Complex Dynamic Behaviors in an Ivlev-type Stage-structured Predator-prey System Concerning Impulsive Control Strategy

Shunyi Li, Zhifang He, Xiangui Xue

Abstract—An Ivlev-type predator-prey system and stage-structured for predator concerning impulsive control strategy is considered. The conditions for the locally asymptotically stable prey-eradication periodic solution is obtained, by using Floquet theorem and small amplitude perturbation skills——when the impulsive period is less than the critical value. Otherwise, the system is permanence. Numerical examples show that the system considered has more complicated dynamics, including high-order quasi-periodic and periodic oscillating, period-doubling and period-halving bifurcation, chaos and attractor crisis, etc. Finally, the biological implications of the results and the impulsive control strategy are discussed.

Keywords—Stage-structured predator-prey system, Impulsive, Permanence, Bifurcation, Chaos.

I. INTRODUCTION

IN the natural world, there are many species whose individual members have a life history that takes them through two stages—immature and mature. Wang and Chen [1] introduced single-species stage-structured model without time delay in 1997, and found that an orbitally asymptotically stable periodic orbit exists in that model. Furthermore, the stage-structured population model without time delay are investigated by many authors [2]. But, there are few papers study the predator-prey system with stage-structured model without time delay and impulsive effect. Song and Xiang [3] considered a two-prey one-predator models with stage structure for the predator and impulsive effects. They show that there exists a globally asymptotically stable pest-eradication periodic solution when the impulsive period is less than some critical value, otherwise, the system is uniformly permanence. Wang, Xu and Feng [4], [5] studied a stage-structured predator-prey system with Holling type-II functional response concerning impulsive control strategy, and the sufficient conditions for existence of a globally stable pest-eradication periodic solution and permanence of the system, numerical simulations are given.

In this paper, we consider an Ivlev-type stage-structured predator-prey system with a constant periodic releasing for the predator and spraying pesticide for prey at fixed moment

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[6]

$$
\left\{
\begin{array}{l}
\left.
\begin{array}{l}
x'(t) = x(t)(a - bx(t)) - (1 - e^{-\alpha x(t)})y_2(t) \\
y_1'(t) = k(1 - e^{-\alpha x(t)})y_2(t) - (m + d_1)y_1(t) \\
y_2'(t) = my_1(t) - d_2 y_2(t)
\end{array}
\right\} t \neq nT, \\
\left.
\begin{array}{l}
\Delta x(t) = -px(t), \\
\Delta y_1(t) = q_1, \\
\Delta y_2(t) = q_2,
\end{array}
\right\} t = nT,
\end{array}
\right.
$$

(1)

where x, y_1, y_2 represent the prey, immature and mature predator populations, respectively; a, b, α, k, d are positive. a is the intrinsic rates of increase of the prey, d_1 and d_2 are the death rate of the immature and mature predator populations, a/b is the carrying capacity of the prey, $k(0 < k < 1)$ is the rate of conversing prey into predator. $m(0 < m < 1)$ is the maturity rate of the predator population. $\Delta x(t) = x(t^+) - x(t), \Delta y_i(t) = y_i(t^+) - y_i(t)(i = 1, 2)$, T is the periodic of the impulsive. $n \in \mathbb{N}, \mathbb{N} = \{1, 2, \cdots\}$, $0 < p < 1$ is the proportionality constant which represents the rate of mortality due to the applied pesticide. $q_i \geq 0 (i = 1, 2)$ is the number of predator released each time.

The paper arranged as follows. Some notations and lemmas are given in the next Section. In Section 3, using the Floquet theory of impulsive equation and small amplitude perturbation skills, we prove the local stability of prey-eradication periodic solution, and give the condition of permanence. The numerical examples and analysis are given in Section 4, moreover, these results are discussed briefly in Section 5.

