Method of moments applied to a cuboidal cavity resonator: Effect of gravitational field produced by a black hole

Arti Vaish and Harish Parthasarathy

Abstract—This paper deals with the formulation of Maxwell's equations in a cavity resonator in the presence of the gravitational field produced by a blackhole. The metric of space-time due to the blackhole is the Schwarzschild metric. Conventionally, this is expressed in spherical polar coordinates. In order to adapt this metric to our problem, we have considered this metric in a small region close to the blackhole and expressed this metric in a cartesian system locally.

Keywords—Method of moments, General theory of relativity, Electromagnetism, Metric tensor, schwarzchild metric, Wave Equation.

I. INTRODUCTION

THE law of electromagnetic wave propagation inside a cavity resonator is governed by the Helmholtz equation. This equation is derived from the standard wave equation by assuming sinusoidal time dependence [1], [2]. Such an equation is valid if the space-time manifold is flat, i.e., Minkowskian. In the presence of a gravitational field, according to Einstein's general theory of relativity, the space-time manifold becomes curved and the geometry of such a curved manifold is described by a Riemannian metric [3], [4]. Consequently, the wave equation in such a curved space-time needs to be modified to account for the curvature [5]. In addition, the assumption of sinusoidal time dependence gives a modified Helmholtz equation [6], [7].

A. Black hole: Background

The simplest possible black hole is one that has mass but neither charge nor angular momentum. These black holes are often referred to as Schwarzschild black holes after the physicist Karl Schwarzschild who discovered this solution in 1915 [8]. The Reissner-Nordstrom solution describes a black hole with electric charge, while the Kerr solution yields a rotating black hole. The most general known stationary black hole solution is the Kerr-Newman metric having both charge and angular momentum [9]. All these general solutions share the property that they converge to the Schwarzschild solution at distances that are large compared to the ratio of charge and angular momentum to mass. While the mass of a black hole can take any positive value, the other two properties

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charge and angular momentum are constrained by the mass [10]. In natural units, the total charge Q and the total angular momentum J are expected to satisfy for a black hole of mass M. Black holes saturating this inequality are called extremal [11]. In 1974, Stephen Hawking showed that black holes are not entirely black but emit small amounts of thermal radiation [12]. He got this result by applying quantum field theory in a static black hole background. The result of his calculations is that a black hole should emit particles in a perfect black body spectrum. This effect has become known as Hawking radiation. Since Hawking's result many others have verified the effect through various methods .

The temperature of the emitted black body spectrum is proportional to the surface gravity of the black hole [13]-[16]. For a Schwarzschild black hole this is inversely proportional to the mass. Consequently, large black holes are very cold and emit very little radiation.

B. Black hole: Introduction

The Schwarzschild metric is valid for the objects like black holes. Black holes form in the early phases of the evolution of the universe as a consequences of the strong fluctuations in the dense primordial medium. These are expected to have small masses and are called mini black holes. More massive black holes form later on during phase transitions. They also form when massive bodies that are no longer pressure supported by radiation, collapse under the gravitational force [17]. Black holes are described by the general theory of relativity. According to this theory when a large amount of mass is present in a sufficiently small region of space, all paths through space are warped inwards towards the center of the volume, preventing all matter and radiation within it from escaping. General relativity describes a black hole as a region of empty space with a point like singularity at the center and an event horizon at the outer edge. Research on this subject shows that when the effects of quantum mechanics are taken into account, rather than holding captured matter forever, black holes may slowly leak a form of thermal energy called Hawking radiation and have a finite life [18], [19], [20]. According to Einsteins general theory of relativity, as mass is added to a degenerate star a sudden collapse will take place and the intense gravitational field of the star will close in on itself. Such a star then forms a black hole in the universe. According to the theory of special relativity two concepts introduced by Albert Einstein are needed to explain here [3].

The first is that time and space are not two independent concepts, but are interrelated forming a single continuum, spacetime. This continuum has some special properties, as an object is not free to move around spacetime , instead it must always move forwards in time, and not only must an object move forwards in time, it also cannot change its position faster than the speed of light. The second concept is the base of general relativity; mass deforms the structure of this spacetime. The effect of a mass on spacetime can informally be described as tilting the direction of time towards the mass. As a result, objects tend to move towards masses. This is experienced as gravity. This tilting effect becomes more pronounced as the distance to the mass becomes smaller. At some point close to the mass the tilting becomes so strong that all the possible paths an object can take lead towards the mass [21]. According to the "No Hair" theorem a black hole has only three independent physical properties: mass, charge and angular momentum [22]. Any two black holes that share the same values for these properties are completely indistinguishable.

