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Magnetoviscous Effects on Axi-Symmetric Ferrofluid Flow over a Porous Rotating Disk with Suction/Injection

Vikas Kumar

Abstract**—**The present study is carried out to investigate the magneto-viscous effects on incompressible ferrofluid flow over a porous rotating disc with suction or injection on the surface of the disc subjected to a magnetic field. The flow under consideration is axi-symmetric steady ferrofluid flow of electrically non-conducting fluid. Karman's transformation is used to convert the governing boundary layer equations involved in the problem to a system of non linear coupled differential equations. The solution of this system is obtained by using power series approximation. The flow characteristics i.e. radial, tangential, axial velocities and boundary layer displacement thickness are calculated for various values of MFD (magnetic field dependent) viscosity and for different values of suction injection parameter. Besides this, skin friction coefficients are also calculated on the surface of the disk. The results thus obtained are presented numerically and graphically in the paper.

Keywords**—**Axi-symmetric, ferrofluid, magnetic field, porous rotating disk.

I. Introduction

FOR centuries, many fascinating materials have been attracting the scientists and researchers due to their extraordinary physical properties and technological usage. Ferrofluid is one of such smart materials, which are not available free state in nature, but are to be synthesized. These fluids have variety of applications in the field of sciences and engineering etc., which are being commercialized.

Ferrofluids are widely used in sealing of computer hard disk drives, rotating X-ray tubes, rotating shafts and rods. These are used as lubricants in bearing and dumpers. Also ferrofluids are used as heat controller in electric motors and hi-fi speaker systems without the need of change in their geometrical shape [1]. Ferrofluids are being greatly used in many magnetic fluid based scientific devices like sensors, densimeters, accelerometer, pressure transducers etc. and also in actuating machines like electromechanical converters and energy converters etc [2]. In field of biomedicine also, they have been found very useful. There is an idea to use ferrofluids for cancer treatment by heating the tumor soaked in ferrofluids by means of an alternating magnetic field [3], [4].

There are rotationally symmetric flows of the incompressible fluids in the field of fluid mechanics, having all three velocity components; radial, tangential and vertical in space different from zero. In such types of flow, the variables are independent of the angular coordinates and the angular velocity is uniform at large distance from the disk. We consider this type of flow for an incompressible ferrofluid; when the plate is subjected to the magnetic field $(H_r, 0, H_z)$. Rosensweig [5] has given an authoritative introduction to the research on magnetic liquids in his monograph and studied the effect of magnetization, resulting in interesting information.

A study of flow within the boundary layer and its effect on the general flow around the body, in detail, are given in Schlichting [6]. The pioneering study of ordinary viscous fluid flow due to the infinite rotating disc was carried by Karman [7]. He introduced the famous transformation which reduces the governing partial differential equations into ordinary differential equations. Cochran [8] obtained asymptotic solutions for the steady hydrodynamic problem formulated by Karman. Many researchers have extended this problem and improved the solution in various ways [9], [10].

Kavenuke et al. [11] formulated a mathematical governing the fluid flow between a fixed impermeable disk and a rotating porous disk. Frusteri and Osalusi [12] examined the laminar convective and slip flow of an electrically conducting Newtonian fluid with variable properties over a rotating porous disk. Ram et al. [13] discussed the various fluid characteristics of ferrofluid flow in porous medium with rotating disk.

