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On One Mathematical Model for Filtration of Weakly Compressible Chemical Compound in the Porous Heterogeneous 3D Medium. Part I: Model Construction with the Aid of the Ollendorff Approach

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Abstract—A filtering problem of almost incompressible liquid chemical compound in the porous inhomogeneous 3D domain is studied. In this work general approaches to the solution of two-dimensional filtering problems in ananisotropic, inhomogeneous and multilayered medium are developed, and on the basis of the obtained results mathematical models are constructed (according to Ollendorff method) for studying the certain engineering and technical problem of filtering the almost incompressible liquid chemical compound in the porous inhomogeneous 3D domain. For some of the formulated mathematical problems with additional requirements for the structure of the porous inhomogeneous medium, namely, its isotropy, spatial periodicity of its permeability coefficient, solution algorithms are proposed. Continuation of the current work titled "On one mathematical model for filtration of weakly compressible chemical compound in the porous heterogeneous 3D medium. Part II: Determination of the reference directions of anisotropy and permeabilities on these directions" will be prepared in the shortest terms by the authors.

Keywords—Porous media, filtering, permeability, elliptic PDE.

I. Introduction

LIQUID filtering processes occur in porous environment, which, depending on its physical, chemical and mechanical properties, belongs to the group of anisotropic (filtering properties at each point of environment are equal in all directions, i.e. the corresponding functions in mathematical models are scalar functions) or ananisotropic (filtering properties at each point of environment are different in each direction, i.e. the corresponding functions in mathematical models are vector functions) materials. Likewise, porous environment layers can be divided into two types - "active/productive" and "passive/non-productive" layers. In addition, in real processes (objects, phenomena) "active/productive" layers do not only show anisotropic or ananisotropic, homogeneous or inhomogeneous filtering properties, but are always flexuous and with variable thickness.

Similar problems appear in the processes of acquiring liquid energy feedstock (oil, gas; for instance, see [1], [2]); when operating hydro-technical and hydro-ameliorative constructions (for instance, see [3]-[7]), as well as when designing and constructing them; in the fight against the problem of pollution and salification of agricultural areas by ground waters (for instance, see [8], and in other dynamic processes, described by 2D elliptic equations. Solution of such problems requires elaboration of filtering process theory in those models of porous medium, which are most adequate to the natural conditions.

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II. Formulation of the direct linear filtering problem in the anisotropic porous environment

Suppose that our studied inhomogeneous porous domain is an anisotropic structure having a periodic volume (not compulsory with a constant period), and the main element of the structure is a rectangular prism. The coefficient of permeability written as a product

$$k(x,y,z) = K(\alpha,\beta,\gamma) = K^{\{1\}}(\alpha)\, K^{\{2\}}(\beta)\, K^{\{3\}}(\gamma), \tag{1}$$

where $\alpha = \alpha(x,y,z)$, $\beta = \beta(x,y,z)$, $\gamma = \gamma(x,y,z)$ are auxiliary functions of arguments, which, firstly, define the geometry of the periodic structure of the porous environment, and periods by α, β and γ are dimensions of periodic structure elements (rectangular prisms), which form a porous region; and secondly, satisfies these conditions:

1.

$$\langle \nabla\alpha, \nabla\beta\rangle = \langle \nabla\alpha, \nabla\gamma\rangle = \langle \nabla\beta, \nabla\gamma\rangle \equiv 0, \tag{2}$$

where $\langle\cdot,\cdot\rangle$ denotes a scalar derivative;

2.

$$\begin{gathered}
\max\Bigg\{ \int\limits_{\alpha(x,y,z)}^{\alpha(x,y,z)+T_{per.}(\alpha(x,y,z))} \left|\frac{\partial}{\partial\alpha(x,y,z)}\Big[x(\alpha,\beta,\gamma)\,\vec{i}_1 \right. \\
\left. + y(\alpha,\beta,\gamma)\,\vec{i}_2 + z(\alpha,\beta,\gamma)\,\vec{i}_3\Big]\right| d\alpha, \\
\int\limits_{\beta(x,y,z)}^{\beta(x,y,z)+T_{per.}(\beta(x,y,z))} \left|\frac{\partial}{\partial\beta(x,y,z)}\Big[x(\alpha,\beta,\gamma)\,\vec{i}_1 \right. \\
\left. + y(\alpha,\beta,\gamma)\,\vec{i}_2 + z(\alpha,\beta,\gamma)\,\vec{i}_3\Big]\right| d\beta, \\
\int\limits_{\gamma(x,y,z)}^{\gamma(x,y,z)+T_{per.}(\gamma(x,y,z))} \left|\frac{\partial}{\partial\gamma(x,y,z)}\Big[x(\alpha,\beta,\gamma)\,\vec{i}_1 \right. \\
\left. + y(\alpha,\beta,\gamma)\,\vec{i}_2 + z(\alpha,\beta,\gamma)\,\vec{i}_3\Big]\right| d\gamma \Bigg\} \ll L.
\end{gathered} \tag{3}$$