II. PRELIMINARIES

Let $\mathbb{R}_+ = [0, \infty), \mathbb{R}_+^3 = \{x \in R^3 | x \geq 0\}$. Denote $f = (f_1, f_2, f_3)$ the map defined by the right hand of the first three equations of system (1), and $\mathbb{N}$ be the set of all non-negative integers. Let $V : R_+ \times \mathbb{R}_+^3 \to \mathbb{R}_+$, then V is said to belong to class V_0 if

(1) V is continuous in $(t, x) \in (nT, (n+1)T] \times \mathbb{R}_+^3$ and for each $x \in R_+^3, n \in N, \lim_{(t,y) \to (nT^+, x)} V(t, y) = V(nT^+, x)$ exists.

(2) V is locally Lipschitzian in x.

Definition 1 *Let $V \in V_0$ then for $(t, x) \in (nT, (n+1)T] \times \mathbb{R}_+^3$, the upper right derivative of $V(t, x)$ with respect to the impulsive differential system (1) is defined as $D^+V(t, x) = \lim_{h \to 0^+} \sup \frac{1}{h}[V(t + h, x + hf(t, x)) - V(t, x)]$.*

Definition 2 *System (1) is said to be permanent if there exist two positive constants m, M and T_0 such that each positive*

solution $(x(t), y_1(t), y_2(t))$ of the system (1) satisfies $m \leq x(t) \leq M, m \leq y_i(t) \leq M(i = 1, 2)$, for all $t > T_0$.

The solution of system (1) is a piecewise continuous function $x : \mathbb{R}_+ \mapsto \mathbb{R}^3_+$, $x(t)$ is continuous on $(nT, (n + 1)T], n \in \mathbb{N}$ and $x(nT^+) = \lim_{t \to nT^+} x(t)$ exists, the smoothness properties of f guarantee the global existence and uniqueness of solutions of system (1), for details see [7], [8].

Lemma 1 [7], [8] *Let $X(t)$ is a solution of system (1) with $X(0^+) \geq 0$, then $X(t) \geq 0$ for all $t \geq 0$ and further $X(t) > 0$ for all $t \geq 0$ if $X(0^+) > 0$.*

Finally, we give some basic properties about the following subsystem of system (1)

$$\begin{cases} y_1'(t) = -(m + d)y_1(t), \\ y_2'(t) = my_1(t) - dy_2(t), \\ \Delta y_1(t) = q_1, \Delta y_2(t) = q_2, \end{cases} \begin{aligned} & t \neq nT, \\ & t = nT, \end{aligned} \quad (2)$$

Clearly, system (2) has a periodic solution [4], [5], $t \in (nT, (n + 1)T]$,

$$\tilde{y}_1(t) = \frac{q_1 \exp[-(d_1 + m)(t - nT)]}{1 - \exp[-(d_1 + m)T]},$$

$$\tilde{y}_2(t) = \left[\frac{q_2}{1 - e^{-d_2 T}} + \frac{m d_1}{(d_1 + m - d_2)(1 - e^{-(d_1 + m)T})} \right.$$
$$\left. \times \left(1 + \frac{e^{-d_2 T} - e^{-(d_1 + m)T}}{1 - e^{-d_2 T}} \right) \right] e^{-d_2(t - nT)}$$
$$- \frac{m}{d_1 + m - d_2} \tilde{y}_1(t),$$

where

$$\tilde{y}_1(0^+) = \frac{p_1}{1 - e^{-(d_1 + m)T}},$$

$$\tilde{y}_2(0^+) = \frac{q_2}{1 - e^{-d_2 T}} +$$
$$\frac{m q_1 (e^{-d_2 T} - e^{-(d_1 + m)T})}{(d_1 + m - d_2)(1 - e^{-d_2 T})(1 - e^{-(d_1 + m)T})},$$

Lemma 2 [4], [5] *Let $(\tilde{y}_1(t), \tilde{y}_2(t))$ be a positive periodic solution of (2) and every solution $(y_1(t), y_2(t))$ of (2) with $y_i(0) > 0 (i = 1, 2)$, we have $|y_i(t) - \tilde{y}_i(t)| \to 0 (i = 1, 2)$ when $t \to \infty$.*

Therefore, we obtain the pest-eradication periodic solution $(0, \tilde{y}_1(t), \tilde{y}_2(t))$ for $t \in (nT, (n + 1)T]$.