In general, magnetization is a function of magnetic field, temperature and density of the fluid. This leads to convection of ferrofluid in the presence of the magnetic field gradient. Viscosity is also one of the astounding rheological properties of ferrofluid influencing convection flow problems. Detail accounts of magneto-viscous effects in ferrofluids have been given in a monograph by Odenbach [14]. Sunil et al. [15] studied the effect of MFD viscosity on thermosolutal convection in a ferromagnetic fluid saturating a porous medium. Nanjundappa et al. [16] studied Benard-Marangoni ferroconvection in a ferrofluid layer in the presence of uniform vertical magnetic field with MFD viscosity. Ram et al. [17], [18] used power series approximation method to solve the non-linear partial differential equations governing the ferrofluid flow over a rotating disk and discussed the effect of MFD viscosity and porosity on the velocity components and pressure profile. Ram and Kumar [19] investigated the effects of field dependent viscosity on ferrofluid flow saturating the porous medium over a rotating disk. Hall effects on the viscous incompressible fluid due to non-coaxial rotations of an oscillating porous disk and a fluid at infinity are studied by

Vikas Kumar is with National Institute of Technology, Kurukshetra, Haryana 136119 India (phone: 91-94660-19417; fax: 91-1744-238050; e-mail: vksingla.nitkkr@yahoo.com).

Ghara et al. [20]. The ferrofluid flow with heat transfer over a stretchable rotating disk under the influence of field dependent viscosity is investigated by Ram and Kumar [21].

In the present problem, we take cylindrical coordinates (r, θ, z) where z-axis is normal to the plane and this axis is being considered as the axis of rotation. We have presented the boundary layer equations together with boundary conditions. These equations together with the Maxwell's relations are solved theoretically as well as numerically. The effects of MFD viscosity and suction/injection parameters in a circular layer of revolving ferrofluid with rotating disk are studied and various types of ferrofluid responses are considered. This problem, to the best of our knowledge, has not been investigated yet.

II. Mathematical Formulation and Solution of the Problem

Here the ferrofluid flow over a porous rotating disk is studied and cylindrical polar coordinates are used for mathematical formulation of the problem. The flow under consideration is subjected to an externally applied uniform magnetic field $\vec{H}$ with components $(H_r, 0, H_z)$ which is generated by placing a permanent magnet below the disk. The schematic describing the coordinate system and flow model is given in Fig. 1.

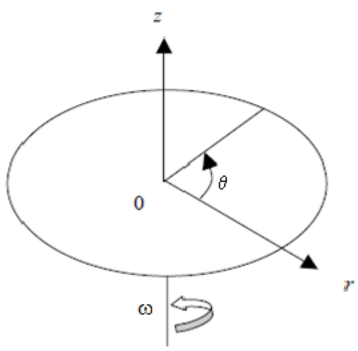


Fig. 1 Schematic diagram

The constitutive equations for the ferrofluid flow over a porous rotating disk with the effects of magnetic field dependent viscosity are given as:

Equation of continuity

$$\nabla \cdot \vec{q} = 0 \tag{1}$$

Equation of motion

$$\rho\left(\frac{\partial \vec{q}}{\partial t} + (\vec{q} \cdot \nabla)\vec{q}\right) = -\nabla p' + \mu_0(\vec{M} \cdot \nabla \vec{H}) + \mu_m \nabla^2 \vec{q} \tag{2}$$

Maxwell's relations

$$\nabla \times \vec{H} = \vec{0}; \quad \nabla \cdot (\vec{H} + \vec{M}) = 0 \tag{3}$$

Assumptions

$$\vec{M} = \chi \vec{H}, \ \vec{M} \times \vec{H} = \vec{0} \tag{4}$$

Here the magnetic field dependent viscosity is given by $\mu_m = \mu(1 + \vec{\delta} \cdot \vec{B})$, where μ is the reference dynamic viscosity, $\vec{\delta}$ the coefficient of viscosity variation and $\vec{B}$ the magnetic induction.

