)\dot{\phi^*}+a_{16}\left(1+\gamma_1\frac{\partial}{\partial t}\right)\dot{C}-\nabla^2T=0 \tag{19}$$

$$\nabla^4\phi+a_{17}\left(1+\tau_1\frac{\partial}{\partial t}\right)\nabla^2T+a_{18}\left(\frac{\partial}{\partial t}+\varepsilon\tau^0\frac{\partial^2}{\partial t^2}\right)C-a_{19}\left(1+\tau^1\frac{\partial}{\partial t}\right)\nabla^2C=0 \tag{20}$$

$$a_2\nabla^2\psi-\ddot{\psi}+a_3\phi_2=0, \tag{21}$$

$$\nabla^2\phi_2-2a_6\phi_2-a_6\nabla^2\psi=a_7\ddot{\phi}_2, \tag{22}$$

Here $\nabla^2=\frac{\partial^2}{\partial x_1^2}+\frac{\partial^2}{\partial x_3^2}$ is the Laplacian operator. Now, we define the Laplace transform:

$$\bar{f}(x,z,s)=\int_0^\infty f(x,z,t)e^{-st}dt. \tag{23}$$

So, the (10)-(15) become:

$$(a_1+a_2)\nabla^2\bar{\phi}-s^2\bar{\phi}+a_4\overline{\phi^*}-(1+\tau_1 s)\bar{T}-a_2(1+\tau^1 s)\bar{C}=0 \tag{24}$$

$$\left(\nabla^2\text{-}a_8\text{-}a_{12}s^2\right)\overline{\phi^*}\text{-}a_9\nabla^2\bar{\phi}+a_{10}(1+\tau_1 s)\bar{T}+a_{11}\left(1+\tau^1 s\right)\bar{C}=0 \tag{25}$$

$$a_{13}(s+\tau_0 s^2)\bar{T}+a_{14}(s+\varepsilon\tau_0 s^2)\nabla^2\bar{\phi}+a_{15}(s+\varepsilon\tau_0 s^2)\overline{\phi^*}+a_{16}(s+\gamma_1 s^2)\bar{C}-\nabla^2\bar{T}=0 \tag{26}$$

$$\nabla^4\bar{\phi}+a_{17}(1+\tau_1 s)\nabla^2\bar{T}+a_{18}(s+\varepsilon\tau^0 s^2)\bar{C}-a_{19}(1+\tau^1 s)\nabla^2\bar{C}=0 \tag{27}$$

$$a_2\nabla^2\bar{\psi}-s^2\bar{\psi}+a_3\overline{\phi_2}=0, \tag{28}$$

$$\nabla^2\overline{\phi_2}-2a_6\overline{\phi_2}-a_6\nabla^2\bar{\psi}=s^2a_7\overline{\phi_2}, \tag{29}$$

now, we define the Fourier's transform:

$$\hat{f}(z,\xi,s)=\int_{-\infty}^{\infty}\bar{f}(x,z,s)e^{i\xi x}dx \tag{30}$$

Applying Fourier's transform to (24)-(29), we obtain the equations:

$$(a_1+a_2)\left(\frac{d^2}{d\,x_3^2}-\xi^2\right)\hat{\phi}-s^2\hat{\phi}+a_4\hat{\phi^*}-(1+\tau_1 s)\hat{T}-a_2(1+\tau^1 s)\hat{C}=0 \tag{31}$$

$$\left(\frac{d^2}{d\,x_3^2}-\xi^2-a_8-a_{12}s^2\right)\widehat{\phi^*}-a_9\left(\frac{d^2}{d\,x_3^2}-\xi^2\right)\hat{\phi}+a_{10}(1+\tau_1 s)\hat{T}+a_{11}(1+\tau^1 s)\hat{C}=0 \tag{32}$$

$$a_{13}(s+\tau_0 s^2)\hat{T}+a_{14}(s+\varepsilon\tau_0 s^2)\left(\frac{d^2}{d\,x_3^2}-\xi^2\right)\hat{\phi}+a_{15}(s+\varepsilon\tau_0 s^2)\widehat{\phi^*}+a_{16}(s+\gamma_1 s^2)\hat{C}-\left(\frac{d^2}{d\,x_3^2}-\xi^2\right)\hat{T}=0 \tag{33}$$

$$\left(\frac{d^2}{d\,x_3^2}-\xi^2\right)^2\hat{\phi}+a_{17}(1+\tau_1 s)\left(\frac{d^2}{d\,x_3^2}-\xi^2\right)\hat{T}+a_{18}(s+\varepsilon\tau^0 s^2)\hat{C}-a_{19}(1+\tau^1 s)\left(\frac{d^2}{d\,x_3^2}-\xi^2\right)\hat{C}=0, \tag{34}$$

$$a_2\left(\frac{d^2}{d\,x_3^2}-\xi^2\right)\hat{\psi}-s^2\hat{\psi}+a_3\widehat{\phi_2}=0, \tag{35}$$

$$\left(\frac{d^2}{d\,x_3^2}-\xi^2\right)\widehat{\phi_2}-2a_6\widehat{\phi_2}-a_6\left(\frac{d^2}{d\,x_3^2}-\xi^2\right)\hat{\psi}=s^2a_7\widehat{\phi_2}. \tag{36}$$

on solving (24)-(27), we obtain:

$$\left[\mathrm{A}_0\frac{d^8}{d\,x_3^8}+\mathrm{A}_1\frac{d^6}{d\,x_3^6}+\mathrm{A}_2\frac{d^4}{d\,x_3^4}+\mathrm{A}_3\frac{d^2}{d\,x_3^2}+\mathrm{A}_5\right]\left(\hat{\phi},\widehat{\phi^*},\hat{T},\hat{C}\right)=0,$$