where $T_{per.}\left(\omega\left(x,y,z\right)\right)$ is a period of an argument function $\omega=\{\alpha;\beta;\gamma\}$, and if along one of directions of $\omega=\{\alpha;\beta;\gamma\}$ the coefficient of permeability $k\left(x,y,z\right)$ is constant, then the period $T_{per.}\left(\omega\left(x,y,z\right)\right)$ can be equal to $T_{per.}\left(\bar{\omega}\right)$, $\bar{\omega}=\{\alpha;\beta;\gamma\}/\omega;$ L is a descriptive size of a filtering domain; $\omega\left(\alpha,\beta,\gamma\right)=\sigma^{-1}\left(x,y,z\right)$, $\omega\left(x,y,z\right)=\sigma\left(\alpha,\beta,\gamma\right)$.

3.

$$\min\left\{\int\limits_{\alpha(x,y,z)}^{\alpha(x,y,z)+T_{per.}(\alpha(x,y,z))}\left|\frac{\partial}{\partial\alpha(x,y,z)}\left[x\left(\alpha,\beta,\gamma\right)\vec{i}_1+y\left(\alpha,\beta,\gamma\right)\vec{i}_2+z\left(\alpha,\beta,\gamma\right)\vec{i}_3\right]\right|d\alpha,\right.$$
$$\int\limits_{\beta(x,y,z)}^{\beta(x,y,z)+T_{per.}(\beta(x,y,z))}\left|\frac{\partial}{\partial\beta(x,y,z)}\left[x\left(\alpha,\beta,\gamma\right)\vec{i}_1+y\left(\alpha,\beta,\gamma\right)\vec{i}_2+z\left(\alpha,\beta,\gamma\right)\vec{i}_3\right]\right|d\beta, \tag{4}$$
$$\left.\int\limits_{\gamma(x,y,z)}^{\gamma(x,y,z)+T_{per.}(\gamma(x,y,z))}\left|\frac{\partial}{\partial\gamma(x,y,z)}\left[x\left(\alpha,\beta,\gamma\right)\vec{i}_1+y\left(\alpha,\beta,\gamma\right)\vec{i}_2+z\left(\alpha,\beta,\gamma\right)\vec{i}_3\right]\right|d\gamma\right\}$$
$$\gg\max\left\{\Delta_1,\Delta_2,\Delta_3\right\}.$$

where $\Delta_i\ \left(i=\overline{1,3}\right)$ is a distance between the arc end of the $\omega=\{\alpha;\beta;\gamma\}$-th coordinate line and the end of the corresponding i-th $\left(i=\overline{1,3}\right)$ edge of the structure element (rectangular prism), i.e. if each point $O\left(x,y,z\right)$ of the structure element (rectangular prism) is considered as a local reference point and, if the ends of three edges $OA_i\ \left(i=\overline{1,3}\right)$ of the rectangular prism are marked by points $A_i\left(x,y,z\right)\ \left(i=\overline{1,3}\right)$, and the ends of three arcs $\left(i=\overline{1,3}\right)$ of the corresponding curvilinear prism are marked by points $\overline{A}_i\left(\alpha,\beta,\gamma\right)\ \left(i=\overline{1,3}\right)$, then the value of $\Delta_i\ \left(i=\overline{1,3}\right)$ is determined as $\Delta_i\overset{def}{\equiv}\left|A_i-\bar{A}_i\right|\ \left(i=\overline{1,3}\right)$.

The condition 1 means that in the porous region surfaces of the level $\alpha\left(x,y,z\right)=\alpha_0\equiv const$, $\beta\left(x,y,z\right)=\beta_0\equiv const$ and $\gamma\left(x,y,z\right)=\gamma_0\equiv const$ create a system of triorthogonal surfaces; the condition 2 means that in the whole filtering domain the arc length

$$\int\limits_{\omega(x,y,z)}^{\omega(x,y,z)+T_{per.}(\omega(x,y,z))}\left|\frac{\partial}{\partial\omega(x,y,z)}\left[x\left(\alpha,\beta,\gamma\right)\vec{i}_1+y\left(\alpha,\beta,\gamma\right)\vec{i}_2+z\left(\alpha,\beta,\gamma\right)\vec{i}_3\right]\right|d\omega$$

