Acoustic Instabilities on Swirling Flames

T. Parra, R. Z. Szasz, C. Duwig, R. Pérez, V. Mendoza, F. Castro

***Abstract*—**The POD makes possible to reduce the complete high-dimensional acoustic field to a low-dimensional subspace where different modes are identified and let reconstruct in a simple way a high percentage of the variance of the field.

Rotating modes are instabilities which are commonly observed in swirling flows. Such modes can appear under both cold and reacting conditions but that they have different sources: while the cold flow rotating mode is essentially hydrodynamic and corresponds to the wellknown PVC (precessing vortex core) observed in many swirled unconfined flows, the rotating structure observed for the reacting case inside the combustion chamber might be not hydrodynamically but acoustically controlled. The two transverse acoustic modes of the combustion chamber couple and create a rotating motion of the flame which leads to a self-sustained turning mode which has the features of a classical PVC but a very different source (acoustics and not hydrodynamics).

***Keywords*—**Acoustic field, POD, swirling flames.

I. Introduction

THE study of the acoustic waves involves several issues which have to be considered: noise generation, wave propagation, combustion-acoustic wave interaction and solid-structure-acoustic wave interaction among others.

There are several sources of acoustic waves in a gas turbine combustion chamber. First, the pressure fluctuations due to the vortices in a turbulent flow field act as acoustic sources. Second, fluctuations in the velocity field and / or species concentration cause fluctuations in the heat release rate which lead to variable expansion rate of the fluid, and by this to acoustic waves. Third, the presence of solid bodies (e.g. a flame holder) in high-speed flows generates as well noise.

Not all the fluctuations generated in a flow field are propagated as acoustic waves. To study the propagation of fluctuations in a flow field it is useful to decompose the fluctuations in three canonical types of disturbances: vortical, entropy and acoustic. While the acoustic waves are transported with the speed of sound, the vorticity and entropy disturbances are transported with the velocity of the flow. Furthermore, the vorticity and the entropy disturbances are convected out from the domain at an open exit. The acoustic disturbances propagate as waves, thus they are reflected. Additionally acoustic waves refract if there are jumps in the flow conditions and diffract at obstacles.

The acoustic fluctuations produce oscillations of the combustion chamber walls. The oscillations of the walls, however, act as noise sources themselves, exciting acoustic waves. The phenomenon is extremely dangerous, if the acoustic fluctuations of the flow match a resonant frequency of the solid structure, since it may lead to failure of the device.

T. Parra, R. Perez and F. Castro are with the Dept. Energy Engineering & Fluid Mechanics, University of Valladolid. Paseo del Cauce 59, 47011 Valladolid, Spain (Phone: (+34) 983 184536 Email: terpar@eii.uva.es).

R. Z. Szasz, C. Duwig are with the Division of Fluid Mechanics, Department of Energy Sciences. Lund University, Sweden.

V. Mendoza is with the Dept. of Mechanical Engineering, Universidad de Tarapacá, Chile.

II. Numerical Modeling of Noise Generation and Propagation

A. Lighthill's Analogy

Lighthill's acoustic analogy was one of the first attempts to describe acoustic wave generation and propagation. It is based on the assumption of scale separation between the acoustic and the flow fields. The basic idea is to decompose the flow variables ϕ into a part describing a quiescent medium, ϕ_0 and a fluctuant part ϕ'.

$$\phi = \phi_0+\phi' \qquad (1)$$

Starting from the set of compressible Navier-Stokes equations (continuity and momentum conservation), one can derive a wave equation describing the propagation of acoustic perturbations as follows. Subtracting the spatial derivative of the momentum equations from the time derivative of the continuity equation, one obtains:

$$\rho_{,tt} - \left(\rho u_i u_j\right)_{,ij} = \left(p\delta_{ij} - \sigma_{ij}\right)_{,ij} \qquad (2)$$

If c_0 denotes the constant speed of sound in the quiescent medium, the equation can be rewritten as:

$$\rho_{,tt} - c_0^2\rho_{,ii} = \left(\rho u_i u_j\right)_{,ij} + \left((p - c_0^2\rho)\delta_{ij} - \sigma_{ij}\right)_{,ij} \qquad (3)$$