The flow is subjected to following boundary conditions

$$\left.\begin{array}{l} q_r = 0, \ q_\theta = 1, \ \ q_z = s \quad at \quad z = 0 \\ q_r, \, q_\theta \to 0 \ and \ \ q_z \to -C \ \ as \ z \to \infty \end{array}\right\} \tag{5}$$

On considering the assumptions that the flow is steady $\left(i.e. \ \frac{\partial}{\partial t}(\,) = 0\right)$ and axisymmetric $\left(i.e. \ \frac{\partial}{\partial \theta}(\,) = 0\right)$, negligible variation in magnetic field in axial direction, the boundary layer approximation $-\frac{1}{\rho}\frac{\partial p}{\partial r} + \frac{\mu_0}{\rho}\left|\vec{M}\right|\frac{\partial}{\partial r}\left|\vec{H}\right| = -r\omega^2$ in radial direction and using the Karman's similarity transformations

$$\begin{array}{l} q_r = r\omega\, E(\alpha), \ q_\theta = r\omega\, F(\alpha), \ q_z = \sqrt{\upsilon\omega}\, G(\alpha), \\ p = \rho\omega\upsilon\, P(\alpha) \ \ where \ \ \alpha = \sqrt{\frac{\omega}{\upsilon}}\, z. \end{array} \tag{6}$$

We get a system of non linear coupled ordinary differential equations in the dimensionless variables *E, F, G* and *P* as:

$$G' + 2E = 0 \tag{7}$$

$$kE'' - GE' - E^2 + F^2 - 1 = 0 \tag{8}$$

$$kF'' - GF' - 2EF = 0 \tag{9}$$

$$P' - kG'' + GG' = 0 \tag{10}$$

The boundary conditions for the flow reduces to

$$\left.\begin{array}{l} E(0) = 0, \ \ F(0) = 1, \ \ G(0) = w, \ P(0) = P_0 \\ E, \ F, \ P \to 0 \ and \ \ G \to -c \ \ as \ \alpha \to \infty \end{array}\right\} \tag{11}$$

where k and w are the MFD viscosity parameter and suction-injection parameter.

Cochran suggested the solution for such type of system of differential equation in the form of power series approximations in terms of $\exp\left(-\frac{c}{k}\alpha\right)$ as

$$E(\alpha) \approx \sum_{i=1}^{\infty} A_i e^{-i\frac{c}{k}\alpha} \tag{12}$$

$$F(\alpha) \approx \sum_{i=1}^{\infty} B_i e^{-i\frac{c}{k}\alpha} \tag{13}$$

$$G(\alpha) \approx G(\infty) + \sum_{i=1}^{\infty} C_i e^{-i\frac{c}{k}\alpha} \tag{14}$$

$$(P - P_0)(\alpha) \approx \sum_{i=1}^{\infty} D_i e^{-i\frac{c}{k}\alpha} \tag{15}$$

Let $E'(0) = a$ and $F'(0) = b$. Using this supposition and (7)-(10), we get the following boundary conditions for the approximate solution:

$$E''(0) = \frac{wa}{k};\ E'''(0) = \frac{w^2 a}{k^2} \tag{16}$$

$$F''(0) = wb;\ F'''(0) = 2a + w^2 b \tag{17}$$

$$G'(0) = 0;\ G''(0) = -2a;\ G'''(0) = \frac{-2wa}{k} \tag{18}$$

$$P'(0) = -2ak;\ P''(0) = 0;\ P'''(0) = 4b \tag{19}$$

We calculate the values of the first four coefficients $A_1, A_2, A_3, A_4, A_5;\ B_1, B_2, B_3, B_4, B_5;\ C_1, C_2, C_3, C_4, C_5;$ D_1, D_2, D_3, D_4 and D_5, numerically in MATLAB using (12)-(15), the boundary conditions (11), additional boundary conditions (16)-(19) and the values a = 0.54, b = -0 .62 and c = 0.886 from Cochran [8] and obtained the power series expressions for dimensionless velocities and pressure. Then, we draw the graphs for velocity components and the pressure with Karman's dimensionless parameter α.

