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Globally Exponential Stability and Dissipativity Analysis of Static Neural Networks with Time Delay

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Abstract—The problems of globally exponential stability and dissipativity analysis for static neural networks (NNs) with time delay is investigated in this paper. Some delay-dependent stability criteria are established for static NNs with time delay using the delay partitioning technique. In terms of this criteria, the delay-dependent sufficient condition is given to guarantee the dissipativity of static NNs with time delay. All the given results in this paper are not only dependent upon the time delay but also upon the number of delay partitions. Two numerical examples are used to show the effectiveness of the proposed methods.

Keywords—Globally exponential stability, Dissipativity, Static neural networks, Time delay.

I. Introduction

In recent years, recurrent neural networks (RNNs) have been extensively broadly researched because of their applications in many fields, such as signal processing, associative memory, pattern recognition, combination optimization and so on. In implementation of artificial neural networks, time delays are unavoidable because of finite switching speeds of the amplifiers. It is known long that delays may result in oscillation and instability [1]. So it is important to investigate the stability of delayed RNNs. Delayed RNNs have been extensively used in various engineering and scientific fields, for example, solving linear projection equations and quadratic optimization problems [2], component detection of medical signal [3], automatic control engineering [4], analysis of moving images or speech [5], biological simulation [6] and so on. In recent years, a variety of competing sufficient stability conditions for delayed RNNs have been accumulated [7]-[15].

As already reported in [16], [17], RNNs can be classified as static neural networks and local field networks. These two types of RNNs cannot always be equivalently transformed to one another [18], [19], so that the aforementioned results for the local field type are not effective for the static one and new stability criteria need to be explored. For delay-free SNNs, the global asymptotic stability (GAS) and/or global exponential stability (GES) problems have been discussed in [19], [20]. For the delayed ones, several delay-independent conditions of the GAS have been proposed in terms of linear matrix inequalities (LMIs) in [21]-[25]. To reduce the inherent conservatism included in the delay-independent results, Shao [23], [24] and Zuo [25], together with Du and Lam [26] and Lu [27], still have made efforts to explore delay-dependent conditions of the GAS of delayed SNNs via a direct Lyapunov method. The GES of delayed SNNs also has been researched in [28] by a similar method.

As we all known, the choice of activation functions is another main factor influencing the ability and performance of NNs, as well as their corresponding applications. For example, Morita [29] pointed out that, compared with the usual sigmoidal activation functions, nonmonotonic activation functions can remarkably improve the networks absolute capacity for associative memory applications. For simplifying theoretical derivation, some assumptions are often made for the activation functions in the addressed NN model. The most frequently used ones include the common Lipschitz condition and the nondecreasing Lipschitz condition. For SNNs, the results in [19], [21], [23]-[27] are based on the nondecreasing Lipschitz condition, while those in [20], [22] are based on the common Lipschitz one. In [30], the authors first introduced a generalized condition, which places more general restrictions on the slope bounds of the activation functions and involves the previous two types as special cases. Under the generalized condition, many results have reported for delayed NNs of the local field type [31]-[34]. For delayed SNNs under this condition, the constant delay case has been investigated in [35].

On the other hand, the theory of dissipative systems plays an important role in system and control areas, hence, it has been attracting a great number of attention. In [36], it has been shown that the dissipative theory gives a framework for the design and analysis of control systems utilizing an input-output description based on energy-related considerations. Take advantage of the LMI approach, many interesting and important results on dissipativity analysis and synthesis have been reported for different kinds of dynamic systems. For instance, the problem of reliable dissipative control has been investigated in [37] for a type of stochastic hybrid systems in terms of the LMI approach, and linear state feedback controllers and impulsive controllers are designed such that, for all admissible uncertainties as well as actuator failure occurring among a prespecified subset of

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actuators, the stochastic hybrid system is stochastically robustly stable and strictly $(\mathcal{Q}, \mathcal{S}, \mathcal{R})-$dissipative. In terms of LMIs, delay-dependent sufficient conditions have been established in [38], which ensure singular systems with time delay to be admissible and dissipative. In [39] the problem of delay-dependent $(\mathcal{Q}, \mathcal{S}, \mathcal{R}) - \alpha-$dissipativity analysis has been investigated for local field NNs, and sufficient conditions were given to guarantee the local field NNs are dissipative. Most recently, Wu [40] was concerned with the problems of $(\mathcal{Q}, \mathcal{S}, \mathcal{R}) - \gamma-$dissipativity analysis for static NNs with time delay.

In this paper, based on the free-weighting matrix technique and the delay partitioning technique, the problems of stability and $(\mathcal{Q}, \mathcal{S}, \mathcal{R}) - \lambda - \gamma-$dissipativity analysis are considered for static NNs with time delay. First, the globally exponential stability criteria is established for static NNs with time-varying delay. Then, the delay-dependent sufficient condition is given to guarantee the dissipativity of this networks. All the results given in this paper are delay-dependent as well as partition-dependent.

Notation: Throughout this paper, let $\mathbb{R}^n$ denotes the n-dimensional Euclidean space, and $\mathbb{R}^{m\times n}$ is the set of all $m\times n$ real matrices, and let Z^T, Z^{-1}, $\lambda_M(Z)$ and $\lambda_m(Z)$ denote the transpose, the inverse, the largest eigenvalue and the smallest eigenvalue, respectively. The notation $X > Y (X \geq Y)$, where X and Y are symmetric matrices, means that $X - Y$ is positive definite(positive semi-definite). I and 0 denotes an identity matrix and a zero matrix with appropriate dimension. In block matrices, we use " $*$ " to denote the symmetric terms, and $diag\{\dots\}$ stands for a block-diagonal matrix. The space of square-integrable vector functions over $[0, \infty)$ is denoted by $L_2[0, \infty)$.

II. Problem statement

Consider the following static NN with time delay:

$$\begin{cases} \dot{z}(t) = -Az(t) + f_1(Wz(t) + J) + f_2(Wz(t - d(t)) + J) \\ z(t) = \phi(t), t \in [-d_2, 0] \end{cases} \tag{1}$$

where $A = diag\{a_1, a_2, \dots, a_n\} > 0$, $W = [\ \hat{W}_1^T \ \ \hat{W}_2^T \ \ \cdots \ \ \hat{W}_n^T\ ]^T$ is the delayed connection weight matrix, $J = [\ J_1 \ \ J_2 \ \ \cdots \ \ J_n\ ]^T$ represents the external inputs, $z(t) = [\ z_1(t) \ \ z_2(t) \ \ \cdots \ \ z_n(t)\ ]^T$ is the state vector associated with the n neurons, $f_i(Wz(t)) = [f_{i1}(\ \hat{W}_1 z(t)) \ \ f_{i2}(\hat{W}_2 z(t)) \ \ \cdots \ \ f_{in}(\hat{W}_n z(t))\ ]^T (i = 1, 2)$ is the activation function of neurons, $\phi(t)$ is the initial condition, and $d(t)$ is the time delay and satisfies

$$0 < d_1 \leq d(t) \leq d_2, \ \dot{d}(t) \leq \mu. \tag{2}$$

where d_1, d_2 and μ are known real constants, and $d_2 > d_1$.

It is assumed that each activation function $f_{ij}(\cdot)$ in (1) is bounded and satisfies

$$0 \leq \frac{f_{1j}(s_1) - f_{1j}(s_2)}{s_1 - s_2} \leq l_j, s_1 \neq s_2 \in \tag{3}$$

$$0 \leq \frac{f_{2j}(s_1) - f_{2j}(s_2)}{s_1 - s_2} \leq b_j, s_1 \neq s_2 \in \tag{4}$$

where $l_j > 0$, $b_j > 0$ are known real constants. This assumption guarantees that there is an equilibrium point u^* of the NN(1). Let $x(t) = z(t) - u^*$, $g_i(Wx(t)) = f_i(W(x(t) + u^*) + J) - f_i(Wu^* + J)$, and $\varphi(t) = \phi(t) - u^*$. Then NN (1) can be expressed as

$$\begin{cases} \dot{x}\left(t\right) = -Ax\left(t\right) + g_1\left(Wx\left(t\right)\right) + g_2\left(Wx\left(t - d\left(t\right)\right)\right) \\ x\left(t\right) = \varphi\left(t\right), t \in \left[-d_2, 0\right]. \end{cases} \tag{5}$$

It can be shown that the activation function $g_{ij}(\cdot)$ is bounded and satisfies

$$0 \leq \frac{g_{1j}(s)}{s} \leq l_j, g_{1j}(0) = 0 \tag{6}$$

$$0 \leq \frac{g_{2j}(s)}{s} \leq b_j, g_{2j}(0) = 0. \tag{7}$$

The definition of exponential stability is now given.

