Performance Analysis of Wavelet Based Multiuser MIMO OFDM

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Abstract—Wavelet analysis has some strong advantages over Fourier analysis, as it allows a time-frequency domain analysis, allowing optimal resolution and flexibility. As a result, they have been satisfactorily applied in almost all the fields of communication systems including OFDM which is a strong candidate for next generation of wireless technology. In this paper, the performances of wavelet based Multiuser Multiple Input and Multiple Output Orthogonal Frequency Division Multiplexing (MU-MIMO OFDM) systems are analyzed in terms of BER. It has been shown that the wavelet based systems outperform the classical FFT based systems. This analysis also unfolds an interesting result, where wavelet based OFDM system will have a constant error performance using Regularized Channel Inversion (RCI) beamforming for any number of users, and outperforms in all possible scenario in a multiuser environment. An extensive computer simulations show that a PAPR reduction of up to 6.8dB can be obtained with M=64.

I. INTRODUCTION

OFDM has been chosen for high data rate communications and has been widely deployed in many wireless communication standards such as Digital Video Broadcasting (DVB) and worldwide interoperability for microwave access (mobile WiMAX) due to its spectral efficiency and strong immunity to multipath fading. OFDM is a multi-carrier modulation scheme which converts a broadband frequency-selective channel into parallel flat-fading narrowband sub-channels. Cyclic prefix is added to each symbol to mitigate the inter-symbol interference caused by multipath wireless channel [1], and hence leads to spectral inefficiency. Cyclic prefix also causes ripples in the power spectral density of the ultra-wideband signals thus resulting in a transmit power back-off. Since OFDM uses orthogonal subcarriers to transmit the signals, the linear combination of the modulated subcarriers can have a very large amplitude fluctuations resulting in a large (Peak-to-Average) PAPR which is a well-known limitation of OFDM that leads to a severe nonlinear distortion in practical implementations of high power amplifier (HPA). Wavelet based modulation, on the other hand, can satisfy orthogonality criterion by wavelet filter banks [2]. We can have all the benefits of OFDM even if we replace all the traditional sinusoid carriers of Fourier based OFDM with suitable wavelets. Wavelet Based systems have been shown to have better immunity to impulse and

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narrowband noises than Fourier OFDM [3], [4] also the interference power can be mitigated to a large extent [5]. Moreover, Wavelet based OFDM doesn't require any cyclic prefix, hence increases spectral efficiency, does not produce the ripples, reduces complexity, leading to a better symbol rate, hence no power wastage for redundancy. Wavelet filter-bank approach brings some distinctive advantages for system implementation such as FPGAs (Field Programmable Gate Arrays). Also the performance of equalization in wavelet system is better than conventional OFDM [6]. Wavelet packet modulation will have much lower side lobes in transmitted signals which reduce the inter-carrier interference (ICI) and narrowband interference (NBI) [7]. In addition to spectral efficiency wavelet systems will have a very less PAPR which will increase HPA efficiency.

In Multi-user MIMO (MU-MIMO) wireless system, several co-channel users with multiple antennae intend to communicate with a base station designed with multiple antennas in the same frequency and time slot. In this multi-user communication system, multiple antennas allow the users to transmit their own data stream in the uplink at the same time. It also allows the base station to transmit the multiple user data streams to be decoded by each user in the downlink. In such cases, it becomes necessary to design a transmission system which is capable of suppressing the co-channel interference (CCI) at the user end. The interference arises due to the fact that the antenna might be serving multiple users. Multiuser detection technique is used to design transmit beam-forming vectors for end users while limiting the co-channel interference (CCI) from other users. Among the various MUDs, the classic MMSE (minimum mean square error) exhibits a low complexity at the cost of a limited performance. But optimum maximum likelihood (ML) MUD is capable of achieving the best performance, though it increases complexity. Several works have been proposed which almost nullify the effect of CCI at each user [8], [9], but it increases the computational complexity extensively. The easy way to deal with such problem in multiuser systems is linear precoder, include the channel inversion (also known as zero-forcing) [10] and the regularized channel inversion [11]. The principle of these approaches is that if the interference is known in advance at the transmitter, it is possible to design the code to compensate for it. Thus, the capacity of an interference channel can be made to be the same as the channel with no interference.