of the $\omega=\{\alpha,\beta,\gamma\}$ −th coordinate line, which corresponds to the period $T_{per.}\left(\omega\right)$, $\omega=\{\alpha;\beta;\gamma\}$, is infinitesimal in comparison to its descriptive size L; the condition 3 means that in the curvilinear prism, limited by surfaces $\alpha=\alpha_0\equiv const$ and $\alpha=\alpha_0+T_{per.}\left(\alpha\left(x,y,z\right)\right)$; $\beta=\beta_0\equiv const$ and $\beta=\alpha_0+T_{per.}\left(\alpha\left(x,y,z\right)\right)$; $\gamma=\alpha_0\equiv const$ and $\gamma=\gamma_0+T_{per.}\left(\gamma\left(x,y,z\right)\right)$, are lengths with the length

$$\int\limits_{\omega(x,y,z)}^{\omega(x,y,z)+T_{per.}(\omega(x,y,z))}\left|\frac{\partial}{\partial\omega(x,y,z)}\left[x\left(\alpha,\beta,\gamma\right)\vec{i}_1+y\left(\alpha,\beta,\gamma\right)\vec{i}_2+z\left(\alpha,\beta,\gamma\right)\vec{i}_3\right]\right|d\omega$$

are slightly deviated from the corresponding ends of the tangent lines (to be more exact – line segments).

Simplifying conditions (2)-(4) allow considering an elementary curvilinear prism as elementary rectangular prism with edges $OA_i\ \left(i=\overline{1,3}\right)$, with lengths equal to

$$L\left(\omega\right)\overset{def}{\equiv}\int\limits_{\omega(x,y,z)}^{\omega(x,y,z)+T_{per.}(\omega(x,y,z))}\left|\frac{\partial}{\partial\omega(x,y,z)}\left[x\left(\alpha,\beta,\gamma\right)\vec{i}_1+y\left(\alpha,\beta,\gamma\right)\vec{i}_2+z\left(\alpha,\beta,\gamma\right)\vec{i}_3\right]\right|d\omega,\quad\omega=\{\alpha;\beta;\gamma\}. \tag{5}$$

Let us denote this elementary rectangular prism as an elementary approximately-averaged prism.

III. Construction of a Simplified Model on the Basis of Ollendorff Method (see [2], [9], [10])

It is necessary to construct a simplified model, which describes the main filtering properties in anisotropic environments with a coefficient of permeability found by the formula (1), and where the functional coefficients in filtering equations are continuous with respect to space variable functions, which are not compulsory periodic. The general fluid filtering model in inhomogeneous environments with a periodically changing permeability contains filtering equations, where functional coefficients are fast oscillating functions and, in general, are piecewise continuous functions. So, finding of analytical solution for the corresponding initial and boundary problem gets more sophisticated (see fundamental monograph [1] and also [7], [8]. Since in relation to all three homogeneous flows, which are perpendicular to the surfaces of the level $\alpha\left(x,y,z\right)=\alpha_0\equiv const$, $\beta\left(x,y,z\right)=\beta_0\equiv const$ and $\gamma\left(x,y,z\right)=\gamma_0\equiv const$, the porous medium with the permeability coefficient (1) has various filtering properties, then the approximating porous medium with curvilinear layers in the model has to be replaced with a "fictive" ananisotropic environment, which has to have fully identical filtering properties in relation to all three above mentioned flows (generally speaking, not anymore one-dimensional). Depending on such filtering flow approximation level – at a structure element level (i.e. within the limits of an elementary approximately-averaged rectangular prism) or at a filtering level in general - it is possible to talk about approximation methods, namely, about Ollendorff local homogeneously-ananisotropic approximation method (obviously, at first [2], see also [1], [9], [10]) or about Leibenzon integral homogeneously-ananisotropic approximation method (presumably, at first [3], see also [1], [4]-[6], [9]-[11]). Notice that local and integral homogenously-ananisotropic approximation methods set various permeability values in ananisotropic models along $\omega=\{\alpha;\beta;\gamma\}$ coordinate lines (see [1], [6]). Taking into account the assumptions (3)-(5), we will apply Ollendorff method for our problem to construct a filtering model in porous curvilinear layer environment with permeability, which changes periodically in space and the coefficient of which is set by the formula (1). For this purpose let us compare all three one-dimensional flows, which are perpendicular to the surfaces of the level $\alpha(x,y,z) = \alpha_0 \equiv const$, $\beta(x,y,z) = \beta_0 \equiv const$, $\gamma(x,y,z) = \gamma_0 \equiv const$ in an elementary approximately-averaged rectangular prism (a single structure element), which is filled by a liquid (inhomogeneous environment) with permeability coefficients in the form (1), and in the same elementary approximately-averaged rectangular prism, which is filled by a homogeneous environment with rectilinear anisotropy. Because of the condition (3) it is possible to consider that within the boundaries of an elementary structure unit (elementary approximately-averaged rectangular prism) density $\rho(\alpha,\beta,\gamma;t)$, dynamic viscosity $\mu(\alpha,\beta,\gamma;t)$ and other physical characteristics of the liquid are constant, and, moreover, filtering process within the boundaries of this elementary structure unit is done at one moment, i.e. all physical characteristics within the boundaries of an elementary approximately-averaged rectangular prism do not depend on time (more precise mathematical interpretation of this statement/assumption by using Dirac's delta function is stated in the work [12]; see also [13], [14]). So, in the local Cartesian coordinate system $\alpha\times\beta\times\gamma$ (locality is understood this way: for each elementary structure unit its own Cartesian coordinate system is introduced) we can write such widely known liquid filtering equations (see, for example, [1], [12]):