Definition 1 : System (1) is said to be globally exponentially stable if there exist constants $k > 0$ and $K > 1$ such that

$$\|x\left(t\right)\| \leq K \left(\sup_{-d_2 \leq \theta \leq 0} \|\varphi\left(\theta\right)\| \right) e^{-kt}$$

where k is called the exponential convergence rate.

Clearly, the equilibrium point of system (1) is exponentially stable if and only if the zero solution of system (5) is exponentially stable.

One propose in this paper is to investigate the globally exponentially stability of the static NN(5).

When an external disturbance appears in the NN (5), we have the following NN:

$$\begin{cases} \dot{x}(t) = -Ax(t) + g_1\left(Wx\left(t\right)\right) + g_2\left(Wx\left(t - d\left(t\right)\right)\right) + \omega(t) \\ y(t) = g_1(Wx(t)) + g_2(Wx(t)) \\ x(t) = \varphi(t), t \in [-d_2, 0] \end{cases} \tag{8}$$

where $y(t)$ is the output of the NN, and $\omega(t)$ is the disturbance input which cannot be fully measured and is not completely known beforehand. In this paper, it is assumed that $\omega(t) \in L_2[0, \infty)$, which implies that it is a function of finite energy.

We are now in a position to introduce the definition on dissipativity. Let the energy supply function of the NN (8) be defined by

$$G(\omega, y, \tau) = \langle y, \mathcal{Q}y \rangle_\tau + 2\langle y, \mathcal{S}\omega \rangle_\tau + \langle \omega, \mathcal{R}\omega \rangle_\tau \ \ \forall \tau \geq 0 \tag{9}$$

where $\mathcal{Q}$, $\mathcal{S}$, and $\mathcal{R}$ are real matrices with $\mathcal{Q}$, $\mathcal{R}$ symmetric, and $\langle a, b \rangle_\tau = \int_0^\tau a^T b dt$. Without loss of generality, it is assumed that $\mathcal{Q} \leq 0$.

Definition 2 : NN(8) is said to be strictly $(\mathcal{Q}, \mathcal{S}, \mathcal{R}) - \lambda - \gamma-$dissipative if, for some scalars $\lambda, \gamma > 0$, the inequality

$$\lambda \langle \omega, \omega \rangle_\tau \geq G(\omega, y, \tau) \geq \gamma \langle \omega, \omega \rangle_\tau \ \forall \tau \geq 0 \tag{10}$$

holds under zero initial condition for any nonzero disturbance $\omega \in L_2[0, \infty)$.

Another propose of this paper is to establish delay-dependent condition such that the NN(8) is globally exponentially stable and strictly $(\mathcal{Q}, \mathcal{S}, \mathcal{R}) - \lambda - \gamma-$dissipative.

Before moving on, the following results are required.

Lemma 1 [41], [42]: Assuming that function $h(\cdot)$ satisfies inequality (6)-(7), then the following inequality hold:

$$\int_v^u \left(h\left(s\right)-h\left(v\right)\right)ds \leq \left(u-v\right)\left(h\left(u\right)-h\left(v\right)\right). \qquad (11)$$

Lemma 2 (Jensen Inequality) [43], [44]: For any positive definite symmetric constant matrix $M \in \mathbb{R}^{n\times n}$, the scalars $r_1 < r_2$ and vector function $\omega : [r_1, r_2] \to \mathbb{R}^n$ such that the concerned integrations are well defined, then the following inequality holds:

$$\begin{aligned}&\left(\int_{r_1}^{r_2}\omega(s)ds\right)^T M\int_{r_1}^{r_2}\omega(s)ds\\ &\leq (r_2-r_1)\int_{r_1}^{r_2}\omega(s)^T M\omega(s)ds. \qquad (12)\end{aligned}$$

Lemma 3 [45]: For any matrix $\begin{bmatrix} M & S \\ * & M\end{bmatrix} \geq 0$, scalars d_1, d_2, $d(t)$ satisfying $d_1 \leq d(t) \leq d_2$, vector function $\dot{x}(t+\cdot) : [-d_2, -d_1] \to \mathbb{R}^n$ such that the integrations $\int_{t-d_2}^{t-d_1}\dot{x}(\alpha)^T M\dot{x}(\alpha)d\alpha$, $\int_{t-d_2}^{t-d_1}\dot{x}(\alpha)d\alpha$, and $\int_{t-d_2}^{t-d(t)}\dot{x}(\alpha)d\alpha$ are well defined, then

$$\begin{aligned}&-\left(d_2-d_1\right)\int_{t-d_2}^{t-d_1}\dot{x}(\alpha)^T M\dot{x}\left(\alpha\right)d\alpha \leq \begin{pmatrix} x(t-d_1)\\ x(t-d(t))\\ x(t-d_2)\end{pmatrix}^T \qquad (13)\\ &\times\begin{bmatrix} -M & M-S & S\\ * & -2M+S+S^T & -S+M\\ * & * & -M\end{bmatrix}\times\begin{pmatrix} x(t-d_1)\\ x(t-d(t))\\ x(t-d_2)\end{pmatrix}\end{aligned}$$

Remark 1: Lemma 3 is a special case of [45, Th. 1], which is presented in a form more convenient for the present application.

Lemma 4 [41], [46]: For any two vectors x and y and a positive definite matrix Q with compatible dimensions, the following matrix inequality holds:

$$2x^T y \leq x^T Qx + y^T Q^{-1}y. \qquad (14)$$

III. Stability Analysis

In this section, the delay partitioning technique will be developed to investigate the stability problem for the NN (5).

Theorem 1: Given an integer $m > 0$, the NN(5) with (2) is globally exponential stable if there exist matrices $P > 0$, $Z_i > 0\,(i = 1, 2, \ldots, m+1)$, $\begin{bmatrix} Q_1 & V_1 & U_1\\ * & Q_2 & V_2\\ * & * & Q_3\end{bmatrix} > 0$, $\begin{bmatrix} Y_1 & M_1 & N_1\\ * & Y_2 & M_2\\ * & * & Y_3\end{bmatrix} > 0$, $\begin{bmatrix} Y_4 & M_3 & N_2\\ * & Y_5 & M_4\\ * & * & Y_6\end{bmatrix} > 0$, $\begin{bmatrix} Z_{m+1} & U_2\\ * & Z_{m+1}\end{bmatrix} \geq 0$, and diagonal matrices $S = diag\{s_1, s_2, \ldots, s_n\} > 0$, $R = diag\{r_1, r_2, \ldots, r_n\} > 0$, $D_i > 0\,(i = 1, 2, \ldots, m+3)$ such that

$$\Xi = \begin{bmatrix}
\Xi_{11} & \Xi_{12} & \Xi_{13} & \Xi_{14} & 0 & 0 & \Xi_{17} & \Xi_{18} & 0\\
* & \Xi_{22} & \Xi_{23} & 0 & \Xi_{25} & 0 & 0 & \Xi_{28} & 0\\
* & * & \Xi_{33} & 0 & 0 & \Xi_{36} & 0 & 0 & \Xi_{39}\\
* & * & * & \Xi_{44} & 0 & 0 & \Xi_{47} & \Xi_{48} & 0\\
* & * & * & * & \Xi_{55} & 0 & 0 & \Xi_{58} & 0\\
* & * & * & * & * & \Xi_{66} & 0 & 0 & \Xi_{69}\\
* & * & * & * & * & * & \Xi_{77} & \Xi_{78} & 0\\
* & * & * & * & * & * & * & \Xi_{88} & 0\\
* & * & * & * & * & * & * & * & \Xi_{99}
\end{bmatrix} < 0 \qquad (15)$$