III. EXTINCTION AND PERMANENCE

Theorem 1 *Let $(x(t), y_1(t), y_2(t))$ be any solution of system (1), then $(0, \tilde{y}_1(t), \tilde{y}_2(t))$ is locally asymptotically stable provided that*

$$T < \frac{\alpha[m q_1 + (m + d_1)q_2]}{a d_2(d_1 + m)} + \frac{1}{a} \ln \left(\frac{1}{1 - p} \right) := T_{\max}.$$

Proof. Define $x(t) = u(t), y_i(t) = v_i(t) + \tilde{y}_i(t) (i = 1, 2)$,

there may be written

$$\begin{pmatrix} u(t) \\ v_1(t) \\ v_2(t) \end{pmatrix} = \Phi(t) \begin{pmatrix} u(0) \\ v_1(0) \\ v_2(0) \end{pmatrix}, \quad 0 \leq t < T,$$

where $\Phi(t)$ satisfies

$$\frac{d\Phi}{dt} = \begin{pmatrix} a - \alpha \tilde{y}_2(t) & 0 & 0 \\ k\alpha \tilde{y}_2(t) & -(m + d) & 0 \\ 0 & m & -d \end{pmatrix} \Phi(t)$$

and $\Phi(0) = I$, the identify matrix. The linearization of the last three equation of (1) become

$$\begin{pmatrix} u(nT^+) \\ v_1(nT^+) \\ v_2(nT^+) \end{pmatrix} = \begin{pmatrix} -p & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} u(nT) \\ v_1(nT) \\ v_2(nT) \end{pmatrix}$$

Hence, if each eigenvalues of

$$M = \begin{pmatrix} -p & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \Phi(T)$$

have absolute values less than one, then the periodic solution $(0, \tilde{y}_1(t), \tilde{y}_2(t))$ is locally stable. Since all eigenvalues of M are

$$\mu_1 = \exp(-d_2 T) < 1,$$
$$\mu_2 = \exp[-(d_1 + m)T] < 1,$$
$$\mu_3 = (1 - p) \exp \left(\int_0^T (a - \alpha \tilde{y}_2(t))dt \right),$$

$|\mu_3| < 1$ if and only if $T < T_{\max}$. According to Floquet theory of impulsive differential equation [7], [8], the prey-eradication solution $(0, \tilde{y}_1(t), \tilde{y}_2(t))$ is locally stable. This completes the proof.

Theorem 3.2. *There exists a constant $M > 0$, such that $x(t) \leq M, y_i(t) \leq M(i = 1, 2)$ for each solution $X(t) = (x(t), y_1(t), y_2(t))$ of system (1) with all t large enough.*

Proof. Let $V(t) = kx(t) + y_1(t) + y_2(t)$. We calculate the upper right derivative of $V(t)$ along a solution of system (1.4) and get the following impulsive differential equation

$$\begin{cases} D^+ V(t)|_{(1)} + \lambda V(t) = kx(t)(a + \lambda - bx(t)) \\ \qquad - (d - \lambda)(y_1(t) + y_2(t)), t \neq nT \\ V(t^+) \leq V(t) + q_1 + q_2, t = nT \end{cases}$$

Let $0 < \lambda < d = \min\{d_1, d_2\}$, then the first equation of above equations is bounded. Select λ_0 and M_0 such that

$$\begin{cases} D^+ V(t) \leq -\lambda_0 V(t) + M_0, & t \neq nT \\ V(t^+) \leq V(t) + q_1 + q_2, & t = nT, \end{cases}$$

where λ_0 and M_0 are two positive constant. According to comparison theorem of impulsive differential equation [7], [8], we have

$$V(t) \leq \frac{M_0}{\lambda_0} + \left(V(0^+) - \frac{M_0}{\lambda_0} \right) \exp(-\lambda_0 t)$$
$$- \frac{q_1 + q_2}{1 - \exp(-\lambda_0 T)} \exp(-\lambda_0 T)$$
$$+ \frac{q_1 + q_2}{1 - \exp(-\lambda_0 T)} \exp[-\lambda_0(t - nT)]$$

where $t \in (nT, (n+1)T]$. Hence

$$\lim_{t \to \infty} V(t) \le \frac{M_0}{\lambda_0} + \frac{q_1 + q_2}{1 - \exp(-\lambda_0 T)}.$$

Therefore $V(t)$ is ultimately bounded. We obtain that each positive solution of (1) is uniformly ultimately bounded. This completes the proof.