In this paper, we analyze the performance of multiuser MIMO wavelet based OFDM systems employing CI & RCI transmit beamforming in their downlinks considering 2 to 6 users. The organization of this paper is as follows: required

models, involved techniques and mathematical analysis are reviewed in Section II. Simulations are presented in Section III. Finally, Section IV concludes the paper.

II. PRELIMINARIES

A. Fourier vs. Wavelet Based OFDM

Fourier based classical OFDM system has been a popular choice for wireless transmission for a long time. In Fourier analysis we break up a signal into a set of an infinite sum of sinusoidal, sine and cosine, and exploit the orthogonality relationship among them.

$$x(n) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X_k e^{j\frac{2\pi}{N}kn}, 0 \leq n \leq N - 1 \qquad (1)$$

OFDM signals may be generated by an N-point Inverse Fast Fourier Transform (IFFT) in the transmitter, and the Fast Fourier Transform (FFT) is employed at the receiver to restore the signal. Let us consider a block of N symbols $X_N = \{X_k, k = 0,1,\ldots,N-1\}$ is formed with each symbol modulating the corresponding subcarrier from a set of orthogonal subcarriers, where X_k is the symbol carried by the k_{th} sub-carrier. Therefore, the discrete-time complex OFDM symbol can be written as

Wavelet transform is a decomposition of the signal by a low-pass (LP) and a high-pass (HP) filter. Half of the frequency components have been filtered out at filter outputs and hence can be down-sampled. We get approximation and detail coefficients from g(n) and h(n) filters respectively which are the wavelet's half-band low pass filter and half-band high pass filter impulse responses.

$$\psi_{lpf}(n) = \sum_{k=-\infty}^{k=+\infty} s(k)g(2n - k) \qquad (2)$$

$$\psi_{hpf}(n) = \sum_{k=-\infty}^{k=+\infty} s(k)h(2n - k) \qquad (3)$$

In wavelet decomposition the details as well as the approximations can be split into a second level details and approximations. These two sets of coefficients are obtained by performing convolution between the input signals and wavelet filter coefficients. Decomposition process is repeated by a series of high and low pass filters until we are left with a coefficient sequence of wavelets that are orthogonal in nature, the original signal is then reconstructed by performing the reverse operation of this decomposition [2]. One thing about wavelet packet analysis that attracts communication system is accurate reconstruction using wavelet coefficients.

B. Mathematical Model for Multi-User MIMO System

Considering K number of users in a multi-user MIMO system, we assume that the base station and each user are equipped with N_T and N_M antennas, respectively. Then the uplink channel known as a multiple access channel (MAC) for K independent users; the received signal at the receiver is given as

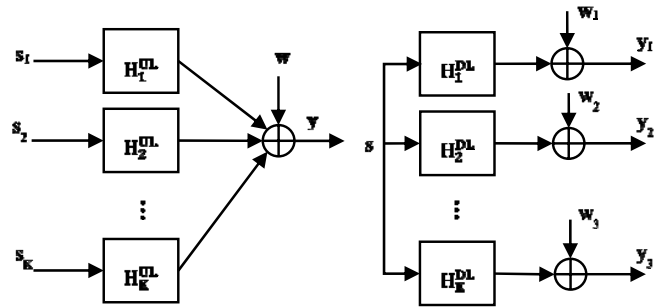$$y = H_1^{UL}x_1 + H_2^{UL}x_2 + \cdots + H_K^{UL}x_K + z$$

Fig. 1 Uplink and Downlink channel model for Multiuser MIMO system

Downlink channel in a multiuser environment is known as a broadcast channel in which **s** is the transmit signal from the base station and y_u is the received signal at the uth user, u=1, 2… K. Let H_u^{DL} represent the channel gain between the user and the base station. In broadcast channel, the received signal at the uth user is expressed as $y_u = H_u^{DL}s + w_u$, where w_u is the additive noise at the uth user for u=1,2… K. The main difficulty in transmitting data over multiuser downlink channel is that, the coordinated signal detection on the receiver side is not straightforward, and this is why an interference cancellation method is required before transmitting the signal on the channel. Different transmission methods are there to solve this problem, like block diagonalization, dirty paper coding (DPC), and Tomlinson-Harashima precoding (THP), but these non-linear techniques increase computational complexity. The most straightforward way is to apply a linear precoder to the data symbols of all users before transmission to control the interference.