where

$$\begin{aligned}
\Xi_{11} &= 2ke_1^T Pe_1 - e_1^T PAe_1 - e_1^T APe_1 + 4ke_1^T W^T RLWe_1\\
&+4ke_1^T W^T RBWe_1 - e_1^T W^T LRWAe_1 - e_1^T AW^T RLWe_1\\
&-e_1^T W^T BRWAe_1 - e_1^T AW^T RBWe_1 + W_1^T Q_1 W_1\\
&-e^{-2k\frac{d_1}{m}}W_2^T Q_1 W_2 + \left(\frac{d_1}{m}\right)^2 e_1^T A\left(\sum_{i=1}^m Z_i\right)Ae_1\\
&-e^{-2kd_1}\sum_{i=1}^m \left(e_i - e_{i+1}\right)^T Z_i\left(e_i - e_{i+1}\right) + e_1^T Y_1 e_1 + e_1^T Y_4 e_1\\
&+\tilde{d}^2 e_1^T AZ_{m+1}Ae_1 - e^{-2kd_2}e_{m+1}^T Z_{m+1}e_{m+1}\\
\Xi_{12} &= e^{-2kd_2}e_{m+1}^T Z_{m+1} - e^{-2kd_2}e_{m+1}^T U_2\\
\Xi_{13} &= e^{-2kd_2}e_{m+1}^T U_2\\
\Xi_{14} &= e_1^T Pe_1 + 2ke_1^T W^T Se_1 - e_1^T AW^T Se_1 + e_1^T W^T LRWe_1\\
&+e_1^T W^T BRWe_1 - 2ke_1^T W^T Re_1 + e_1^T AW^T Re_1 + W_1^T V_1 W_1\\
&-e^{-2k\frac{d_1}{m}}W_2^T V_1 W_2 - \left(\frac{d_1}{m}\right)^2 e_1^T A\left(\sum_{i=1}^m Z_i\right)e_1 + e_1^T M_1 e_1\\
&+e_1^T M_3 e_1 - \tilde{d}^2 e_1^T AZ_{m+1}e_1 + \sum_{i=1}^{m+1} e_i^T W^T LD_i e_i\\
\Xi_{17} &= 2ke_1^T W^T Se_1 - e_1^T AW^T Se_1 - 2ke_1^T W^T Re_1\\
&+e_1^T AW^T Re_1 + W_1^T U_1 W_1 - e^{-2k\frac{d_1}{m}}W_2^T U_1 W_2 + e_1^T N_1 e_1\\
&+e_1^T N_2 e_1 + \sum_{i=1}^{m+1} e_i^T W^T BD_i e_i\\
\Xi_{18} &= e_1^T P + e_1^T W^T LRW + e_1^T W^T BRW\\
&-\left(\frac{d_1}{m}\right)^2 e_1^T A\left(\sum_{i=1}^m Z_i\right) - \tilde{d}^2 e_1^T AZ_{m+1}\\
\Xi_{22} &= -\left(1-\mu\right)e^{-2kd_2}Y_1 - 2e^{-2kd_2}Z_{m+1} + e^{-2kd_2}U_2 + e^{-2kd_2}U_2^T\\
\Xi_{23} &= -e^{-2kd_2}U_2 + e^{-2kd_2}Z_{m+1}\\
\Xi_{25} &= -\left(1-\mu\right)e^{-2kd_2}M_1 + W^T LD_{m+2}\\
\Xi_{28} &= -\left(1-\mu\right)e^{-2kd_2}N_1 + W^T BD_{m+2}\\
\Xi_{33} &= -e^{-2kd_2}Y_4 - e^{-2kd_2}Z_{m+1}\\
\Xi_{36} &= -e^{-2kd_2}M_3 + W^T LD_{m+3}\\
\Xi_{39} &= -e^{-2kd_2}N_2 + W^T BD_{m+3}\\
\Xi_{44} &= e_1^T SWe_1 + e_1^T W^T Se_1 - e_1^T RWe_1 - e_1^T W^T Re_1
\end{aligned}$$

$$+W_1^T Q_2 W_1 - e^{-2k\frac{d_1}{m}} W_2^T Q_2 W_2$$
$$+\left(\frac{d_1}{m}\right)^2 e_1^T \left(\sum_{i=1}^{m} Z_i\right) e_1 + e_1^T Y_2 e_1 + e_1^T Y_5 e_1$$
$$+\tilde{d}^2 e_1^T Z_{m+1} e_1 - 2\sum_{i=1}^{m+1} e_i^T D_i e_i$$
$$\Xi_{47} = e_1^T W^T S e_1 - e_1^T W^T R e_1 + W_1^T V_2 W_1$$
$$-e^{-2k\frac{d_1}{m}} W_2^T V_2 W_2 + e_1^T M_2 e_1 + e_1^T M_4 e_1$$
$$\Xi_{48} = e_1^T SW - e_1^T RW + \left(\frac{d_1}{m}\right)^2 e_1^T \left(\sum_{i=1}^{m} Z_i\right) + \tilde{d}^2 e_1^T Z_{m+1}$$
$$\Xi_{55} = -(1-\mu) e^{-2kd_2} Y_2 - 2D_{m+2}$$
$$\Xi_{58} = -(1-\mu) e^{-2kd_2} M_2$$
$$\Xi_{66} = -e^{-2kd_2} Y_5 - 2D_{m+3}$$
$$\Xi_{69} = -e^{-2kd_2} M_4$$
$$\Xi_{77} = W_1^T Q_3 W_1 - e^{-2k\frac{d_1}{m}} W_2^T Q_3 W_2 + e_1^T Y_3 e_1 + e_1^T Y_6 e_1$$
$$-2\sum_{i=1}^{m+1} e_i^T D_i e_i$$
$$\Xi_{78} = e_1^T SW - e_1^T RW$$
$$\Xi_{88} = \left(\frac{d_1}{m}\right)^2 \sum_{i=1}^{m} Z_i - (1-\mu) e^{-2kd_2} Y_3 + \tilde{d}^2 Z_{m+1}$$
$$-2D_{m+2}$$
$$\Xi_{99} = -e^{-2kd_2} Y_6 - 2D_{m+3}$$
$$L = diag\{l_1, l_2, \ldots, l_n\}$$
$$B = diag\{b_1, b_2, \ldots, b_n\}$$
$$W_1 = \begin{bmatrix} I_{mn} & 0_{mn\times n} \end{bmatrix}$$
$$W_2 = \begin{bmatrix} 0_{mn\times n} & I_{mn} \end{bmatrix}$$
$$\tilde{d} = d_2 - d_1$$
$$e_l = \begin{bmatrix} 0_{n\times(l-1)n} & I_n & 0_{n\times(m+1-l)n} \end{bmatrix}, l=1,2,\ldots,m+1.$$

Proof: Construct the following Lyapunov-Krasovskii functional candidate for NN(5):

$$V(x(t)) = \sum_{i=1}^{6} V_i(x(t)) \tag{16}$$

where

$$V_1(x(t)) = e^{2kt} x(t)^T P x(t) + 2e^{2kt} \sum_{i=1}^{n} s_i \int_0^{\hat{W}_i x(t)} g_{1i}(s)\, ds$$
$$+2e^{2kt} \sum_{i=1}^{n} s_i \int_0^{\hat{W}_i x(t)} g_{2i}(s)\, ds$$
$$+2e^{2kt} \sum_{i=1}^{n} r_i \int_0^{\hat{W}_i x(t)} (l_i s - g_{1i}(s))\, ds$$
$$+2e^{2kt} \sum_{i=1}^{n} r_i \int_0^{\hat{W}_i x(t)} (b_i s - g_{2i}(s))\, ds$$

$$V_2(x(t)) = \int_{t-\frac{d_1}{m}}^{t} e^{2ks} \begin{bmatrix} \eta_1(s) \\ \eta_2(s) \\ \eta_3(s) \end{bmatrix}^T \begin{bmatrix} Q_1 & V_1 & U_1 \\ * & Q_2 & V_2 \\ * & * & Q_3 \end{bmatrix}$$
$$\times \begin{bmatrix} \eta_1(s) \\ \eta_2(s) \\ \eta_3(s) \end{bmatrix} ds$$
$$V_3(x(t)) = \frac{d_1}{m} \sum_{i=1}^{m} \int_{-\frac{i}{m}d_1}^{-\frac{i-1}{m}d_1} \int_{t+\alpha}^{t} e^{2ks} \dot{x}(s)^T Z_i \dot{x}(s)\, ds d\alpha$$
$$V_4(x(t)) = \int_{t-d(t)}^{t} e^{2ks} \begin{bmatrix} x(s) \\ g_1(Wx(s)) \\ g_2(Wx(s)) \end{bmatrix}^T \begin{bmatrix} Y_1 & M_1 & N_1 \\ * & Y_2 & M_2 \\ * & * & Y_3 \end{bmatrix}$$
$$\times \begin{bmatrix} x(s) \\ g_1(Wx(s)) \\ g_2(Wx(s)) \end{bmatrix} ds$$
$$V_5(x(t)) = \int_{t-d_2}^{t} e^{2ks} \begin{bmatrix} x(s) \\ g_1(Wx(s)) \\ g_2(Wx(s)) \end{bmatrix}^T \begin{bmatrix} Y_4 & M_3 & N_2 \\ * & Y_5 & M_4 \\ * & * & Y_6 \end{bmatrix}$$
$$\times \begin{bmatrix} x(s) \\ g_1(Wx(s)) \\ g_2(Wx(s)) \end{bmatrix} ds$$
$$V_6(x(t)) = \tilde{d} \int_{-d_2}^{-d_1} \int_{t+\alpha}^{t} e^{2ks} \dot{x}(s)^T Z_{m+1} \dot{x}(s)\, ds d\alpha.$$

where

$$\eta_1(t) = \begin{bmatrix} x(t) \\ x(t-\frac{1}{m}d_1) \\ x(t-\frac{2}{m}d_1) \\ \vdots \\ x(t-\frac{m-1}{m}d_1) \end{bmatrix}, \eta_2(t) = \begin{bmatrix} g_1(Wx(t)) \\ g_1(Wx(t-\frac{1}{m}d_1)) \\ g_1(Wx(t-\frac{2}{m}d_1)) \\ \vdots \\ g_1(Wx(t-\frac{m-1}{m}d_1)) \end{bmatrix},$$
$$\eta_3(t) = \begin{bmatrix} g_2(Wx(t)) \\ g_2(Wx(t-\frac{1}{m}d_1)) \\ g_2(Wx(t-\frac{2}{m}d_1)) \\ \vdots \\ g_2(Wx(t-\frac{m-1}{m}d_1)) \end{bmatrix}.$$