Theorem 3 *System (1) is permanent if*

$$T > \frac{\alpha[mq_1 + (m + d_1)q_2]}{a d_2 (d_1 + m)} + \frac{1}{a} \ln \left(\frac{1}{1 - p} \right) := T_{\max}.$$

Proof. Suppose $X(t) = (x(t), y_1(t), y_2(t))$ is a solution of system (1) with $X(0) > 0$. From theorem 2, we may assume $x(t) < M, y_i(t) < M (i = 1, 2)$ and $M > a/b, t \ge 0$. Let $\varepsilon_i (i = 2, 3)$ small enough and

$$m_2 = \frac{q_1[-(d_1 + m)T]}{1 - \exp[-(d_1 + m)T]} - \varepsilon_2,$$

$$m_3 = \frac{q_2 \exp(-d_2 T)}{1 - \exp(-d_2 T)} - \varepsilon_3, \ \varepsilon_i > 0 (i = 1, 2)$$

According to lemma 2, we have $y_{i-1}(t) > m_i (i = 2, 3)$ for all t large enough. We will find $m_1 > 0$ such that $x(t) > m_1$ for all t large enough in following two steps.

Step 1. Since $T > T_{\max}$, we can select $m_4 > 0, \varepsilon_1 > 0$ small enough such that

$$0 < m_4 < \frac{(a - \alpha \varepsilon_1)T - T_{\max}}{(b + k\alpha^2 m M C)T},$$

$$\eta = (1 - p) \exp \left(\int_0^T [a - bm_4 - \alpha(\tilde{y}_2(t) + m\delta C + \varepsilon_1)dt] \right) > 1,$$

where $\delta = k\alpha M m_4$ and

$$C = \frac{2 - e^{-d_2 T}}{d_2(d_1 + m)(1 - e^{-d_2 T})} + \frac{3(1 - e^{-d_2 T}) + e^{-(d_1 + m)T}}{(d_1 + m - d_2)(d_1 + m)(1 - e^{-d_2 T})},$$

We will prove there exists $t_1 > 0$ such that $x(t_1) > 0$. Otherwise, according to the above assumption, we get $y_1'(t) < \delta - (m + d)y_1(t)$. Consider the comparison system,

$$\begin{cases} \left. \begin{array}{l} y_1'(t) = \delta - (m + d_1)y_1(t), \\ y_2'(t) = my_1(t) - d_2 y_2(t), \end{array} \right\} & t \ne nT, \\ \Delta y_1(t) = q_1, \Delta y_2(t) = q_2, & t = nT, \end{cases} \quad (3)$$

By comparison theorem of impulsive differential equation [7], [8] and the lemma 2, we have $y_2(t) \le \tilde{y}_2(t) + \delta mC + \varepsilon_1$ when t large enough, $t \in (nT, (n+1)T]$.

Hence, we can obtain

$$\begin{cases} x'(t) \ge x(t)[a - bm_4 - \alpha(\tilde{y}_2(t) + \delta mC + \varepsilon_1)], & t \ne nT, \\ \Delta x(t) = -px(t), & t = nT. \end{cases} \quad (4)$$

Integrating (4) on $(nT, (n+1)T]$, we have

$$\begin{aligned} & x((n+1)T) \\ & \ge x(nT)(1 - p) \\ & \quad \times \exp \left(\int_0^T [a - bm_4 - \alpha(\tilde{y}_2(t) + m\delta C + \varepsilon_1)dt] \right) \\ & = \eta x(nT) \end{aligned}$$

Then $x((n+h)T) \ge x(nT)\eta^h \to +\infty$ as $h \to +\infty$, which is a contradiction to the boundedness of $x(t)$. Hence, there exists a $t_1 > 0$ such that $x(t_1) > m_4$.