and on solving (31)-(34), we obtain:

$$\left[\frac{d^4}{d\,x_3^4}+\mathrm{A}_6\frac{d^2}{d\,x_3^2}+\mathrm{A}_7\right]\left(\widehat{\phi_2},\hat{\psi}\right)=0,$$

the solution of the above system satisfying the radiation conditions that $\left(\hat{\phi},\widehat{\phi^*},\hat{T},\hat{C},\widehat{\phi_2},\hat{\psi}\right)\to 0$ as $x_3\to\infty$ are given as following:

$$\left(\hat{\phi},\widehat{\phi^*},\hat{T},\hat{C}\right)=\sum_{i=1}^{4}(1,\alpha_{1i},\alpha_{2i},\alpha_{3i})M_i e^{-m_i x_3},$$

$$\left(\widehat{\phi_2},\hat{\psi}\right)=\sum_{i=5}^{6}(1,\beta_{1i})M_i' e^{-m_i x_3},$$

here,

$$\alpha_{1i}=\frac{D_{1i}}{D_{0i}}\ \alpha_{2i}=\frac{D_{2i}}{D_{0i}}\ \alpha_{3i}=\frac{D_{3i}}{D_{0i}} \qquad i=1,2,3,4$$

$$\beta_{1i}=-\frac{\delta_3\left(m_i^2-k^2\right)}{\left(m_i^2-k^2\right)+\delta_4} \qquad i=5,6$$

IV. Boundary Conditions

Consider normal and tangential force acting on the surface $x_3=0$ along with vanishing of couple stress, microstress, mass diffusion and temperature gradient at the boundary and considering insulated and infinite boundary at $x_3=0$. Mathematically this can be written as:

$$t_{33}=-F_1\delta(x)\delta(t)$$
$$t_{31}=-F_2\delta(x)\delta(t), m_{32}=0, \lambda_3=0, \frac{\partial T}{\partial x_3}=F_3\delta(x)\delta(t),$$
$$\frac{\partial C}{\partial x_3}=F_4\delta(x)\delta(t)$$

Here F_1 and F_2 are the magnitude of the applied force.On applying the Laplace transform and then Fourier transform the above conditions reduces to:

$$\widehat{t_{33}}=-\widehat{t_{31}}=-F_2\widehat{m_{32}}=0, \widehat{\lambda_3}=0, \frac{\partial\hat{T}}{\partial x_3}=F_3,$$
$$\frac{\partial\hat{C}}{\partial x_3}=F_4 \tag{37}$$

Using these boundary conditions and solving the linear equations formed, we obtain:

$$\widehat{t_{33}} = \sum_{i=1}^{6} G_{1i} e^{-m_i x_3},\ i = 1,2,\ldots,6 \quad (38)$$

$$\widehat{t_{31}} = \sum_{i=1}^{6} G_{2i} e^{-m_i x_3},\ i = 1,2,\ldots,6 \quad (39)$$

$$\widehat{m_{32}} = \sum_{i=1}^{6} G_{3i} e^{-m_i x_3},\ i = 1,2,\ldots,6 \quad (40)$$

$$\widehat{\lambda_3} = \sum_{i=1}^{6} G_{4i} e^{-m_i x_3},\ i = 1,2,\ldots,6 \quad (41)$$

$$\widehat{u_1} = \sum_{i=1}^{6} G_{5i} e^{-m_i x_3},\ i = 1,2,\ldots,6 \quad (42)$$

$$\widehat{u_3} = \sum_{i=1}^{6} G_{6i} e^{-m_i x_3},\ i = 1,2,\ldots,6 \quad (43)$$

$$\hat{T} = \sum_{i=1}^{6} G_{7i} e^{-m_i x_3} e^{-(kx_1 - \omega t)}\ i = 1,2,\ldots,6 \quad (44)$$

$$\hat{C} = \sum_{i=1}^{6} G_{8i} e^{-m_i x_3},\ i = 1,2,\ldots,6 \quad (45)$$

Here $G_{ij}, i = 1,2,\ldots,6,\ j = 1,2,\ldots,8$ are the constants.

Case I- Normal Stress

To obtain the expressions due to normal stress we must set $F_2 = 0$ in the boundary conditions (37).

Case II- Tangential Stress

To obtain the expressions due to tangential stress we must set $F_1 = 0$ in the boundary conditions (37).

Particular cases:

(i) If we take $\tau_1 = \tau^1 = 0,\ \varepsilon = 1,\ \gamma_1 = \tau_0$, in (38)-(45), we obtain the corresponding expressions of stresses, displacements and temperature distribution for L-S theory.

(ii) If we take $\varepsilon = 0,\ \gamma_1 = \tau^0$ in (38)-(45), the corresponding expressions of stresses, displacements and temperature distribution are obtained for G-L theory.

(iii) Taking $\tau^0 = \tau^1 = \tau_0 = \tau_1 = \gamma_1 = 0$ in (38)-(45), yield the corresponding expressions of stresses, displacements and temperature distribution for Coupled theory of thermoelasticity.

Special cases:

(a) Microstretch Thermoelastic Solid:
If we neglect the diffusion effect in (38)-(45), we obtain the corresponding expressions of stresses, displacements and temperature for microstretch thermoelastic solid.

(b) Micropolar Thermoelastic Diffusive Solid:
If we neglect the microstretch effect in (38)-(45), we obtain the corresponding expressions of stresses, displacements and temperature for micropolar thermoelastic diffusive solid.

V. Inversion of Transform

The transformed displacements, stresses and pore pressures are functions of the parameters of Laplace and Fourier transforms s and ξ respectively and hence are of the form $f(s, \xi, z)$. To obtain the solution of the problem in the physicaldomain, we must invert the Laplace and Fourier transform by using the method applied by Kumar [16].

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