C. Linear Precoder: CI, RCI & MPCI Beamformer

We consider the downlink of a multiuser system, where the base station with N transmit antennas simultaneously sends independent data symbols s_k, $1 \leq k \leq K$, to K users, each with one receive antenna. To reduce the amount of interuser interference at the receiver, the data symbols of all users will be jointly processed by an N × K transmit beamforming matrix **P**. The transmitted signals across N transmit antennas are x = **Ps**, where s = $[s_1, s_2\ldots s_k]^T$, and are constrained to have a fixed transmitted power. The receive data at the kth user is given by

$$y_k = \sum_{n=1}^{N} h_{k,n} x_n + w_k \qquad (4)$$

where $x_n = [x]_n$, is the signal sent from nth antenna and w_k is an independent sample of a zero-mean complex Gaussian random variable with variance $\sigma^2/2$ per complex dimension. The coefficient $h_{k,n}$ is the fading gain for the path between antenna

n and user k. combining all the received signals from all K users into a vector $= [y_1, y_2, \ldots, y_K]^T$, we can write

$$\mathbf{y} = \mathbf{Hx} + \mathbf{w} \qquad (5)$$

where $\mathbf{w} = [w_1, w_2, \ldots, w_K]^T$, and H is the K×N matrix of downlink channel gains, with $h_{k,n}$ as elements. In case where the zero-forcing (ZF) equalization is employed [10], the corresponding weight matrix is given as

$$\mathbf{P}_{ZF} = \beta\mathbf{H}^{-1} \qquad (6)$$

where,$\beta = \sqrt{\dfrac{N_T}{\text{Trace}(\mathbf{H}^{-1}(\mathbf{H}^{-1})^{\mathbf{H}})}}$

To compensate for the effect of amplification by a factor of b at the transmitter, the received signal must be divided by β via automatic gain control (AGC) at the receiver.Then the received signal can be given as

$$y = \frac{1}{\beta}(\mathbf{HP}_{ZF}s + \mathbf{z}) = \frac{1}{\beta}(\mathbf{H}\beta\mathbf{H}^{-1}s + \mathbf{z}) = \tilde{s} + \tilde{\mathbf{w}} \qquad (7)$$

It has been proposed in [11] that the RCI beamformer gives a much better error performance than the channel inversion. The regularization parameter that is used to scale the identity matrix in the transmit beamforming matrix, that prevents the channel inversion problem when the channel matrix is ill conditioned [11].

$$\mathbf{P}_{RCI} = c\mathbf{H}^{\mathbf{H}}(\mathbf{HH}^{\mathbf{H}} + \alpha\mathbf{I})^{-1} \qquad (8)$$

where c is a normalizing constant and α is the regularizing constant. Let h_k denote the kth row of $\mathbf{H}$, which is a $1 \times N$ vector containing the fading gains for the paths between N_T transmit antennas and the receiver of user k. It should be also noted that, for the α→0 case in Eq. (8), the RCI beamformer reduces to the Moore-Penrose channel inversion (MPCI) beamformer, where P becomes as

$$\mathbf{P}_{MPCI} = \lim_{\alpha\to 0} c\mathbf{H}^{\mathbf{H}}(\mathbf{HH}^{\mathbf{H}} + \alpha\mathbf{I})^{-1} = \lim_{\alpha\to 0}\mathbf{c}(\mathbf{H}^{\mathbf{H}}\mathbf{H} + \alpha\mathbf{I})^{-1}\mathbf{H}(9)$$

$$y_k = ch_k(\mathbf{H}^{\mathbf{H}}\mathbf{H} + \alpha\mathbf{I})^{-1}\mathbf{H}^{\mathbf{H}}s + w_k$$
$$= ch_k(\mathbf{H}^{\mathbf{H}}\mathbf{H} + \alpha\mathbf{I})^{-1}h_k^H s_k \sum_{j\neq k}^{K} ch_k(\mathbf{H}^{\mathbf{H}}\mathbf{H} + \alpha\mathbf{I})^{-1}h_k^H s_k$$
$$+ w_k$$