$$\frac{\partial}{\partial\alpha}\left\{K(\alpha,\beta,\gamma)\frac{\partial p(\alpha,\beta,\gamma)}{\partial\alpha}\right\}+\frac{\partial}{\partial\beta}\left\{K(\alpha,\beta,\gamma)\frac{\partial p(\alpha,\beta,\gamma)}{\partial\beta}\right\}$$
$$+\frac{\partial}{\partial\gamma}\left\{K(\alpha,\beta,\gamma)\frac{\partial p(\alpha,\beta,\gamma)}{\partial\gamma}\right\}=0, \tag{6}$$

where $p(\alpha,\beta,\gamma)$ is pressure, and

$$\vec{\vartheta} = -\frac{K(\alpha,\beta,\gamma)}{\mu(\alpha,\beta,\gamma)}\cdot\nabla p, \tag{7}$$

is the Darcy's law, where $\vec{\vartheta} = \vec{\vartheta}(\alpha,\beta,\gamma)$ defines a filtering velocity field within the elementary approximately-averaged rectangular prism.

Then for a one-dimensional filtering flux, which flows along the coordinate line within the elementary approximately-averaged rectangular prism, from (6) we can write $\frac{\partial}{\partial\alpha}\left\{K^{\{1\}}(\alpha)\cdot\frac{\partial p(\alpha,\beta,\gamma)}{\partial\alpha}\right\}=0$, and after integration it gives

$$p(\alpha,\beta,\gamma)=\left[K^{\{1\}}(\alpha)\frac{\partial p(\alpha,\beta,\gamma)}{\partial\gamma}\right]\Big|_{\alpha=0}$$
$$\times\int\limits_0^{\alpha}\frac{d\alpha_1}{K^{\{1\}}(\alpha_1)}+p(\alpha,\beta,\gamma)|_{\alpha=0}.$$

Assuming that a function $p(\alpha,\beta,\gamma)|_{\alpha=L(\alpha)}$ is defined, from the last formula we can find that

$$\left[K^{\{1\}}(\alpha)\frac{\partial p(\alpha,\beta,\gamma)}{\partial\gamma}\right]\Big|_{\alpha=0}$$
$$=\frac{p(\alpha,\beta,\gamma)|_{\alpha=L(\alpha)}+p(\alpha,\beta,\gamma)|_{\alpha=0}}{\int\limits_0^{L(\alpha)}\frac{d\alpha_1}{K^{\{1\}}(\alpha_1)}}$$

Thus, we have got

$$p(\alpha,\beta,\gamma)=\frac{1}{\int\limits_0^{L(\alpha)}\frac{d\alpha_1}{K^{\{1\}}(\alpha_1)}}\Bigg\{p(\alpha,\beta,\gamma)|_{\alpha=L(\alpha)}$$
$$+p(\alpha,\beta,\gamma)|_{\alpha=0}\left[\int\limits_0^{\alpha}\frac{d\alpha_1}{K^{\{1\}}(\alpha_1)}-\int\limits_0^{L(\alpha)}\frac{d\alpha_1}{K^{\{1\}}(\alpha_1)}\right]\Bigg\} \tag{8}$$

where $L(\alpha)=|OA_1|$ is determined by the formula (5) and denotes the length of the elementary approximately-averaged rectangular prism edge OA_1 (we remind the according to the above mentioned construction/assumption the edge OA_1 is located at α-coordinate line; the edge OA_2 is located at β-coordinate line; the edge OA_3 is located at γ-coordinate line); $K^{\{1\}}(\cdot)$ is a function from (1).