Step 2. If $x(t_1) \ge m_4$ for all $t > t_1$, then our aim is obtained. Hence, we only need to consider those solution which leave the region $\{X(t) \in \mathbb{R}_3^+ \,|\, x(t) < m_4\}$ and re-enter again. Let $t^* = \inf_{t > t_1} \{x(t) < m_4\}$. Then t^* would be impulsive point or non-impulsive point.

1) If t^* an impulsive point, $t^* = n_1 T, n_1 \in \mathbb{N}$. Then $x(t) \ge m_4$ for $t \in [t_1, t^*]$ and $(1 - p)m_4 < x(t^{*+}) = (1 - p)x(t^*) < m_4$. Select $n_2, n_3 \in \mathbb{N}$, such that

$$n_2 T > -\frac{1}{d} \ln \frac{\varepsilon_4}{C_1}, \ (1 - p)^{n_2} \exp(T\sigma)\eta^{n_3} > 1,$$

where $\sigma = a - bm_4 - \alpha M < 0$ and

$$\begin{aligned} C_1 = \Bigg| & y_2(0) - \frac{q_2}{1 - e^{-d_2 T}} - \frac{mq_1 \left(e^{-d_2 T} - e^{-(d_1 + m)T} \right)}{(1 - e^{-d_2 T})(1 - e^{-(d_1 + m)T})} \\ & + m \left(y_1(0) - y_1(0^+) \right) \\ & \times \left[\frac{1 - e^{-(d_1 + m)T}}{(d_1 + m - d_2)(1 - e^{-d_2 T})} - \frac{e^{-d_2 T} - e^{-(d_1 + m)T}}{1 - e^{-d_2 T}} \right] \Bigg| \end{aligned}$$

Let $\overline{T} = (n_2 + n_3)T$. Then, there exist a $t_2 \in [t^*, t^* + \overline{T}]$ such that $x(t_2) \ge m_4$. Otherwise $x(t) < m_4, t_2 \in [t^*, t^* + \overline{T}]$. Similarly to (3), when $t \in [t^* + n_2 T, t^* + \overline{T}]$, we have $|y_2(t) - \tilde{y}_2(t) - m\delta C| \le C_1 e^{-n_2 d_2 T}$. Hence $y_2(t) \le \tilde{y}_2(t) + m\delta C + \varepsilon_4$, which implies (4) holds for $t^* + n_2 T \le t \le t^* + \overline{T}$. As in *step 1*, we have $x(t^* + \overline{T}) \ge x(t^* + n_2 T)\eta^{n_3}$. On the other hand, the first and the fourth equation of (1) give

$$\begin{cases} x'(t) \ge x(t)(a - bm_4 - cM) = \sigma x(t), & t \ne nT, \\ \Delta x(t) = -px(t), & t = nT, \end{cases}$$

Integrating the above equation on $[t^*, t^* + n_2 T]$, we can get $x(t^* + n_2 T) \ge m_4 (1 - p)^{n_2} \exp(n_2 T\sigma)$, thus $x(t^* + \overline{T}) \ge m_4 (1 - p_2)^{n_2} \exp(n_2 T\sigma)\eta^{n_3} > m_4$, a contradiction.

Let $\bar{t} = \inf_{t \ge t^*} \{x(t) \ge m_4\}$, then $x(t) < m_4$ and $x(\bar{t}) \ge m_4$ for $t \in [t^*, \bar{t}]$, by $x'(t) \ge \sigma x(t)$, we have $x(t) \ge m_4 (1 - p)^{n_2 + n_3} \exp(\sigma(n_2 + n_3)T) \doteq m_1$. For $t > \bar{t}$, the same arguments can be continued since $x(\bar{t}) \ge m_4$.