The equation is a non-homogeneous wave equation. The source term due to viscous effects is usually small in comparison with the other source terms, thus the last term on the RHS equation can be neglected. Applying the decomposition $\phi = \phi_0+\phi'$ one obtains the Lighthill's analogy in terms of the acoustic density fluctuations.

$$(\rho')_{,tt} - c_0^2(\rho')_{,ii} = \left(T_{ij}\right)_{,ij} \qquad (4)$$

where T_{ij} is the Lighthill stress tensor. Lighthill's acoustic analogy describes the propagation of density fluctuation waves with a constant speed of sound c_0. The source in the equation has two terms which represent two different mechanism for sound generation. The first term represents the noise generated by the unsteadiness of the flow, while the second term represents the compressibility effects.

$$(T_{ij})_{,ij} = (\rho u_i u_j)_{,ij} + \left((p - c_0^2\rho)\delta_{ij}\right)_{,ij} \quad (5)$$

Lighthill's analogy is a relativity simple method to describe the acoustic field. It is specially efficient for low – Mach number cases, where direct computation of the acoustic field would require large computational resources. The computation of the flow field, providing the source term, can be decoupled from the computation of the acoustic field. Bailly et al. computed the flow field using RANS-based methods for turbulence modelling. RANS-based methods, however, have limited accuracy, since the source terms for Lighthill's acoustic analogy are decoupled based on the mean flow quantities.

The major limitation of Lighthill's analogy is that it cannot account for sound generated by solid surfaces. Furthermore, the acoustic density fluctuation obtained from Lighthill's analogy includes the fluctuations of the flow field itself (e.g. due to vorticity) which should be propagated by the convection speed of the flow concluding that the accuracy of the method is reduced in regions with strong fluctuations of the flow variables.

In this work, the Lighthill's acoustic analogy has been used to compute the acoustic field generated by one swirling jet in a typical gas turbine combustor. The flow has been computed using LES for turbulence modeling [1] and [2]. Since the jet was situated in a confined environment, the acoustic field has been solved in a different grid from that of the flow. The flow was considered isothermal which lead to small values of the entropy source term (second term of the equation) as compared to the source generated by the unsteadiness of the flow. The acoustic source term has been computed at each timestep. Using different grid for the flow and the acoustic computations has the inconvenient that interpolations were needed to transfer the source term from the flow grid to the acoustic grid.

B. Governing Equations

The acoustic flow computations were base on Lighthill's acoustic analogy

$$(\rho')_{,tt} - \frac{1}{M^2}(\rho')_{,ii} = (T_{ij})_{,ij} \quad (6)$$

In the equation ρ' is the acoustic density fluctuation, M the Mach number in the undisturbed medium and T_{ij} the acoustic source term given by

$$(T_{ij})_{,ij} = (\bar{\rho}\tilde{u}_i\tilde{u}_j)_{,ij} + \left(\left(\bar{p} - \frac{1}{M^2}\bar{\rho}\right)\delta_{ij}\right)_{,ij} \quad (7)$$

The source term was given by the flow solver, which resolved the semi-compressible equations of continuity, momentum and energy [3].

For the spatial discretization, a Cartesian staggered grid is used. The Cartesian grid is chosen because of the advantages it presents in comparison with other methods: the computational cost per cell is highly reduced, faster convergence, easy implementation of higher. Order discretization schemes, low storage requirements, fast grid generation. These advantages make Cartesian grids suitable for LES computations. The major disadvantage of the method is that non brick-shaped geometries are difficult to approximate with high accuracy.

For the spatial discretization, the second order space derivative has been discretized using a second order central scheme.

III. POD (Proper Orthogonal Decomposition)

POD is a powerful tool for analyzing data since it let identify patterns in data, and express the data in such a way as to highlight their similarities and differences [4]. The other advantage of POD is that once you have found these patterns in the data, and you can compress the data, i.e. by reducing the number of dimensions, without much loss of information.