Evaluating the derivative of $V(x(t))$ along the solution of NN(5), we obtain

$$\dot{V}_1(x(t)) = e^{2kt}\{2kx(t)^T P x(t) + 2x(t)^T P \dot{x}(t)$$
$$+4k\sum_{i=1}^{n} s_i \int_0^{\hat{W}_i x(t)} g_{1i}(s)\, ds + 2g_1(Wx(t))^T SW\dot{x}(t)$$
$$+4k\sum_{i=1}^{n} s_i \int_0^{\hat{W}_i x(t)} g_{2i}(s)\, ds + 2g_2(Wx(t))^T SW\dot{x}(t)$$
$$+4k\sum_{i=1}^{n} r_i \int_0^{\hat{W}_i x(t)} (l_i s - g_{1i}(s))\, ds$$
$$+2(LWx(t) - g_1(Wx(t)))^T RW\dot{x}(t)$$
$$+4k\sum_{i=1}^{n} r_i \int_0^{\hat{W}_i x(t)} (b_i s - g_{2i}(s))\, ds$$
$$+2(BWx(t) - g_2(Wx(t)))^T RW\dot{x}(t)\}$$

$$\begin{aligned}
&\le e^{2kt}\{2kx(t)^T Px(t)+2x(t)^T P\dot{x}(t)\\
&+4kx(t)^T W^T Sg_1(Wx(t))+2g_1(Wx(t))^T SW\dot{x}(t)\\
&+4kx(t)^T W^T Sg_2(Wx(t))+2g_2(Wx(t))^T SW\dot{x}(t)\\
&+4kx(t)^T W^T R(LWx(t)-g_1(Wx(t)))\\
&+2(LWx(t)-g_1(Wx(t)))^T RW\dot{x}(t)\\
&+4kx(t)^T W^T R(BWx(t)-g_2(Wx(t)))\\
&+2(BWx(t)-g_2(Wx(t)))^T RW\dot{x}(t)\}\\
&=e^{2kt}\{\theta_1(t)^T(2ke_1^T Pe_1-e_1^T(PA+AP)e_1\\
&+4ke_1^T W^T RLWe_1+4ke_1^T W^T RBWe_1-e_1^T W^T LRWAe_1\\
&-e_1^T AW^T RLWe_1-e_1^T W^T BRWAe_1-e_1^T AW^T RBWe_1)\\
&\times\theta_1(t)+\theta_1(t)^T(2e_1^T Pe_1+4ke_1^T W^T Se_1-2e_1^T AW^T Se_1\\
&+2e_1^T W^T LRWe_1+2e_1^T W^T BRWe_1-4ke_1^T W^T Re_1\\
&+2e_1^T AW^T Re_1)\theta_2(t)+\theta_1(t)^T(4ke_1^T W^T Se_1-2e_1^T AW^T Se_1\\
&-4ke_1^T W^T Re_1+2e_1^T AW^T Re_1)\theta_3(t)+\theta_1(t)^T(2e_1^T P\\
&+2e_1^T W^T LRW+2e_1^T W^T BRW)g_2(Wx(t-d(t)))\\
&+\theta_2(t)^T(e_1^T(SW+W^T S)e_1-e_1^T(RW+W^T R)e_1)\theta_2(t)\\
&+\theta_2(t)^T\left(2e_1^T W^T Se_1-2e_1^T W^T Re_1\right)\theta_3(t)\\
&+\theta_2(t)^T\left(2e_1^T SW-2e_1^T RW\right)g_2(Wx(t-d(t)))\\
&+\theta_3(t)^T\left(2e_1^T SW-2e_1^T RW\right)g_2(Wx(t-d(t)))\} \qquad (17)
\end{aligned}$$

where Lemma 1 is applied and

$$\theta_1(t)=\begin{bmatrix}\eta_1(t)\\ x(t-d_1)\end{bmatrix},\theta_2(t)=\begin{bmatrix}\eta_2(t)\\ g_1(Wx(t-d_1))\end{bmatrix},$$
$$\theta_3(t)=\begin{bmatrix}\eta_3(t)\\ g_2(Wx(t-d_1))\end{bmatrix}.$$

$$\begin{aligned}
&\dot{V}_2(x(t))=e^{2kt}\begin{bmatrix}\eta_1(t)\\ \eta_2(t)\\ \eta_3(t)\end{bmatrix}^T\begin{bmatrix}Q_1 & V_1 & U_1\\ * & Q_2 & V_2\\ * & * & Q_3\end{bmatrix}\begin{bmatrix}\eta_1(t)\\ \eta_2(t)\\ \eta_3(t)\end{bmatrix}\\
&-e^{2k\left(t-\frac{d_1}{m}\right)}\begin{bmatrix}\eta_1\left(t-\frac{d_1}{m}\right)\\ \eta_2\left(t-\frac{d_1}{m}\right)\\ \eta_3\left(t-\frac{d_1}{m}\right)\end{bmatrix}^T\begin{bmatrix}Q_1 & V_1 & U_1\\ * & Q_2 & V_2\\ * & * & Q_3\end{bmatrix}\\
&\times\begin{bmatrix}\eta_1\left(t-\frac{d_1}{m}\right)\\ \eta_2\left(t-\frac{d_1}{m}\right)\\ \eta_3\left(t-\frac{d_1}{m}\right)\end{bmatrix}\\
&=e^{2kt}\begin{bmatrix}W_1\theta_1(t)\\ W_1\theta_2(t)\\ W_1\theta_3(t)\end{bmatrix}^T\begin{bmatrix}Q_1 & V_1 & U_1\\ * & Q_2 & V_2\\ * & * & Q_3\end{bmatrix}\begin{bmatrix}W_1\theta_1(t)\\ W_1\theta_2(t)\\ W_1\theta_3(t)\end{bmatrix}\\
&-e^{2k\left(t-\frac{d_1}{m}\right)}\begin{bmatrix}W_2\theta_1(t)\\ W_2\theta_2(t)\\ W_2\theta_3(t)\end{bmatrix}^T\begin{bmatrix}Q_1 & V_1 & U_1\\ * & Q_2 & V_2\\ * & * & Q_3\end{bmatrix}\\
&\times\begin{bmatrix}W_2\theta_1(t)\\ W_2\theta_2(t)\\ W_2\theta_3(t)\end{bmatrix} \qquad (18)
\end{aligned}$$

$$\begin{aligned}
&\dot{V}_3(x(t))=\left(\frac{d_1}{m}\right)^2 e^{2kt}\dot{x}(t)^T\left(\sum_{i=1}^{m}Z_i\right)\dot{x}(t)\\
&-\frac{d_1}{m}\sum_{i=1}^{m}\int_{t-\frac{i}{m}d_1}^{t-\frac{i-1}{m}d_1}e^{2ks}\dot{x}(s)^T Z_i\dot{x}(s)\,ds\\
&\le\left(\frac{d_1}{m}\right)^2 e^{2kt}\dot{x}(t)^T\left(\sum_{i=1}^{m}Z_i\right)\dot{x}(t)\\
&-\frac{d_1}{m}e^{2k(t-d_1)}\sum_{i=1}^{m}\int_{t-\frac{i}{m}d_1}^{t-\frac{i-1}{m}d_1}\dot{x}(s)^T Z_i\dot{x}(s)\,ds\\
&\le\left(\frac{d_1}{m}\right)^2 e^{2kt}\{\theta_1(t)^T e_1^T A\left(\sum_{i=1}^{m}Z_i\right)Ae_1\theta_1(t)\\
&-2\theta_1(t)^T e_1^T A\left(\sum_{i=1}^{m}Z_i\right)e_1\theta_2(t)\\
&-2\theta_1(t)^T e_1^T A\left(\sum_{i=1}^{m}Z_i\right)g_2(Wx(t-d(t)))\\
&+2\theta_2(t)^T e_1^T\left(\sum_{i=1}^{m}Z_i\right)e_1\theta_2(t)\\
&+2\theta_2(t)^T e_1^T\left(\sum_{i=1}^{m}Z_i\right)g_2(Wx(t-d(t)))\\
&+g_2(Wx(t-d(t)))^T\left(\sum_{i=1}^{m}Z_i\right)g_2(Wx(t-d(t)))\}\\
&-e^{2k(t-d_1)}\sum_{i=1}^{m}\theta_1(t)^T(e_i-e_{i+1})^T Z_i(e_i-e_{i+1})\theta_1(t)\\
&\qquad (19)
\end{aligned}$$

where Lemma 2 is applied.