2) $t^* \ne nT, n \in \mathbb{N}$. Then $x(t) \ge m_4$ for $t \in (t_1, t^*)$ and $x(t^*) = m_4$. Suppose $t^* \in (n_1' T, (n_1' + 1)T)$, there are two possible case for $t \in (t^*, (n_1' + 1)T]$.

(i) $x(t) \le m_4$ for all $t \in (t^*, (n_1' + 1)T]$. Similar to case 1), we can prove that there must be a $t_2' \in ((n_1' + 1)T, (n_1' + 1)T + t]$, such that $x(t_2') > m_4$. Let $\tilde{t} = \inf_{t > t^*} \{x(t) > m_4\}$, then $x(t) \le m_4$ for $t \in (t^*, \tilde{t})$ and $x(\tilde{t}) = m_4$. For $t \in (t^*, \tilde{t})$, we have $x(t) \ge m_4 (1 - p)^{n_2 + n_3} \exp[(n_2 + n_3 + 1)\sigma T] :=$

$m_1 < m'_1$. For $t \geq \tilde{t}$, the same arguments can be continued since $x(\tilde{t}) \geq m_4$.

(ii) There exists a $t \in (t^*, (n'_1 + 1)T]$, such that $x(t) > m_4$. Let $\hat{t} = \inf_{t>t^*}\{x(t) \geq m_4\}$, then $x(t) < m_4$ for $t \in [t^*, \hat{t})$ and $x(\hat{t}) = m_4$. For $t \in [t^*, \hat{t})$, we have $x(t) \geq x(t^*) \exp[\sigma(t - t^*)] \geq m_4 \exp(\sigma T) > m_1$. Since $x(\hat{t}) \geq m_4$ for $t > \hat{t}$, the same arguments can be continued. Hence, $x(t) \geq m_1$ for $t > t_1$. The proof is completed.

Note 1. Let

$$f(T) = T - \frac{1}{a} \ln \left(\frac{1}{1-p} \right) - \frac{\alpha[mq_1 + (m + d_1)q_2]}{ad_2(d_1 + m)},$$

since

$$f(0) = -\frac{1}{a} \ln \left(\frac{1}{1-p} \right) - \frac{\alpha[mq_1 + (m + d_1)q_2]}{ad_2(d_1 + m)} < 0,$$

$f(T) \to \infty$ as $T \to \infty$, and $f'(T) = 1 > 0$, so $f(T) = 0$ has a unique positive root, denoted by T_{max}. From theorems 1 and 3 we know T_{max} is a threshold. If $T < T_{max}$, then pest-eradication periodic solution $(0, \tilde{y}_1(t), \tilde{y}_2(t))$ is asymptotically stable. If $T > T_{max}$, then system (1) is permanent.

Note 2. If system (1) without stage-structured, then the unique threshold

$$T'_{max} = \frac{\alpha(q_1 + q_2)}{ad} + \frac{1}{a} \ln \left(\frac{1}{1-p} \right),$$

and

$$T'_{max} - T_{max} = \frac{\alpha}{a} \left[q_1 \left(\frac{1}{d} - \frac{1}{d_2 \left(1 + \frac{d_1}{m}\right)} \right) + q_2 \left(\frac{1}{d} - \frac{1}{d_2} \right) \right],$$

which is little more than T_{max} when $d = d_2$.

IV. NUMERICAL EXAMPLES

Let $a = 5.65, b = 1.45, \alpha = 1.12, k = 0.71, d_1 = 0.25, d_2 = 0.2, m = 0.62, q_1 = 0.25, q_2 = 0.3, p = 0.75$ with $X(0) = (1.0, 1.0, 1.0)$. From theorem 1 we know that the prey-eradication periodic solution is asymptotically stable provided that $T = 0.71 < T_{max} = 0.7193$. A typical prey-eradication periodic solution of the system (1) is shown in Fig. 1, where we observe how the variable $y_1(t)$ and $y_2(t)$ oscillates in a stable cycle. In contrast, the prey $x(t)$ rapidly decreases to zero. According to theorem 3, if the impulsive periodic $T = 0.73 > T_{max} = 0.7193$, the prey eradication solution becomes unstable, then the prey and predator can coexist on a stable positive periodic solution (see Fig. 2).