The full liquid filtering flux, which flows through the side surface of the elementary approximately-averaged rectangular prism, which is perpendicular to α-coordinate line can be calculated by the formula

$$Q_\alpha \stackrel{def}{\equiv} -\frac{p(\alpha,\beta,\gamma)|_{\alpha=L(\alpha)}-p(\alpha,\beta,\gamma)|_{\alpha=0}}{\int\limits_0^{L(\alpha)}\frac{d\alpha_1}{K^{\{1\}}(\alpha_1)}}$$
$$\times\int\limits_0^{L(\beta)}K^{\{2\}}(\beta_1)\,d\beta_1\int\limits_0^{L(\gamma)}\frac{K^{\{3\}}(\gamma_1)}{\mu(\alpha,\beta_1,\gamma_1)}d\gamma_1.$$

By repeating this assumption, we can find full fluid filtering fluxes, which flow through other two side surfaces of the elementary approximately-averaged rectangular prism, which are perpendicular to the β- and γ-coordinate lines, respectively:

$$Q_\beta \stackrel{def}{\equiv} -\frac{p(\alpha,\beta,\gamma)|_{\beta=L(\beta)}-p(\alpha,\beta,\gamma)|_{\beta=0}}{\int\limits_0^{L(\beta)}\frac{d\beta_1}{K^{\{2\}}(\beta_1)}}$$
$$\times\int\limits_0^{L(\alpha)}K^{\{1\}}(\alpha_1)\,d\alpha_1\int\limits_0^{L(\gamma)}\frac{K^{\{3\}}(\gamma_1)}{\mu(\alpha_1,\beta,\gamma_1)}d\gamma_1,$$

$$Q_\gamma \stackrel{def}{\equiv} -\frac{p(\alpha,\beta,\gamma)|_{\gamma=L(\gamma)}-p(\alpha,\beta,\gamma)|_{\gamma=0}}{\int\limits_0^{L(\gamma)}\frac{d\gamma_1}{K^{\{3\}}(\gamma_1)}}$$
$$\times\int\limits_0^{L(\alpha)}K^{\{1\}}(\alpha_1)\,d\alpha_1\int\limits_0^{L(\beta)}\frac{K^{\{2\}}(\beta_1)}{\mu(\alpha_1,\beta_1,\gamma)}d\beta_1.$$

Moreover, if our assumption that within the elementary approximately-averaged rectangular prism the dynamic viscosity of the liquid is constant (as well as other physical characteristics of the liquid) is valid, then in the last three formulae $\mu(\alpha,\beta,\gamma)=\mu_{const}\equiv const$, and therefore this value can be taken out of the integral sign and in addition, in the right sides of these formulae the multiple integrals are reduced to repeated integrals:

$$Q_\alpha \overset{def}{\equiv} -\frac{p(\alpha,\beta,\gamma)|_{\alpha=L(\alpha)} - p(\alpha,\beta,\gamma)|_{\alpha=0}}{\mu_{const.} \int\limits_0^{L(\alpha)} \frac{d\alpha_1}{K^{\{1\}}(\alpha_1)}} \times \int\limits_0^{L(\beta)} K^{\{2\}}(\beta_1)\, d\beta_1 \int\limits_0^{L(\gamma)} K^{\{3\}}(\gamma_1)\, d\gamma_1, \tag{9}$$

$$Q_\beta \overset{def}{\equiv} -\frac{p(\alpha,\beta,\gamma)|_{\beta=L(\beta)} - p(\alpha,\beta,\gamma)|_{\beta=0}}{\mu_{const.} \int\limits_0^{L(\beta)} \frac{d\beta_1}{K^{\{2\}}(\beta_1)}} \times \int\limits_0^{L(\alpha)} K^{\{1\}}(\alpha_1)\, d\alpha_1 \int\limits_0^{L(\gamma)} K^{\{3\}}(\gamma_1)\, d\gamma_1, \tag{10}$$

$$Q_\gamma \overset{def}{\equiv} -\frac{p(\alpha,\beta,\gamma)|_{\gamma=L(\gamma)} - p(\alpha,\beta,\gamma)|_{\gamma=0}}{\mu_{const.} \int\limits_0^{L(\gamma)} \frac{d\gamma_1}{K^{\{3\}}(\gamma_1)}} \times \int\limits_0^{L(\alpha)} K^{\{1\}}(\alpha_1)\, d\alpha_1 \int\limits_0^{L(\beta)} K^{\{2\}}(\beta_1)\, d\beta_1. \tag{11}$$

As it is shown in [1], [15], [16], in the local Cartesian coordinate system $\alpha\times\beta\times\gamma$ within small volumes (in our case it is an elementary approximately-averaged rectangular prism) it is possible to consider that rectilinear ananisotropic porous environment permeability coefficients along the local Cartesian coordinate axis are constant: $k_\alpha \equiv const$, $k_\beta \equiv const$, $k_\gamma \equiv const$. Then fluid filtering equation within small volume of such rectilinear ananisotropic porous environment looks like this (see [1], [15]-[17]):

$$\sum_{i=1}^{3} k_{\omega_i} \frac{\partial^2 p(\omega_1,\omega_2,\omega_3)}{\partial \omega_i^2} = 0; \{\omega_1;\omega_2;\omega_3\} \overset{def}{\equiv} \{\alpha;\beta;\gamma\}. \tag{12}$$