$$\begin{aligned}
&\dot{V}_4(x(t))\le e^{2kt}\begin{bmatrix}x(t)\\ g_1(Wx(t))\\ g_2(Wx(t))\end{bmatrix}^T\begin{bmatrix}Y_1 & M_1 & N_1\\ * & Y_2 & M_2\\ * & * & Y_3\end{bmatrix}\\
&\times\begin{bmatrix}x(t)\\ g_1(Wx(t))\\ g_2(Wx(t))\end{bmatrix}\\
&-(1-\mu)e^{2k(t-d(t))}\begin{bmatrix}x(t-d(t))\\ g_1(Wx(t-d(t)))\\ g_2(Wx(t-d(t)))\end{bmatrix}^T\\
&\times\begin{bmatrix}Y_1 & M_1 & N_1\\ * & Y_2 & M_2\\ * & * & Y_3\end{bmatrix}\begin{bmatrix}x(t-d(t))\\ g_1(Wx(t-d(t)))\\ g_2(Wx(t-d(t)))\end{bmatrix}\\
&\le e^{2kt}\begin{bmatrix}\theta_1(t)\\ \theta_2(t)\\ \theta_3(t)\end{bmatrix}^T\begin{bmatrix}e_1^T Y_1e_1 & e_1^T M_1e_1 & e_1^T N_1e_1\\ * & e_1^T Y_2e_1 & e_1^T M_2e_1\\ * & * & e_1^T Y_3e_1\end{bmatrix}\begin{bmatrix}\theta_1(t)\\ \theta_2(t)\\ \theta_3(t)\end{bmatrix}\\
&-(1-\mu)e^{2k(t-d_2)}\begin{bmatrix}x(t-d(t))\\ g_1(Wx(t-d(t)))\\ g_2(Wx(t-d(t)))\end{bmatrix}^T\begin{bmatrix}Y_1 & M_1 & N_1\\ * & Y_2 & M_2\\ * & * & Y_3\end{bmatrix}\\
&\times\begin{bmatrix}x(t-d(t))\\ g_1(Wx(t-d(t)))\\ g_2(Wx(t-d(t)))\end{bmatrix} \qquad (20)
\end{aligned}$$

$$\dot{V}_5(x(t)) = e^{2kt}\begin{bmatrix} x(t) \\ g_1(Wx(t)) \\ g_2(Wx(t)) \end{bmatrix}^T \begin{bmatrix} Y_4 & M_3 & N_2 \\ * & Y_5 & M_4 \\ * & * & Y_6 \end{bmatrix}$$
$$\times \begin{bmatrix} x(t) \\ g_1(Wx(t)) \\ g_2(Wx(t)) \end{bmatrix} - e^{2k(t-d_2)}\begin{bmatrix} x(t-d_2) \\ g_1(Wx(t-d_2)) \\ g_2(Wx(t-d_2)) \end{bmatrix}^T$$
$$\times \begin{bmatrix} Y_4 & M_3 & N_2 \\ * & Y_5 & M_4 \\ * & * & Y_6 \end{bmatrix}\begin{bmatrix} x(t-d_2) \\ g_1(Wx(t-d_2)) \\ g_2(Wx(t-d_2)) \end{bmatrix}$$
$$= e^{2kt}\begin{bmatrix} \theta_1(t) \\ \theta_2(t) \\ \theta_3(t) \end{bmatrix}^T \begin{bmatrix} e_1^T Y_4 e_1 & e_1^T M_3 e_1 & e_1^T N_2 e_1 \\ * & e_1^T Y_5 e_1 & e_1^T M_4 e_1 \\ * & * & e_1^T Y_6 e_1 \end{bmatrix}$$
$$\times \begin{bmatrix} \theta_1(t) \\ \theta_2(t) \\ \theta_3(t) \end{bmatrix}$$
$$-e^{2k(t-d_2)}\begin{bmatrix} x(t-d_2) \\ g_1(Wx(t-d_2)) \\ g_2(Wx(t-d_2)) \end{bmatrix}^T \begin{bmatrix} Y_4 & M_3 & N_2 \\ * & Y_5 & M_4 \\ * & * & Y_6 \end{bmatrix}$$
$$\times \begin{bmatrix} x(t-d_2) \\ g_1(Wx(t-d_2)) \\ g_2(Wx(t-d_2)) \end{bmatrix} \tag{21}$$

$$\dot{V}_6(x(t)) = \tilde{d}^2 e^{2kt}\dot{x}(t)^T Z_{m+1}\dot{x}(t)$$
$$-\tilde{d}\int_{t-d_2}^{t-d_1} e^{2ks}\dot{x}(s)^T Z_{m+1}\dot{x}(s)\,ds$$
$$\leq \tilde{d}^2 e^{2kt}\dot{x}(t)^T Z_{m+1}\dot{x}(t)$$
$$-\tilde{d}e^{2k(t-d_2)}\int_{t-d_2}^{t-d_1}\dot{x}(s)^T Z_{m+1}\dot{x}(s)\,ds$$
$$\leq \tilde{d}^2 e^{2kt}\{\theta_1(t)^T e_1^T A Z_{m+1} A e_1 \theta_1(t)$$
$$-2\theta_1(t)^T e_1^T A Z_{m+1} e_1 \theta_2(t)$$
$$-2\theta_1(t)^T e_1^T A Z_{m+1} g_2(Wx(t-d(t)))$$
$$+\theta_2(t)^T e_1^T Z_{m+1} e_1 \theta_2(t)$$
$$+2\theta_2(t)^T e_1^T Z_{m+1} g_2(Wx(t-d(t)))$$
$$+g_2(Wx(t-d(t)))^T Z_{m+1} g_2(Wx(t-d(t)))\}$$
$$-e^{2k(t-d_2)}\begin{bmatrix} \theta_1(t) \\ x(t-d(t)) \\ x(t-d_2) \end{bmatrix}^T \begin{bmatrix} e_{m+1} & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix}^T$$
$$\times \begin{bmatrix} Z_{m+1} & -Z_{m+1}+U_2 & -U_2 \\ * & 2Z_{m+1}-U_2-U_2^T & U_2-Z_{m+1} \\ * & * & Z_{m+1} \end{bmatrix}$$
$$\times \begin{bmatrix} e_{m+1} & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix}\begin{bmatrix} \theta_1(t) \\ x(t-d(t)) \\ x(t-d_2) \end{bmatrix} \tag{22}$$

where Lemma 3 is used. On the other hand, by (6)-(7), we have

$$2e^{2kt}\sum_{i=1}^{m+1} g_j\left(Wx\left(t-\frac{i-1}{m}d_1\right)\right)^T D_i$$
$$\times\left(LWx\left(t-\frac{i-1}{m}d_1\right) - g_j\left(Wx\left(t-\frac{i-1}{m}d_1\right)\right)\right) \geq 0 \tag{23}$$

That is

$$\xi_1 = 2e^{2kt}\sum_{i=1}^{m+1}\theta_2(t)^T e_i^T D_i\left(LWe_i\theta_1(t) - e_i\theta_2(t)\right) \geq 0 \tag{24}$$

$$\xi_2 = 2e^{2kt}\sum_{i=1}^{m+1}\theta_3(t)^T e_i^T D_i\left(BWe_i\theta_1(t) - e_i\theta_3(t)\right) \geq 0. \tag{25}$$