The boundary layer displacement thickness is calculated as:

$$d = \frac{1}{r\omega}\int_{z=0}^{\infty} v_\theta\, dz = \int_{\alpha=0}^{\infty} F(\alpha)\, d\alpha \tag{20}$$

The radial stress (τ_r) and tangential shear stress (τ_t) thus generated are calculated using Newtonian formulae as:

$$\tau_r = \left[\mu_m\left(\frac{\partial q_r}{\partial z} + \frac{\partial q_z}{\partial r}\right)\right]_{z=0} = \mu\, k\, \mathrm{R}_e^{1/2}\, \omega\, E'(0)$$

$$\tau_t = \left[\mu_m\left(\frac{\partial q_\theta}{\partial z} + \frac{1}{r}\frac{\partial q_z}{\partial \theta}\right)\right]_{z=0} = \mu\, k\, R_e^{1/2}\omega\, F'(0)$$

So, the radial and tangential skin frictions are, respectively given by

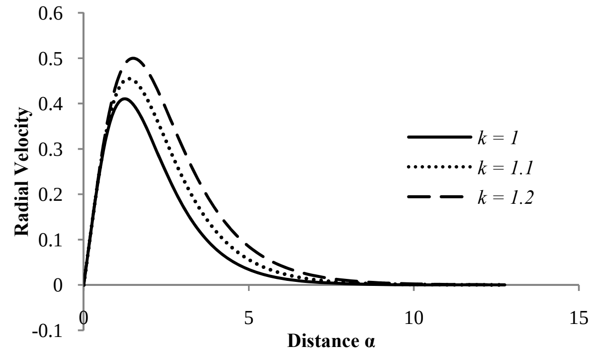
$$R_e^{1/2} C_{f_r} = k\, E'(0) \tag{21}$$

$$R_e^{1/2} C_{f_t} = k\, F'(0) \tag{22}$$

where $R_e = \dfrac{r\omega^2}{\nu}$ is the rotational Reynolds number.

III. Results and Discussion

The problem considered here involves a number of parameters, on the basis of which a wide range of numerical results have been derived, and a small description is made here. In the present problem, a porous disk is rotating with a constant angular velocity (ω) in ferrofluid with magnetic field dependent variable viscosity. The fluid particles which rotate due to the rotation of the disk are in equilibrium due to centrifugal force balanced by the radial pressure gradient and radial magnetization force gradient. Numerical calculations are carried out by considering the coefficient of viscosity variation as isotropic $(\delta_1 = \delta_2 = \delta_3)$ with magnitude 0, 0.01 & 0.02, and magnitude of magnetic field $\vec{B}$ is taken as 10T which results in the viscosity variation parameter k = 1.0, 1.1 & 1.2.



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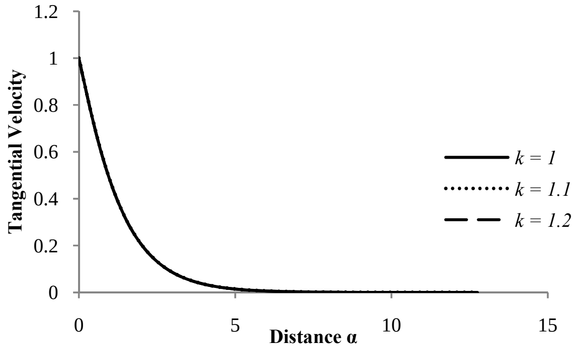

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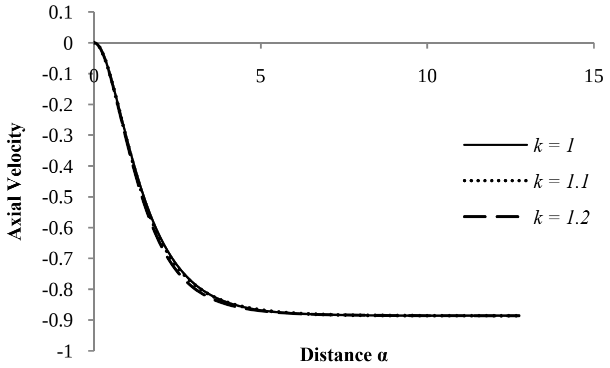