Moreover, the fluid filtering velocity field is found by the tensor Darcy's law:

$$\vartheta_{\omega_i} = -\frac{k_{\omega_i}}{\mu_{const.}} \frac{\partial p}{\partial \omega_i}; \;\; \{\omega_1;\omega_2;\omega_3\} \overset{def}{\equiv} \{\alpha;\beta;\gamma\}. \tag{13}$$

For a one-dimensional filtering flux, which flows along coordinate line within the elementary approximately-averaged rectangular prism, from (12) we obtain that

$$p(\alpha,\beta,\gamma) = p(\alpha,\beta,\gamma)|_{\alpha=0} + \alpha \frac{p(\alpha,\beta,\gamma)|_{\alpha=L(\alpha)} - p(\alpha,\beta,\gamma)|_{\alpha=0}}{L(\alpha)}.$$

So, we can calculate the full fluid filtering flux, which flows through the side surface of the elementary approximately-averaged rectangular prism, which is perpendicular to the coordinate line:

$$\begin{aligned} Q_\alpha &\overset{def}{\equiv} -\frac{k_\alpha}{\mu_{const}} \int\limits_0^{L(\beta)} d\beta_1 \int\limits_0^{L(\gamma)} \frac{\partial p(\alpha,\beta_1,\gamma_1)}{\partial \alpha} d\gamma_1 \\ &= -\frac{k_\alpha}{\mu_{const}} \int\limits_0^{L(\beta)} d\beta_1 \int\limits_0^{L(\gamma)} \left\{ \frac{p(\alpha,\beta,\gamma)|_{\alpha=L(\alpha)} - p(\alpha,\beta,\gamma)|_{\alpha=0}}{L(\alpha)} \right\} d\gamma_1 \\ &= -\frac{k_\alpha}{\mu_{const}} \frac{L(\beta)L(\gamma)}{L(\alpha)} \left\{ p(\alpha,\beta,\gamma)|_{\alpha=L(\alpha)} - p(\alpha,\beta,\gamma)|_{\alpha=0} \right\}. \end{aligned} \tag{14}$$

Absolutely likewise, we can find full fluid filtering fluxes, which flow through other two side surfaces of the elementary approximately-averaged rectangular prism, which are perpendicular to the $\beta-$ and $\gamma-$ coordinate lines, respectively:

$$\begin{aligned} Q_\beta &\overset{def}{\equiv} -\frac{k_\beta}{\mu_{const}} \int\limits_0^{L(\alpha)} d\alpha_1 \int\limits_0^{L(\gamma)} \frac{\partial p(\alpha_1,\beta,\gamma_1)}{\partial \beta} d\gamma_1 \\ &= -\frac{k_\beta}{\mu_{const}} \int\limits_0^{L(\alpha)} d\alpha_1 \int\limits_0^{L(\gamma)} \left\{ \frac{p(\alpha,\beta,\gamma)|_{\beta=L(\beta)} - p(\alpha,\beta,\gamma)|_{\beta=0}}{L(\beta)} \right\} d\gamma_1 \\ &= -\frac{k_\beta}{\mu_{const}} \frac{L(\alpha)L(\gamma)}{L(\beta)} \left\{ p(\alpha,\beta,\gamma)|_{\beta=L(\beta)} - p(\alpha,\beta,\gamma)|_{\beta=0} \right\}, \end{aligned} \tag{15}$$

$$\begin{aligned} Q_\gamma &\overset{def}{\equiv} -\frac{k_\gamma}{\mu_{const}} \int\limits_0^{L(\alpha)} d\alpha_1 \int\limits_0^{L(\beta)} \frac{\partial p(\alpha_1,\beta_1,\gamma)}{\partial \gamma} d\beta_1 \\ &= -\frac{k_\gamma}{\mu_{const}} \int\limits_0^{L(\alpha)} d\alpha_1 \int\limits_0^{L(\beta)} \left\{ \frac{p(\alpha,\beta,\gamma)|_{\gamma=L(\gamma)} - p(\alpha,\beta,\gamma)|_{\gamma=0}}{L(\gamma)} \right\} d\beta_1 \\ &= -\frac{k_\gamma}{\mu_{const}} \frac{L(\alpha)L(\beta)}{L(\gamma)} \left\{ p(\alpha,\beta,\gamma)|_{\gamma=L(\gamma)} - p(\alpha,\beta,\gamma)|_{\gamma=0} \right\}. \end{aligned} \tag{16}$$