We can also get from (6)-(7)

$$\xi_3 = 2e^{2kt}g_1(Wx(t-d(t)))^T D_{m+2}$$
$$\times\left(LWx(t-d(t)) - g_1(Wx(t-d(t)))\right) \geq 0 \tag{26}$$

$$\xi_4 = 2e^{2kt}g_2(Wx(t-d(t)))^T D_{m+2}$$
$$\times\left(BWx(t-d(t)) - g_2(Wx(t-d(t)))\right) \geq 0 \tag{27}$$

and

$$\xi_5 = 2e^{2kt}g_1(Wx(t-d_2))^T D_{m+3}$$
$$\times\left(LWx(t-d_2) - g_1(Wx(t-d_2))\right) \geq 0 \tag{28}$$

$$\xi_6 = 2e^{2kt}g_2(Wx(t-d_2))^T D_{m+3}$$
$$\times\left(BWx(t-d_2) - g_2(Wx(t-d_2))\right) \geq 0. \tag{29}$$

Thus, we have from (17)-(22) and (24)-(29)

$$\dot{V}(x(t)) \leq \dot{V}_1(x(t)) + \dot{V}_2(x(t)) + \dot{V}_3(x(t)) + \dot{V}_4(x(t))$$
$$+\dot{V}_5(x(t)) + \dot{V}_6(x(t)) + \xi_1 + \xi_2 + \xi_3 + \xi_4 + \xi_5 + \xi_6$$
$$= e^{2kt}\rho(t)^T \Xi \rho(t)$$

where

$$\rho(t) = [\ \theta_1(t)^T \quad x(t-d(t))^T \quad x(t-d_2)^T$$
$$\theta_2(t)^T \quad g_1(Wx(t-d(t)))^T \quad g_1(Wx(t-d_2))^T$$
$$\theta_3(t)^T \quad g_2(Wx(t-d(t)))^T \quad g_2(Wx(t-d_2))^T\ ]^T. \tag{30}$$

Therefore, $V(x(t)) \leq V(x(0))$. Furthermore, we obtain

$$V(x(0)) \leq x(0)^T P x(0) + 2\sum_{i=1}^{n} s_i \int_0^{\hat{W}_i x(0)} g_{1i}(s)\,ds$$
$$+2\sum_{i=1}^{n} s_i \int_0^{\hat{W}_i x(0)} g_{2i}(s)\,ds + 2\sum_{i=1}^{n} r_i \int_0^{\hat{W}_i x(0)} (l_i s - g_{1i}(s))\,ds$$
$$+2\sum_{i=1}^{n} r_i \int_0^{\hat{W}_i x(0)} (b_i s - g_{2i}(s))\,ds$$
$$+\int_{-\frac{d_1}{m}}^{0}\begin{bmatrix} \eta_1(s) \\ \eta_2(s) \\ \eta_3(s) \end{bmatrix}^T \begin{bmatrix} Q_1 & V_1 & U_1 \\ * & Q_2 & V_2 \\ * & * & Q_3 \end{bmatrix}\begin{bmatrix} \eta_1(s) \\ \eta_2(s) \\ \eta_3(s) \end{bmatrix} ds$$
$$+\frac{d_1}{m}\sum_{i=1}^{m}\int_{-\frac{i}{m}d_1}^{-\frac{i-1}{m}d_1}\int_{\alpha}^{0}\dot{x}(s)^T Z_i \dot{x}(s)\,ds\,d\alpha$$
$$+\int_{d(0)}^{0}\begin{bmatrix} x(s) \\ g_1(Wx(s)) \\ g_2(Wx(s)) \end{bmatrix}^T \begin{bmatrix} Y_1 & M_1 & N_1 \\ * & Y_2 & M_2 \\ * & * & Y_3 \end{bmatrix}\begin{bmatrix} x(s) \\ g_1(Wx(s)) \\ g_2(Wx(s)) \end{bmatrix} ds$$

$$
+\int_{-d_2}^{0}\begin{bmatrix} x(s) \\ g_1(Wx(s)) \\ g_2(Wx(s)) \end{bmatrix}^T \begin{bmatrix} Y_4 & M_3 & N_2 \\ * & Y_5 & M_4 \\ * & * & Y_6 \end{bmatrix}
\times\begin{bmatrix} x(s) \\ g_1(Wx(s)) \\ g_2(Wx(s)) \end{bmatrix} ds
+\tilde{d}\int_{-d_2}^{-d_1}\int_{\alpha}^{0}\dot{x}(s)^T Z_{m+1}\dot{x}(s)\,dsd\alpha.
$$

From lemma 4, NN (5) and inequality (6)-(7), similar to [41], [47] we have

$$
V(x(0)) \le \Lambda\|\phi(t)-u^*\|^2
$$

where

$$
\begin{aligned}
\Lambda &= \lambda_M(P) + 4\lambda_M\left(W^TW\right) + \lambda_M\left(W^TL^2S^2W\right) \\
&+\lambda_M\left(W^TB^2S^2W\right) + \lambda_M\left(W^TL^2R^2W\right) \\
&+\lambda_M\left(W^TB^2R^2W\right) + \frac{d_1}{m}\{\lambda_M\left(\left(e_1^{-1}\right)^T W_1^T Q_1 W_1 e_1^{-1}\right) \\
&+\lambda_M\left(\left(e_1^{-1}\right)^T W_1^T V_1 Q_2^{-1} V_1^T W_1 e_1^{-1}\right) \\
&+\lambda_M\left(\left(e_1^{-1}\right)^T W_1^T U_1 Q_3^{-1} U_1^T W_1 e_1^{-1}\right) \\
&+\lambda_M\left(W^T L\left(e_1^{-1}\right)^T W_1^T V_2 Q_3^{-1} V_2^T W_1 e_1^{-1} LW\right) \\
&+2\lambda_M\left(W^T L\left(e_1^{-1}\right)^T W_1^T Q_2 W_1 e_1^{-1} LW\right) \\
&+3\lambda_M\left(W^T B\left(e_1^{-1}\right)^T W_1^T Q_3 W_1 e_1^{-1} BW\right)\} \\
&+3\frac{d_1^3}{m}\{\lambda_M(A^T\left(\sum_{i=1}^{m} Z_i\right)A) \\
&+\lambda_M(W^T L\left(\sum_{i=1}^{m} Z_i\right)LW) + \lambda_M(W^T B\left(\sum_{i=1}^{m} Z_i\right)BW)\} \\
&+d_2\{\lambda_M(Y_1) + \lambda_M(Y_4) + \lambda_M\left(M_1 Y_2^{-1} M_1^T\right) \\
&+\lambda_M\left(M_3 Y_5^{-1} M_3^T\right) + \lambda_M\left(N_1 Y_3^{-1} N_1^T\right) + \lambda_M\left(N_2 Y_6^{-1} N_2^T\right) \\
&+2\lambda_M\left(W^T L Y_2 LW\right) + 2\lambda_M\left(W^T L Y_4 LW\right) \\
&+\lambda_M\left(W^T L M_2 Y_3^{-1} M_2^T LW\right) + \lambda_M\left(W^T L M_4 Y_6^{-1} M_4^T LW\right) \\
&+3\lambda_M\left(W^T B Y_3 BW\right) + 3\lambda_M\left(W^T B Y_6 BW\right)\} \\
&+3\tilde{d}^3\{\lambda_M\left(A^T Z_{m+1} A\right) + \lambda_M\left(W^T L Z_{m+1} LW\right) \\
&+\lambda_M\left(W^T B Z_{m+1} BW\right)\}
\end{aligned}
$$

Meanwhile $V(x(t)) \ge e^{2kt}\|x(t)\|^2\lambda_m(P)$, by the Lyapunov stability theory, the proof of Theorem 1 is completed. ■

IV. Dissipativity Analysis

In this section, we will give the results on dissipativity analysis for NN (8) based on the stability conditions that were proposed in Section 3.