We have displayed bifurcation diagrams (See Fig. 3) for the pest population x and the predator population y_1, y_2 for impulsive period T over $[0.8, 8]$, one can easily see that the dynamical behavior is very complicated, which includes: (1) high-order quasi-periodic and periodic oscillating, (2) period-doubling and period-halving bifurcation, (3) chaos and attractor crisis, etc. That is to say, the presence of pulses make the dynamical behaviors of system (1) more complex, and impulsive periodic T has very important influence on system (1).
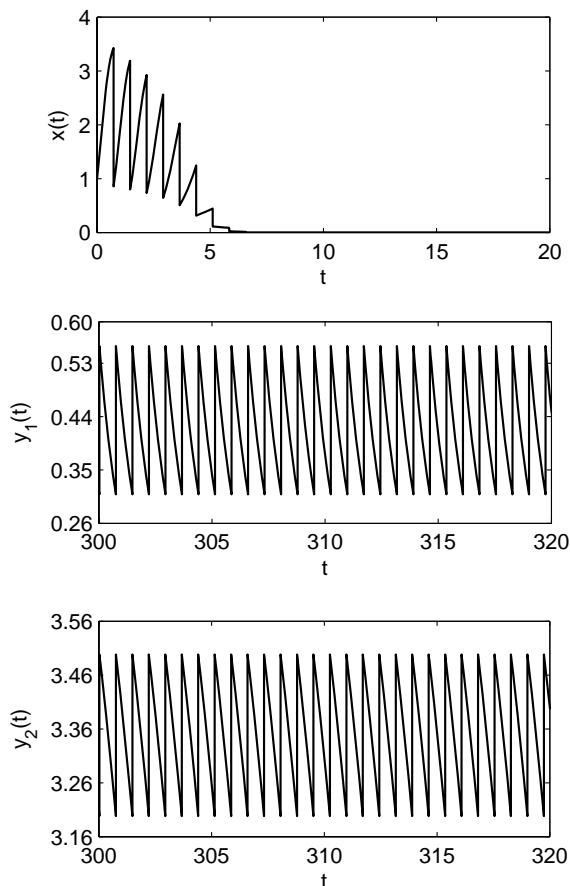

Fig. 1. Time-series of system (1): prey population $x(t)$ eradication and predator population $y_1(t), y_2(t)$ periodic oscillation when $T = 0.71 < T_{max} = 0.7193$.

V. CONCLUSION

In this paper, we have investigated an Ivlev-type predator-prey system with stage-structured and concerning impulsive control strategy for pest control in detail. We have shown that there exists a asymptotically stable pest-eradication periodic solution if the impulsive period is less than the critical value T_{max}. If we choose our impulsive control strategy, in order to drive the pest to extinction, we can determine the impulsive period T according to the effect of the chemical pesticides on the populations and the cost of releasing natural enemies such that $T < T_{max}$. But, in a real world, complete eradication of pest populations is generally not possible, nor is it biologically or economically desirable. A good pest control program should reduce pest population to levels acceptable to the public.