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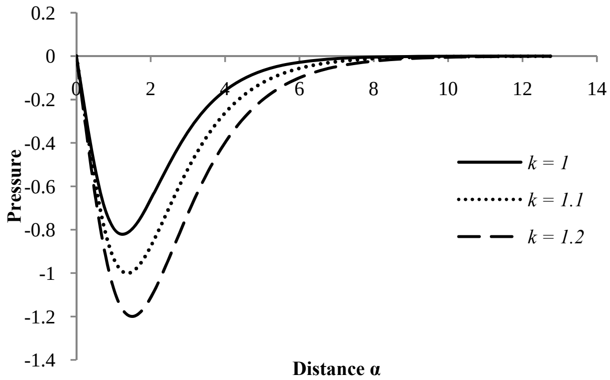

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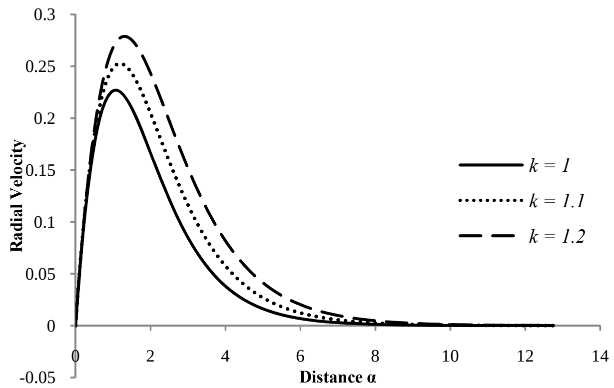
Fig. 2 Effect of MFD viscosity parameter for w = 0 on (a) radial velocity (b) tangential velocity (c) axial velocity (d) pressure



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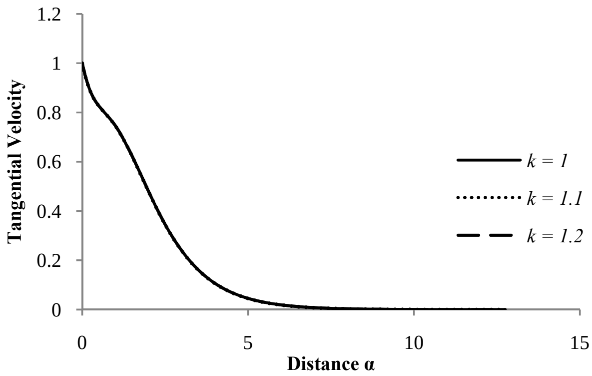

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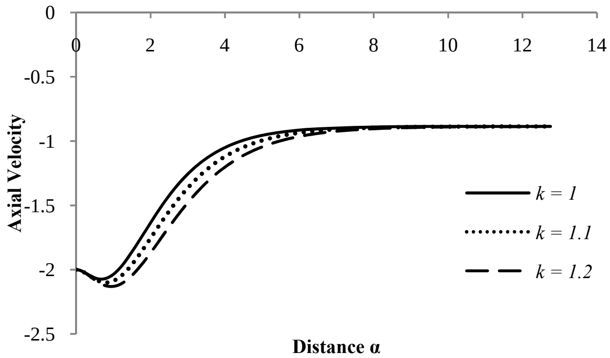

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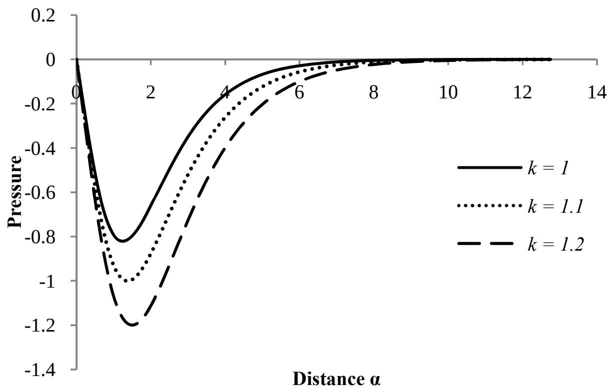