Theorem 2: Given scalars λ, $\gamma > 0$, and an integer $m > 0$, NN (8) with (2) is globally exponential stable and strictly $(\mathcal{Q},\mathcal{S},\mathcal{R}) - \lambda - \gamma$−dissipative if there exist matrices $P > 0$, $Z_i > 0\,(i = 1,2,\ldots,m+1)$, $\begin{bmatrix} Q_1 & V_1 & U_1 \\ * & Q_2 & V_2 \\ * & * & Q_3 \end{bmatrix} > 0$, $\begin{bmatrix} Y_1 & M_1 & N_1 \\ * & Y_2 & M_2 \\ * & * & Y_3 \end{bmatrix} > 0$, $\begin{bmatrix} Y_4 & M_3 & N_2 \\ * & Y_5 & M_4 \\ * & * & Y_6 \end{bmatrix} > 0$, $\begin{bmatrix} Z_{m+1} & U_2 \\ * & Z_{m+1} \end{bmatrix} \ge 0$ and diagonal matrices $S = diag\{s_1,s_2,\ldots,s_n\} > 0$, $R = diag\{r_1,r_2,\ldots,r_n\} > 0$, $D_i > 0\,(i = 1,2,\ldots,m+3)$, such that

$$
\bar{\Xi} = \begin{bmatrix}
\Xi_{11} & \Xi_{12} & \Xi_{13} & \Xi_{14} & 0 & 0 & \Xi_{17} & \Xi_{18} & 0 & \Xi_{1,10} & \Xi_{1,11} \\
* & \Xi_{22} & \Xi_{23} & 0 & \Xi_{25} & 0 & 0 & \Xi_{28} & 0 & 0 & 0 \\
* & * & \Xi_{33} & 0 & 0 & \Xi_{36} & 0 & 0 & \Xi_{39} & 0 & 0 \\
* & * & * & \bar{\Xi}_{44} & 0 & 0 & \bar{\Xi}_{47} & \Xi_{48} & 0 & \Xi_{4,10} & \bar{\Xi}_{4,11} \\
* & * & * & * & \Xi_{55} & 0 & 0 & \Xi_{58} & 0 & 0 & 0 \\
* & * & * & * & * & \Xi_{66} & 0 & 0 & \Xi_{69} & 0 & 0 \\
* & * & * & * & * & * & \bar{\Xi}_{77} & \Xi_{78} & 0 & 0 & \bar{\Xi}_{7,11} \\
* & * & * & * & * & * & * & \Xi_{88} & 0 & \Xi_{8,10} & \Xi_{8,11} \\
* & * & * & * & * & * & * & * & \Xi_{99} & 0 & 0 \\
* & * & * & * & * & * & * & * & * & \Xi_{10,10} & \Xi_{10,11} \\
* & * & * & * & * & * & * & * & * & * & \bar{\Xi}_{11,11}
\end{bmatrix} < 0 \tag{31}
$$

$$
\hat{\Xi} = \begin{bmatrix}
\Xi_{11} & \Xi_{12} & \Xi_{13} & \Xi_{14} & 0 & 0 & \Xi_{17} & \Xi_{18} & 0 & \Xi_{1,10} & \Xi_{1,11} \\
* & \Xi_{22} & \Xi_{23} & 0 & \Xi_{25} & 0 & 0 & \Xi_{28} & 0 & 0 & 0 \\
* & * & \Xi_{33} & 0 & 0 & \Xi_{36} & 0 & 0 & \Xi_{39} & 0 & 0 \\
* & * & * & \hat{\Xi}_{44} & 0 & 0 & \hat{\Xi}_{47} & \Xi_{48} & 0 & \Xi_{4,10} & \hat{\Xi}_{4,11} \\
* & * & * & * & \Xi_{55} & 0 & 0 & \Xi_{58} & 0 & 0 & 0 \\
* & * & * & * & * & \Xi_{66} & 0 & 0 & \Xi_{69} & 0 & 0 \\
* & * & * & * & * & * & \hat{\Xi}_{77} & \Xi_{78} & 0 & 0 & \hat{\Xi}_{7,11} \\
* & * & * & * & * & * & * & \Xi_{88} & 0 & \Xi_{8,10} & \Xi_{8,11} \\
* & * & * & * & * & * & * & * & \Xi_{99} & 0 & 0 \\
* & * & * & * & * & * & * & * & * & \Xi_{10,10} & \Xi_{10,11} \\
* & * & * & * & * & * & * & * & * & * & \hat{\Xi}_{11,11}
\end{bmatrix} < 0 \tag{32}
$$

where Ξ_{11}, Ξ_{12}, Ξ_{13}, Ξ_{14}, Ξ_{17}, Ξ_{18}, Ξ_{22}, Ξ_{23}, Ξ_{25}, Ξ_{28}, Ξ_{33}, Ξ_{36}, Ξ_{39}, Ξ_{48}, Ξ_{55}, Ξ_{58}, Ξ_{66}, Ξ_{69}, Ξ_{78}, Ξ_{88} and Ξ_{99} follow the same definitions as those in Theorem 1, and

$$
\begin{aligned}
&\Xi_{1,10} = -e_1^T A M_1^T \\
&\Xi_{1,11} = e_1^T P + e_1^T W^T LRW + e_1^T W^T BRW \\
&-\left(\frac{d_1}{m}\right)^2 e_1^T A\left(\sum_{i=1}^{m} Z_i\right) - \tilde{d}^2 e_1^T A Z_{m+1} - e_1 A M_2^T \\
&\bar{\Xi}_{44} = \Xi_{44} + e_1^T \mathcal{Q} e_1 \\
&\hat{\Xi}_{44} = \Xi_{44} - e_1^T \mathcal{Q} e_1 \\
&\bar{\Xi}_{47} = \Xi_{47} + e_1^T \mathcal{Q} e_1 \\
&\hat{\Xi}_{47} = \Xi_{47} - e_1^T \mathcal{Q} e_1 \\
&\Xi_{4,10} = e_1^T M_1^T \\
&\bar{\Xi}_{4,11} = e_1^T SW - e_1^T RW + \left(\frac{d_1}{m}\right)^2 e_1^T\left(\sum_{i=1}^{m} Z_i\right) + \tilde{d}^2 e_1^T Z_{m+1} \\
&+e_1^T M_2^T + e_1^T \mathcal{S} \\
&\hat{\Xi}_{4,11} = e_1^T SW - e_1^T RW + \left(\frac{d_1}{m}\right)^2 e_1^T\left(\sum_{i=1}^{m} Z_i\right) + \tilde{d}^2 e_1^T Z_{m+1} \\
&+e_1^T M_2^T - e_1^T \mathcal{S} \\
&\bar{\Xi}_{77} = \Xi_{77} + e_1^T \mathcal{Q} e_1 \\
&\hat{\Xi}_{77} = \Xi_{77} - e_1^T \mathcal{Q} e_1 \\
&\bar{\Xi}_{7,11} = e_1^T SW - e_1^T RW + e_1^T \mathcal{S} \\
&\hat{\Xi}_{7,11} = e_1^T SW - e_1^T RW - e_1^T \mathcal{S} \\
&\Xi_{8,10} = M_1^T \\
&\Xi_{8,11} = \left(\frac{d_1}{m}\right)^2\left(\sum_{i=1}^{m} Z_i\right) + \tilde{d}^2 Z_{m+1} + M_2^T \\
&\Xi_{10,10} = -M_1^T - M_1 \\
&\Xi_{10,11} = M_1 - M_2^T \\
&\bar{\Xi}_{11,11} = \left(\frac{d_1}{m}\right)^2\left(\sum_{i=1}^{m} Z_i\right) + \tilde{d}^2 Z_{m+1} + M_2^T + M_2 + \mathcal{R} - \lambda I
\end{aligned}
$$

$$\hat{\Xi}_{11,11} = \left(\frac{d_1}{m}\right)^2 \left(\sum_{i=1}^{m} Z_i\right) + \tilde{d}^2 Z_{m+1} + M_2^T + M_2 - \mathcal{R} + \gamma I.$$

Proof: It is clear that (31)-(32) holds implies (15) holds.Therefore, NN (8) with $\omega(t) = 0$ is stable according to Theorem 1. To prove the dissipativity performance, first, we consider the Lyapunov-Krasovskii functional candidate (16) and the following index for NN (8):

$$J_{\tau,\lambda} = \int_0^\tau \left[\begin{pmatrix} y \\ \omega \end{pmatrix}^T \begin{pmatrix} \mathcal{Q} & \mathcal{S} \\ * & \mathcal{R} \end{pmatrix} \begin{pmatrix} y \\ \omega \end{pmatrix} - \lambda \omega^T \omega \right] dt \quad (33)$$

On the other hand, obviously

$$\xi_7 = 2\left[\dot{x}(t)^T M_1 + \omega(t)^T M_2\right]\left[-\dot{x}(t) - Ax(t) + g_1(W(x(t))) + g_2(W(x(t-d(t)))) + \omega(t)\right] = 0.$$

Applying a similar analysis method employed in the proof of Theorem 1, we have

$$\dot{V}(x(t)) \le \dot{V}_1(x(t)) + \dot{V}_2(x(t)) + \dot{V}_3(x(t)) + \dot{V}_4(x(t)) + \dot{V}_5(x(t)) + \dot{V}_6(x(t)) + \xi_1 + \xi_2 + \xi_3 + \xi_4 + \xi_5 + \xi_6 + \xi_7$$

and

$$\int_0^\tau \left[\begin{pmatrix} y \\ \omega \end{pmatrix}^T \begin{pmatrix} \mathcal{Q} & \mathcal{S} \\ * & \mathcal{R} \end{pmatrix} \begin{pmatrix} y \\ \omega \end{pmatrix} - \lambda \omega^T \omega + \dot{V}(x(t)) \right] dt \le \int_0^\tau \bar{\theta}(t)^T \Xi \bar{\theta}(t)\, dt \quad (34)$$

where

$$\bar{\theta}(t) = \begin{bmatrix} \rho(t) \\ \dot{x}(t) \\ \omega(t) \end{bmatrix}$$