System (1) is permanent and there exists a nontrivial periodic solution when $T > T_{max}$ close to T_{max} (See Fig. 2). The smaller the period, the fewer the pest. And, in order to keep a small quantity of pests such that below some economic threshold (ET is defined as the pest population level that produces damage equal to the costs of preventing damage) by choosing appropriate impulsive period T and the number of mature predator released q_2, making integrated
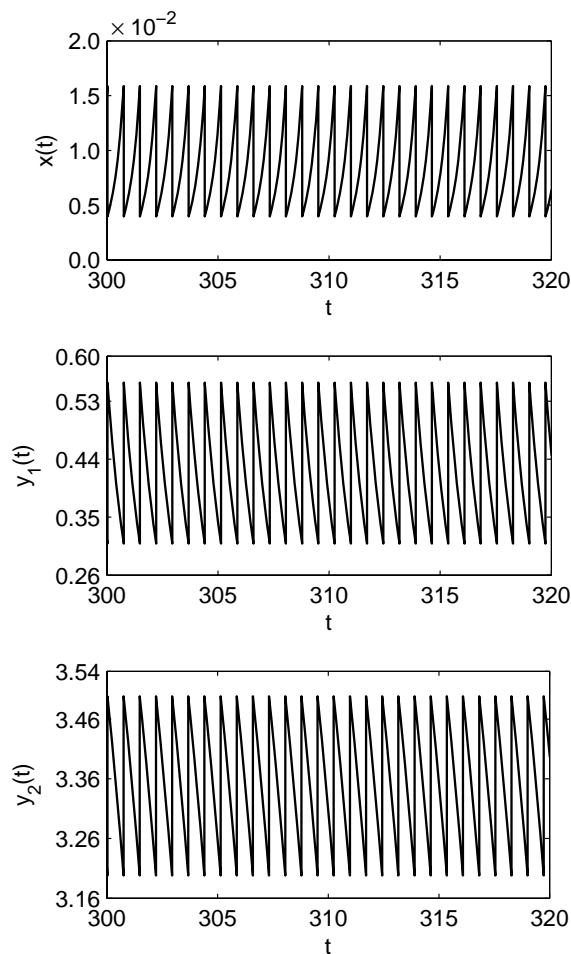
Fig. 2. Time-series of system (1): prey population $x(t)$ periodic oscillation co-existence with predator population $y_1(t), y_2(t)$ when $T = 0.73 > T_{max} = 0.7193$.
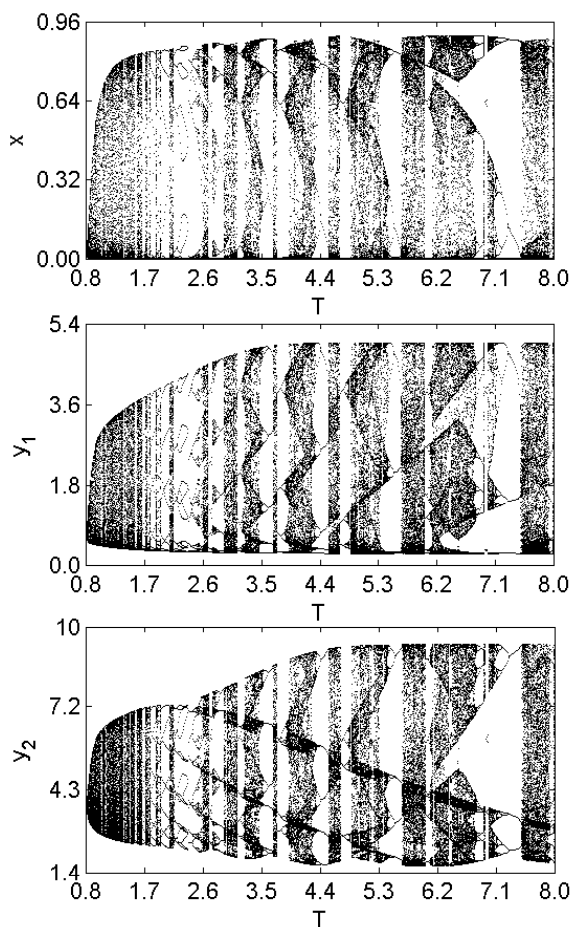

Fig. 3. Bifurcation diagrams of system: prey population $x(t)$, predator population $y_1(t)$ and $y_2(t)$ with T over $[0.8, 8]$. Show the effect of impulsive period T on the dynamical behavior.

pest management strategy every impulsive period. Therefore, the periodic releasing natural enemies and spraying pesticides change the properties of the system without impulses and our results suggest an effective approach in the pest control.

There are some interesting problems: If constant periodic release of predator (natural enemy) and sprays pesticide (or harvests of pest) at different fixed time, how will the period of impulsive effect affect the complexity in the stage-structured predator-prey model Ivlev-type functional response? In a real world, the numbers of releasing natural enemies often change, if we release natural enemies stochastically [9], how does this stochastic noise affect the permanence and extinction of this system? We will continue to study these problems in the future.

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