(d)

Fig. 3 Effect of MFD viscosity parameter for w = -2 (case of suction) on (a) radial velocity (b) tangential velocity (c) axial velocity (d) pressure

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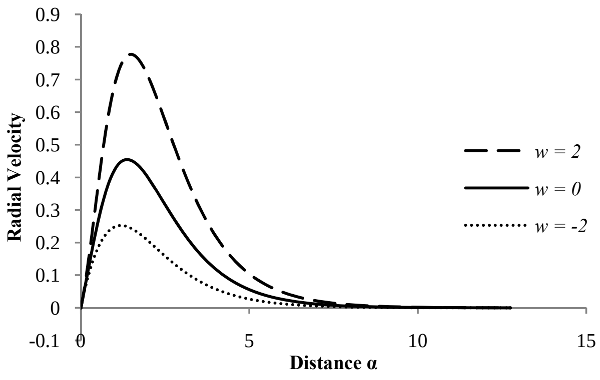

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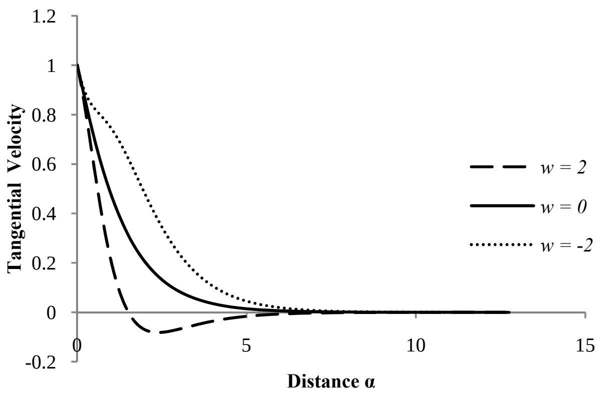

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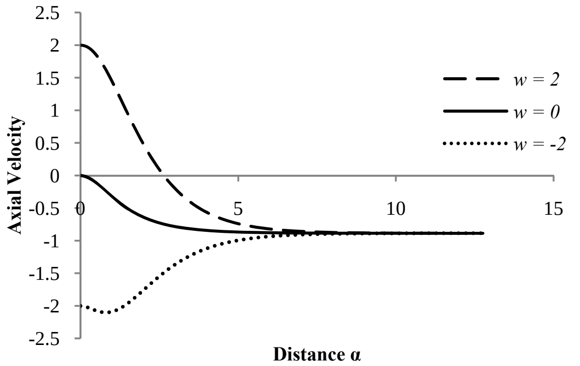

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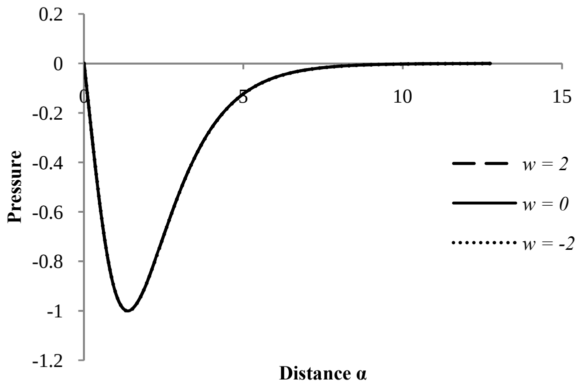

(d)

Fig. 4 Effect of suction/ injection parameter for k = 1.1 on (a) radial velocity (b) tangential velocity (c) axial velocity (d) pressure

Also the values of suction/ injection parameter are taken as w = 2, 0 & -2, where positive value defines the case of injection and negative value is for suction at the wall.