Here are the steps to follow:

- Get as many samples per cycle and as many periods as possible of a cross section (2D snapshot) of a variable, in this case, the fluctuation of pressure. Hence N data set of values
- Subtract the mean from each of the values of all the data points of the cross section.
- Calculate the covariance matrix which is a square matrix NxN for each point.
- Calculate the eigenvectors and eigenvalues of the covariance matrix
- Choosing components and forming a feature mode. The eigenvector with the highest eigenvalue is the principle mode that represents the highest energy of the flow.
- Projecting snapshots on the POD modes to form new data set for the principal modes. Once the main modes have been chosen (eigenvectors of high eigenvalues) to get the mode, take the transpose of the vector and multiply it on the left of the original data set transposed.

The proper orthogonal decomposition (POD) is a method that is used on experimental and numerical results to reduce the data necessary to get the main characteristics of the turbulent flows which contain a wide range of length scales due to eddies of different sizes.

The data is expanded in terms of ortho-normal base vectors such that this representation is optimal in the sense that the required number of terms recovering the energy of the field is minimal. The method can be used to extract spatially dominant features, i.e. coherent structures, from space-time data, in this case data obtained by the acoustic model. The data of pressure fluctuation $p(x_0, y, z, t)$ is decomposed into a set of M empirical, mutually orthogonal eigenfunctions, or POD modes. The random variable p can then be reconstructed from linear combinations of the POD modes which are multiplied with the amplitude coefficients $c_m(t)$. The POD modes are purely spatial correlations with no time dependence. The amplitude coefficients are independent of space and simulate the amplitude of the corresponding POD mode as function of time. Thus, the POD of p into M modes is called p_M and given by:

$$p_M(y,z,t) = \sum_{m=1}^{M} c_m(t)\, \phi_m(y,z) \tag{8}$$

The POD modes are chosen such that the difference between p and pM is minimal on average. The eigenvalue λm is the variance of the random variable set in the direction of the eigenmode ϕ_m. Therefore the eigenvalues are ordered by magnitude, and the set of the largest M eigenvalues contains the most energetic modes ϕ_M: coherent structures. The total energy is then the sum of all eigenvalues. The fraction of energy km represented in the mode ϕm is therefore given by

$$k_m = \frac{\lambda_m}{\sum_{i=1}^{M} \lambda_i} \tag{9}$$

Therefore, it is necessary a high number of samples per period and number of periods to capture the main phenomena and to get the certitude the modes are ranking correctly according the energy spectrum.

Because of the swirling nature of the flame, the modes are coupled by pairs of nearly equal energy, hence mode 1 and 2 are represented together, as well as 3 and 4, 5 and 6 ...

The Proper Orthogonal Decomposition, also known as Principal Component Analysis, is a useful procedure to process large amount of possibly correlated data to obtain the description of the main phenomena through a smaller number of uncorrelated data called modes. Hence it is useful to process experimental and numerical turbulent flow as far as there was enough samples to capture main characteristics of the flow.

From the evidence of the eigenvalue spectrum, it was decided to truncate at twenty eigenvectors, capturing about 95% of the variance. The reconstruction coefficients for each data point snapshots were then passed to the routine, which was therefore presented with a data set in twenty dimensions. The modes themselves are points in the twenty-dimensional reconstruction coefficient space, but they can be visualized by projecting each onto the principal component eigenvectors.

IV. Results

As for postprocess tasks, instantaneous density fluctuation fields and its correspondent RMS have been analyzed, see Fig. 1 and 2. POD has been performed using 200 samples.

Mode 0, Fig. 3, is the averaged field, hence a necessary condition of the number of samples is its symmetrical behavior. Because of the swirling nature of the flow, the modes are coupled by pairs of nearly equal energy, hence mode 1 and 2 are similar, as well as 3 and 4, Figs. 4 and 5 respectively.