The receive signal at the kth user is then given by

$$y_k = ch_k(\mathbf{H}^{\mathbf{H}}\mathbf{H} + \alpha\mathbf{I})^{-1}\mathbf{H}^{\mathbf{H}}s + w_k$$
$$= ch_k(\mathbf{H}^{\mathbf{H}}\mathbf{H} + \alpha\mathbf{I})^{-1}h_k^H s_k \sum_{j\neq k}^{K} ch_k(\mathbf{H}^{\mathbf{H}}\mathbf{H} + \alpha\mathbf{I})^{-1}h_k^H s_k + w_k$$

First term is the desired signal, and second term is the interference caused by other users to the kth user.

A Wavelet based OFDM system with beam former and MIMO configuration is shown in Fig. 2. We consider this system is in a multiuser environment of K interfering users, where k^{th} user decorated with N_R number of antennas is communicating with a base station equipped with N_T number of antennas.
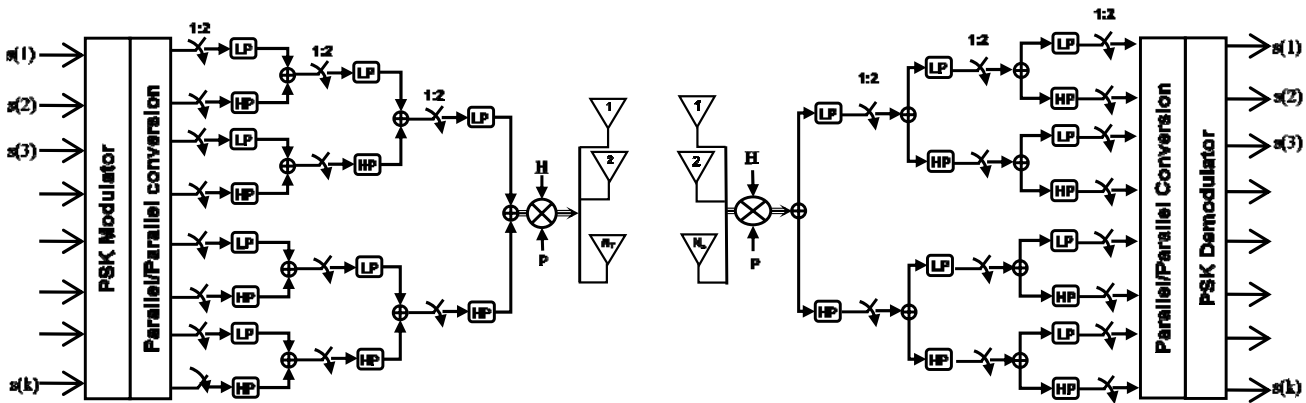

Fig. 2 Wavelet based OFDM transmitter and receiver

D. Wavelet Based Multiuser OFDM with MIMO

On the transmitter side, first a binary phase shift keying (BSPK) modulator is used for mapping s(k) data stream to the symbol stream x(n). After the mapping process a parallel-to-parallel (P/P) converter reshapes the modulated symbols into, for example, N = 8 parallel data streams. This P/P converter makes sure that $N = 2^n$, where n is an integer, so that the transmitter can perform inverse discrete wavelet transform (IDWT) and produce one final sequence in "n"

stages. Sequential two x(n)symbol streams are up-sampled by the up-sampling factor 2, filtered by the wavelet filter g(n) or h(n), respectively, and then summed up. Output stream is then up-sampled by 2, filtered and summed again. The up-sampling and filtering processes continue until one single output stream is obtained. To satisfy the orthogonality criterion of the wavelet filters we use Quadrature Mirror Filter (QMF) bank. According to QMF, the relationship between both filters is given as

$$h(n) = (-1)^n g(L - 1 - n)$$

where L is the filter length of the impulse response g(n) [12]. Stream of wavelet coefficients $\psi(n)$, is then multiplied by N×1beamforming vector **P** and weighted signals are added to form the output signal [12].