Fig. 2 represents the various velocity profiles and pressure profile for varying values of the MFD viscosity parameter k for the case w = 0 (i.e. the case when there is no suction or injection). It is seen that for increasing values of the viscosity parameter radial velocity increases, tangential velocity remains unaffected, axial velocity shows negligible change and pressure gets decreased. Thus radial velocity is directly proportional and axial velocity & pressure are inversely proportional to the coefficient of viscosity variation.

Fig. 3 demonstrates the radial, tangential, axial velocity profiles and pressure profile for various values of viscosity parameter and for w = -2 (i.e. the case of suction on the surface of disk). In this case too, all the velocities and pressure behaves in the same as in the case of impermeable disk. There is also the same relation between the flow characteristics and the coefficient of viscosity variation.

Fig. 4 shows the effects of suction / injection parameter on the velocity profiles and pressure for a specified value of the viscosity parameter k = 1.1. It is depicted that radial and axial velocities get decreased for suction and increased for injection, whereas the tangential velocity behaves conversely i.e. it gets increased for suction and decreased for injection and the pressure remains unchanged for both of the cases.

The radial skin friction coefficients are 0.53993, 0.59397 & 0.64775 for k = 1.0, 1.1 & 1.2 respectively, which means increasing viscosity increases the radial skin friction. And, the tangential skin friction coefficients are 0.620214, 0.619122 & 0.610234 for w = 2, 0 & -2 respectively *i.e.* tangential skin friction increases for injection and decreases for suction at the wall.

We have also calculated the boundary layer displacement thickens for various values of suction/ injection parameter. The values of displacement thickness are 0.668919, 1.908728 & 3.14879 for w = 2, 0 & -2, respectively. Meaning thereby, boundary layer gets thinner with injection and thicker with suction on the surface of the disk. Whereas in case of Benton

[9] for ordinary viscous fluid flow with constant viscosity and over a non porous disk, the displacement thickness was 1.27144, which means that for ferrofluid and field dependent variable viscosity boundary layer gets thicker.

IV. Conclusions

In nut shell, the magnetic field dependent variable viscosity and suction-injection parameter both play significant role in characterizing the ferrofluid flow over a porous rotating disk. And the present investigation is a theoretical motivation explaining the influence of MFD viscosity on various flow characteristics of ferrofluid over a porous rotating disk. On increasing viscosity parameter, radial component of velocity increases; axial velocity and pressure get decrease whereas tangential velocity remains same. Suction at the wall decreases the radial and axial velocity, and tangential velocity increase, whereas for injection, these velocities behave conversely. Also boundary layer gets thicker for suction and thinner for blowing at the surface of the disk.

The present study has background applications in many areas like rotating machinery, lubrication, oceanography and bio-medicine etc.

Nomenclature

$\vec{H}$	Magnetic field intensity
$\vec{M}$	Magnetization
p'	Fluid pressure
p	Reduced fluid pressure
$\vec{q}$	Velocity of ferrofluid
υ	Kinematic viscosity
μ	Reference viscosity of fluid
μ_m	Magnetic field dependent viscosity
μ_0	Magnetic permeability of free space
ρ	Fluid density
χ	Magnetic susceptibility
∇	Gradient operator
α	Dimensionless parameter
k	MFD viscosity parameter
w	Suction/ injection parameter
r	Radial direction
θ	Tangential direction
z	Axial direction
ω	Angular velocity of disk
q_r	Radial velocity
q_θ	Tangential velocity
q_z	Axial velocity

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Vikas Kumar was born in Haryana (India) on 16 December, 1984. The author has earned M.Sc. degree in Applied Mathematics from Kurukshetra University, Kurukshetra, India in the year of 2007. Author's major field of interest is boundary layer flows over infinite disk in ferrohydrodynamics.

He is presently working as Research Scholar in the Department of Mathematics, National Institute of Technolgy Kurukshetra, India. He has 6 years of experience in teaching and research. He has visited several universities and technical institutions to participate in various research activities and to present his research work. He has published 4 research papers in referred/ SCI international journal of considerable impact and 2 papers in conference proceedings.