As it was mentioned above, our considered porous region with permeability 1 has various filtering properties in relation to one-dimensional fluxes, which are perpendicular to the surfaces of the level $\alpha(x,y,z) = \alpha_0 \equiv const$, $\beta(x,y,z) = \beta_0 \equiv const$, and $\gamma(x,y,z) = \gamma_0 \equiv const$. Therefore, when modelling, it is necessary to substitute the porous region with curvilinear surfaces with such "fictive" ananisotropic environment, in order to save identical filtering characteristics in relation to the same fluxes. So, it is necessary to find the corresponding approximating "fictive" ananisotropic environment parameters, so that the filtering properties of the modeled porous anisotropic region, the periodic permeability of which is set by the formula (1), would be identical in relation to all three one-dimensional fluxes. For this purpose let us compare the each formula from formulas (9)-(11) to the corresponding formula from formulas (14)-(16):

• from (9) and (14) we obtain that in the local Cartesian coordinate system $\alpha \times \beta \times \gamma$ within small volumes the rectilinear permeability coefficient k_α of the ananisotropic porous environment along the local Cartesian axis has to be chosen (found, calculated) as

$$k_\alpha = \frac{L(\beta)L(\gamma)}{L(\alpha)} \frac{\int\limits_0^{L(\beta)} K^{\{2\}}(\beta_1)\, d\beta_1 \int\limits_0^{L(\gamma)} K^{\{3\}}(\gamma_1)\, d\gamma_1}{\int\limits_0^{L(\alpha)} \frac{d\alpha_1}{K^{\{1\}}(\alpha_1)}}, \tag{17}$$

• from (10) and (15) we obtain that the permeability of this environment has to be chosen as

$$k_\beta = \frac{L(\alpha)L(\gamma)}{L(\beta)} \frac{\int\limits_0^{L(\alpha)} K^{\{1\}}(\alpha_1)\, d\alpha_1 \int\limits_0^{L(\gamma)} K^{\{3\}}(\gamma_1)\, d\gamma_1}{\int\limits_0^{L(\beta)} \frac{d\beta_1}{K^{\{2\}}(\beta_1)}}, \tag{18}$$

• from (11) and (16) we obtain that the permeability of the k_γ-th porous environment has to be chosen as

$$k_\gamma = \frac{L(\alpha)\,L(\beta)}{L(\gamma)}\frac{\int\limits_0^{L(\alpha)} K^{\{1\}}(\alpha_1)\,d\alpha_1 \int\limits_0^{L(\beta)} K^{\{2\}}(\beta_1)\,d\beta_1}{\int\limits_0^{L(\gamma)} \frac{d\gamma_1}{K^{\{3\}}(\gamma_1)}}. \tag{19}$$

Let us denote that by putting the expressions (12) and (13) directly into the expressions (9)-(11) for each elementary approximately-averaged rectangular prism, we will obtain the following expressions for calculation of Q_α, Q_β, Q_γ :

$$Q_\alpha = -\frac{p(\alpha,\beta,\gamma)|_{\alpha=L(\alpha)} - p(\alpha,\beta,\gamma)|_{\alpha=0}}{\mu_{const.}\int\limits_0^{L(\alpha)}\frac{d\alpha_1}{k_\alpha}}\int\limits_0^{L(\beta)} k_\beta d\beta_1 \int\limits_0^{L(\gamma)} k_\gamma d\gamma_1 =$$
$$= -\frac{k_\alpha k_\beta k_\gamma}{\mu_{const.}}\frac{L(\beta)L(\gamma)}{L(\alpha)}\left\{ p(\alpha,\beta,\gamma)|_{\alpha=L(\alpha)} - p(\alpha,\beta,\gamma)|_{\alpha=0}\right\}, \tag{20}$$

$$Q_\beta = -\frac{p(\alpha,\beta,\gamma)|_{\beta=L(\beta)} - p(\alpha,\beta,\gamma)|_{\beta=0}}{\mu_{const.}\int\limits_0^{L(\beta)}\frac{d\beta_1}{k_\beta}}\int\limits_0^{L(\alpha)} k_\alpha d\alpha_1 \int\limits_0^{L(\gamma)} k_\gamma d\gamma_1 =$$
$$= -\frac{k_\alpha k_\beta k_\gamma}{\mu_{const.}}\frac{L(\alpha)L(\gamma)}{L(\beta)}\left\{ p(\alpha,\beta,\gamma)|_{\beta=L(\beta)} - p(\alpha,\beta,\gamma)|_{\beta=0}\right\}, \tag{21}$$