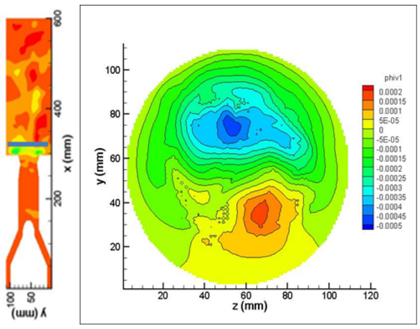

Fig. 1 Instantaneous field of density fluctuations. Left: Longitudinal snapshot. Right: Transversal snapshot @ X = 320 mm

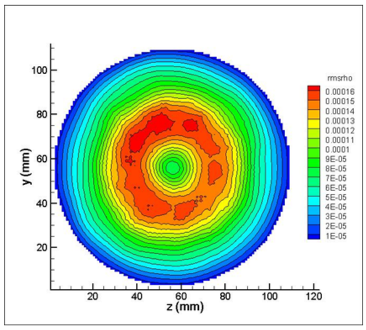

Fig. 2 Density RMS field. Snapshot @ X = 320 mm

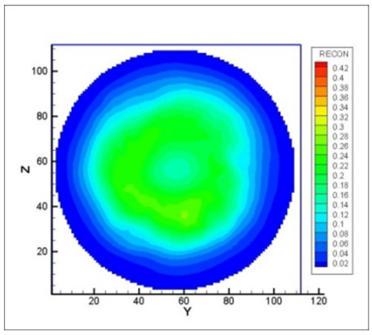

Fig. 3 Mode 0 averaged field. Snapshot @ X = 320 mm

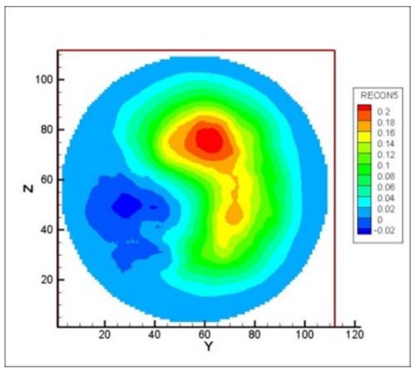

Fig. 4 Modes 1-2 axial vortex. Snapshot @ X = 320 mm

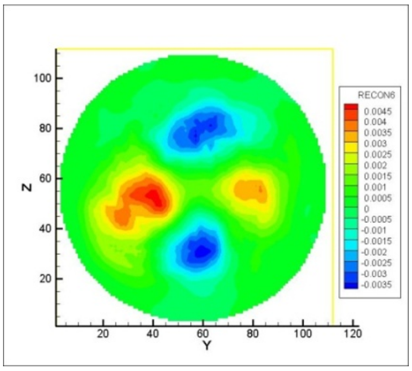

Fig. 5 Modes 3-4 transversal vortex. Snapshot @ X = 320 mm

V. Conclusion

The Proper Orthogonal Decomposition (POD) has being performed on the provisional results. It is a useful tool to perform a realistic identification of the most energetic flow structures.

Acknowledgment

The authors thankfully acknowledge the computer resources, technical expertise and assistance provided by the Barcelona Supercomputing Center (ref. FI-2013-2-0002). Besides, this research was developed in the framework of the project reference ENE2011-25468 from the Spanish Ministry of Science and Innovation.

References

[1] C. Duwig C. and L. Fuchs L. "Large eddy simulation of vortex breakdown/flame interaction" Physics of Fluids 19, 075103 2007.
[2] C. Duwig, B. Gherman, M. Mihaescu, M. Salewski, L. Fuchs. "numerical study of thermo-acoustic waves generation by a swirling flame using a new approach based on large eddy simulation" Proceedings of ASME TURBO EXPO 2005 USA.
[3] R.Z. Szasz, C. Duwig, "Noise generated by a lifted flame in a vitiated co-flow" Proceedings of ASME TURBO EXPO 2008 Germany.
[4] C. Duwig, A. Lacarelle, M. Beutke, C.O. Paschereit, L. Fuchs. Study of the vortex breakdown in a conical swirler using LDV, LES and POD.H. Proceedings of ASME TURBO EXPO 2007 Canada.