III. SIMULATIONS

The systems are simulated under wireless Rayleigh fading channel, with 10^5 randomly generated binary data, N=64 and BPSK mapping, considering a multiuser environment of 2 to 6users. The user data stream is first BPSK modulated and then level 3 wavelet decomposition is done in this approach with a "haar" wavelet function. Hence the low-pass and high-pass decomposition filter coefficients are ([0.7071, 0.7071] and [0.7071,-0.7071]), whereas reconstruction filter coefficients are ([0.7071, 0.7071] and [-0.7071,-0.7071]).

Fig. 3 shows, channel inversion is compared to that of regularized channel inversion for $K=N_T=2, 3, 4, 5$ and 6 and $N_R=1$, in which N_T number of users with the highest channel norm values are selected out of K = 20. As can be seen in Fig. 3 that wavelet based OFDM performs better than FFT based OFDM. For instance, wavelet system improves BER by 1.5dB at error probability 10^{-3}, for 2 users, and 8 dB for 6 users.
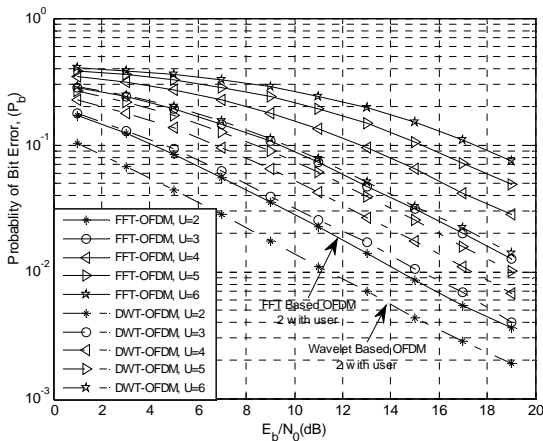

Fig. 3 BER of FFT based and wavelet based MU-MIMOOFDM, 2 to 6 users using channel inversion beamforming

Fig. 4 shows that regularized channel inversion achieves better performance than channel inversion method. Graph shows that, the error performance of wavelet based system is almost steady while FFT based OFDM continues to degrade with the increase in the number of users. For instance, Wavelet based RCI improves 4.7dB over FFT based RCI at 10^{-3} error probability, and performance doesn't change even if the number of user increases. Simulation is shown for 2 to 6 users.
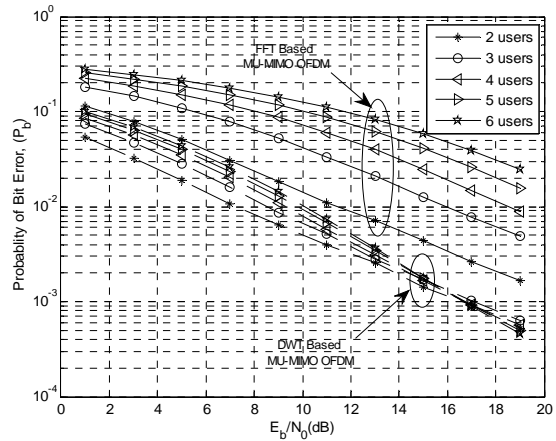

Fig. 4 BER of FFT based and wavelet based MU-MIMO OFDM, 2 to 6 users using RCI beamforming

IV. CONCLUSIONS

This article introduces an effective implementation of wavelet packet transform instead of Fourier transform in multi-carrier communication considering practical cases. Wavelet based analysis is more immune to impulse and narrowband noises than conventional Fourier based OFDM system, also improves spectral efficiency and saves transmission power. In this paper, it has been shown that wavelet based OFDM systems perform better than FFT based OFDM. MATLAB simulation results and mathematical analysis are given to support the statement.

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