$$Q_\gamma = -\frac{p(\alpha,\beta,\gamma)|_{\gamma=L(\gamma)} - p(\alpha,\beta,\gamma)|_{\gamma=0}}{\mu_{const.}\int\limits_0^{L(\gamma)}\frac{d\gamma_1}{k_\gamma}}\int\limits_0^{L(\alpha)} k_\alpha d\alpha_1 \int\limits_0^{L(\beta)} k_\beta d\beta_1 =$$
$$= -\frac{k_\alpha k_\beta k_\gamma}{\mu_{const.}}\frac{L(\alpha)L(\beta)}{L(\gamma)}\left\{ p(\alpha,\beta,\gamma)|_{\gamma=L(\gamma)} - p(\alpha,\beta,\gamma)|_{\gamma=0}\right\}. \tag{22}$$

Obviously, the formulas (20)-(22) substantially differ from the formulas (14)-(16).

Similarly to the assumptions taken prior to the acquisition of the formulas (17)-(19), it is necessary to compare the each of formulas (9)-(11) to the corresponding formulas (20)-(22). Such comparison gives us the following interesting result:

$$k_\alpha k_\beta k_\gamma = \frac{L(\alpha)}{L(\beta)L(\gamma)}\frac{\int\limits_0^{L(\beta)} K^{\{2\}}(\beta_1)d\beta_1 \int\limits_0^{L(\gamma)} K^{\{3\}}(\gamma_1)d\gamma_1}{\int\limits_0^{L(\alpha)}\frac{d\alpha_1}{K^{\{1\}}(\alpha_1)}}$$
$$= \frac{L(\beta)}{L(\alpha)L(\gamma)}\frac{\int\limits_0^{L(\alpha)} K^{\{1\}}(\alpha_1)d\alpha_1 \int\limits_0^{L(\gamma)} K^{\{3\}}(\gamma_1)d\gamma_1}{\int\limits_0^{L(\beta)}\frac{d\beta_1}{K^{\{2\}}(\beta_1)}}$$
$$= \frac{L(\gamma)}{L(\alpha)L(\beta)}\frac{\int\limits_0^{L(\alpha)} K^{\{1\}}(\alpha_1)d\alpha_1 \int\limits_0^{L(\beta)} K^{\{2\}}(\beta_1)d\beta_1}{\int\limits_0^{L(\gamma)}\frac{d\gamma_1}{K^{\{3\}}(\gamma_1)}}.$$

From here it is obvious that

$$\frac{1}{L^2(\alpha)}\int\limits_0^{L(\alpha)} K^{\{1\}}(\alpha_1)\,d\alpha_1 \int\limits_0^{L(\alpha)}\frac{d\alpha_1}{K^{\{1\}}(\alpha_1)}$$
$$= \frac{1}{L^2(\beta)}\int\limits_0^{L(\beta)} K^{\{2\}}(\beta_1)\,d\beta_1 \int\limits_0^{L(\beta)}\frac{d\beta_1}{K^{\{2\}}(\beta_1)}$$
$$= \frac{1}{L^2(\gamma)}\int\limits_0^{L(\gamma)} K^{\{3\}}(\gamma_1)\,d\gamma_1 \int\limits_0^{L(\gamma)}\frac{d\gamma_1}{K^{\{3\}}(\gamma_1)}.$$

IV. Conclusions

In this work the filtering process is studied in non-homogeneous porous environment, which is anisotropic and periodic in space. Assuming that the coefficient of permeability of the porous environment is a multiplicative function, the following results are obtained:

• a mathematical model is elaborated, which describes a process of linear filtering in the anisotropic environment, the geometric form of which is a region curvilinear surfaces;
• a simplifying model is constructed on the basis of Ollendorff method (simplifying Ollendorff procedure), where in the filtering equation the functions-coefficients of the special variables are supposed to be continuous and not definitely periodic;
• for the simplifying model an analytic formula is found describing the full filtering flow of the fluid, which flows through wall surfaces of an elementary approximately averaged prism;
• for a one-dimensional filtering flow, which flows along the horizontal coordinate line within the limits of the elementary approximately averaged prism an analytical formula is found, which determines pressure by knowing the full filtering flow of the fluid;
• coefficients of permeability for the direct ananisotropic porous environment are defined in the analytical form along the local lines of the Cartesian coordinate system.

Continuation of the current work titled "On one mathematical model for filtration of weakly compressible chemical compound in the porous heterogeneous 3D medium. Part II: Determination of the reference directions of anisotropy and permeabilities on these directions" will be prepared in the shortest terms by the authors.

Moreover, in the further research, by using the results obtained in this work, the authors are going to formulate and investigate the inverse problem of stable determination of the permeability coefficient for the porous anisotropic and ananisotropic environments.

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