And $\rho(t)$ follows the same definition as that in (30). We can get from (31)

$$\int_0^\tau \left[\begin{pmatrix} y \\ \omega \end{pmatrix}^T \begin{pmatrix} \mathcal{Q} & \mathcal{S} \\ * & \mathcal{R} \end{pmatrix} \begin{pmatrix} y \\ \omega \end{pmatrix} - \lambda \omega^T \omega + \dot{V}(x(t)) \right] dt \le 0 \quad (35)$$

which implies

$$\int_0^\tau \left[\begin{pmatrix} y \\ \omega \end{pmatrix}^T \begin{pmatrix} \mathcal{Q} & \mathcal{S} \\ * & \mathcal{R} \end{pmatrix} \begin{pmatrix} y \\ \omega \end{pmatrix} - \lambda \omega^T \omega \right] dt \le -\int_0^\tau \dot{V}(x(t)) = -V(x(\tau)) + V(x(0)) \quad (36)$$

Thus, under zero initial condition, we have

$$\lambda \langle \omega, \omega \rangle_\tau \ge G(\omega, y, \tau) \quad \forall \tau \ge 0. \quad (37)$$

Second, we consider the LyapunovCKrasovskii functional candidate (16) and another index for NN (8):

$$J_{\tau,\gamma} = \int_0^\tau \left[\gamma \omega^T \omega - \begin{pmatrix} y \\ \omega \end{pmatrix}^T \begin{pmatrix} \mathcal{Q} & \mathcal{S} \\ * & \mathcal{R} \end{pmatrix} \begin{pmatrix} y \\ \omega \end{pmatrix} \right] dt$$

Applying a similar analysis method employed in the proof of above, we can obtain

$$\int_0^\tau \left[\gamma \omega^T \omega - \begin{pmatrix} y \\ \omega \end{pmatrix}^T \begin{pmatrix} \mathcal{Q} & \mathcal{S} \\ * & \mathcal{R} \end{pmatrix} \begin{pmatrix} y \\ \omega \end{pmatrix} + \dot{V}(x(t)) \right] dt \le \int_0^\tau \bar{\theta}(t)^T \hat{\Xi} \bar{\theta}(t)\, dt$$

So, we can get from (32)

$$\int_0^\tau \left[\gamma \omega^T \omega - \begin{pmatrix} y \\ \omega \end{pmatrix}^T \begin{pmatrix} \mathcal{Q} & \mathcal{S} \\ * & \mathcal{R} \end{pmatrix} \begin{pmatrix} y \\ \omega \end{pmatrix} + \dot{V}(x(t)) \right] dt \le 0$$

which implies

$$\int_0^\tau \left[\gamma \omega^T \omega - \begin{pmatrix} y \\ \omega \end{pmatrix}^T \begin{pmatrix} \mathcal{Q} & \mathcal{S} \\ * & \mathcal{R} \end{pmatrix} \begin{pmatrix} y \\ \omega \end{pmatrix} \right] dt \le -\int_0^\tau \dot{V}(x(t)) = -V(x(\tau)) + V(x(0))$$

Therefore, under zero initial condition, we have

$$G(\omega, y, \tau) \ge \gamma \langle \omega, \omega \rangle_\tau \quad \forall \tau \ge 0.$$

In conclusion, NN(8) is strictly $(\mathcal{Q}, \mathcal{S}, \mathcal{R}) - \lambda - \gamma -$dissipative. This completes the proof. ■

V. Numerical examples

In this section, we will provide two examples to show the effectiveness of the proposed methods.

Example 1. Consider the neutral networks system (5) with

$$A = \begin{bmatrix} 2.7 & 0 \\ 0 & 3.9 \end{bmatrix},\ W = \begin{bmatrix} -12.3 & 2.9 \\ 7.4 & -23.6 \end{bmatrix}$$

and take the activation functions

$$g_{11}(s) = \tanh(0.16s),\ g_{12}(s) = \tanh(0.31s)$$
$$g_{21}(s) = \tanh(0.05s),\ g_{22}(s) = \tanh(0.08s).$$

It is clear that the activation functions satisfy (6)-(7) with

$$L = \begin{bmatrix} 0.16 & 0 \\ 0 & 0.31 \end{bmatrix},\ B = \begin{bmatrix} 0.05 & 0 \\ 0 & 0.08 \end{bmatrix}.$$

We choose $d_1 = 0.1$, $\mu = 0.5$ and $d_2 = 1.1$, applying Theorem 1 of this paper, system (5) in this example is globally exponential stable. The simulations of the neural network (5) with initial function $\varphi(t) = \begin{bmatrix} 1.2 & -1.2 \end{bmatrix}^T$ $t \in [-1.1, 0]$ are shown at Fig.1. We can find from Fig. 1 that the corresponding state responses converge to zero.

Example 2. Consider the neutral networks system (8) with

$$A = \begin{bmatrix} 11.5 & 0 \\ 0 & 13.7 \end{bmatrix},\ W = \begin{bmatrix} -0.8 & -2.9 \\ -7.4 & 5.6 \end{bmatrix}.$$

We choose

$$\mathcal{Q} = \begin{bmatrix} -2 & 0 \\ 0 & -0.5 \end{bmatrix},\ \mathcal{S} = \begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix},\ \mathcal{R} = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix},$$
$$\lambda = 2.1,\ \gamma = 0.2,$$

and take the activation functions

$$g_{11}(s) = \frac{0.3(e^s - e^{-s})}{e^s + e^{-s}},\ g_{12}(s) = \frac{0.2(e^s - e^{-s})}{e^s + e^{-s}}$$
$$g_{21}(s) = \frac{0.5(e^s - e^{-s})}{e^s + e^{-s}},\ g_{22}(s) = \frac{0.1(e^s - e^{-s})}{e^s + e^{-s}}.$$

It is clear that the activation functions satisfy (6)-(7) with

$$L = \begin{bmatrix} 0.3 & 0 \\ 0 & 0.2 \end{bmatrix},\ B = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.1 \end{bmatrix}.$$

We choose $d_1 = 0.3$, $\mu = 0.6$ and $d_2 = 1.5$, applying Theorem 2 of this paper, system (8) in this example is globally exponential stable and dissipativity. The simulations of the neural network (8) with initial function $\varphi(t) = \begin{bmatrix} 1.2 & -1.2 \end{bmatrix}^T t \in [-1.5, 0]$ are shown at Fig. 2. We can find from Fig. 2 that the corresponding state responses converge to zero.

VI. Conclusion

Based on the Lyapunov-Krasovskii functional approach, the Jensen integral inequality, the free-weighting matrix technique and the delay partitioning technique, two sufficient conditions have been derived to guarantee the globally exponential stability and $(\mathcal{Q}, \mathcal{S}, \mathcal{R}) - \lambda - \gamma-$dissipative of static neural networks with time delays. The obtained results can be expressed in the form of LMIs and are easy to verify. Two illustrative examples were also given to demonstrate the effectiveness of the proposed methods.

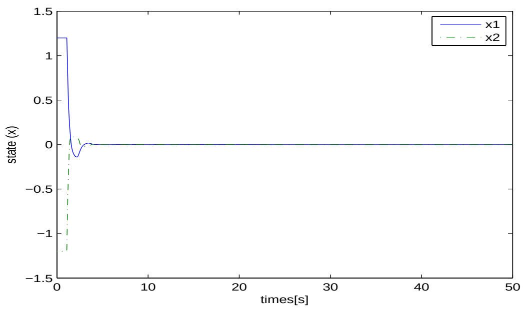

Fig.1 State responses of the NN(5) for Example 1

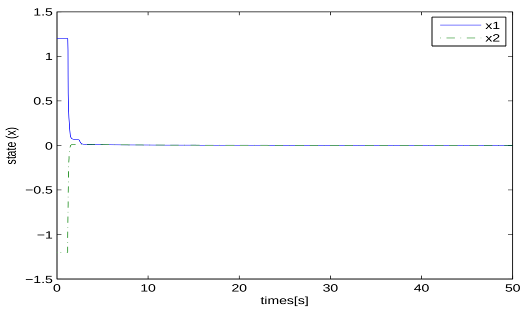

Fig.2 State responses of the NN(8) for Example 2.

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