1) Problem formulation: Suppose (X, Y, Z) are the cartesian coordinates of the centre of our system relative to the centre of the blackhole. Then the metric is expressed in terms of differentials dx, dy, dz where $(X + x, Y + y, Z + z)$ are the cartesian coordinates of out point relative to the centre of the blackhole. The metric is then linearized in the mass of the blackhole yielding a local metric having the form of the conventional Minkowski metric of flat space-time plus a small perturbation about the Minkowski metric due to the presence of the gravitational field of the blackhole. The Maxwell equations are then written down for this metric. The model assumed is a box (cavity resonator) with centre at (X, Y, Z) and the point $(X + x, Y + y, Z + z)$ varies inside the box. The Maxwell equations are expressed in terms of the electromagnetic four potential yielding after applying the Lorentz gauge condition, a perturbed wave equation.

The Schwarzschild metric in spherical coordinate system is given by,

$$
\begin{aligned}
ds^2 &= -C^2 d\tau^2 \\
&= -\left(1 - \frac{2GM}{rC^2}\right) C^2 dt^2 + \left(1 - \frac{2GM}{rC^2}\right)^{-1} \\
&\quad dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2
\end{aligned}
\tag{1}
$$

C. Transformation of Schwarzschild metric in to cartesian coordinates

Relation between spherical and rectangular coordinate Eq. (1) is given by:

$$
r = \sqrt{x^2 + y^2 + z^2}
\tag{2}
$$

$$
\tan \theta = \frac{x^2 + y^2}{z}
\tag{3}
$$

$$
\tan \phi = \frac{y}{x}
\tag{4}
$$

$$
x = r \sin \theta \cos \phi
\tag{5}
$$

$$
y = r \sin \theta \sin \phi
\tag{6}
$$

$$
z = r \cos \theta
\tag{7}
$$

The differential displacement is given by

$$
dl = dr\hat{a}_r + rd\theta\hat{a}_\theta + r\sin\theta d\phi\hat{a}_\phi
\tag{8}
$$

Where

$$
r = \sqrt{x^2 + y^2 + z^2}
\tag{9}
$$

$$
dr = \frac{xdx + ydy + zdz}{r}
\tag{10}
$$

$$
dr^2 = \frac{(xdx + ydy + zdz)^2}{r^2}
\tag{11}
$$

Now adding and subtracting dr^2 in Eq. (1)

$$
\begin{aligned}
ds^2 = {} & -\left(1 - \frac{2GM}{rC^2}\right) C^2 dt^2 + \left(1 - \frac{2GM}{rC^2}\right)^{-1} dr^2 \\
& + dr^2 - dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2
\end{aligned}
\tag{12}
$$

Or

$$
\begin{aligned}
ds^2 = {} & -\left(1 - \frac{2GM}{rC^2}\right) C^2 dt^2 + \left(\left(1 - \frac{2GM}{rC^2}\right)^{-1} - 1\right) dr^2 \\
& + dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2
\end{aligned}
\tag{13}
$$

Here substituting $\frac{GM}{C^2} = 1, C^2 = 1$ in Eq. (13)and the term

$$
dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 = dx^2 + dy^2 + dz^2
\tag{14}
$$

and from Binomial expansion

$$
\begin{aligned}
\left(\left(1 - \frac{2GM}{rC^2}\right)^{-1} - 1\right) dr^2 &= \left(1 + \frac{2}{\sqrt{x^2 + y^2 + z^2}} - 1\right) \\
&\quad \frac{(xdx + ydy + zdz)^2}{r^2}
\end{aligned}
\tag{15}
$$

Now ds^2 in rectangular coordinate system will be given by (substituting $\frac{GM}{C^2} = 1$, and simplify)

$$
\begin{aligned}
ds^2 = {} & -\left(1 - \frac{2}{\sqrt{x^2 + y^2 + z^2}}\right) dt^2 + \frac{(1 + 2x^2)dx^2}{(x^2 + y^2 + z^2)^{3/2}} \\
& + \frac{(1 + 2y^2)dy^2}{(x^2 + y^2 + z^2)^{3/2}} dy^2 + \frac{(1 + 2z^2)dz^2}{(x^2 + y^2 + z^2)^{3/2}} \\
& + \frac{2xydxdy}{(x^2 + y^2 + z^2)^{3/2}} + \frac{2yzdydz}{(x^2 + y^2 + z^2)^{3/2}} \\
& + \frac{2zxdzdx}{(x^2 + y^2 + z^2)^{3/2}}
\end{aligned}
\tag{16}
$$

Now we'll write the components of Schwarzchild metric from Eq.(16). We have

$$
g_{00} = -\left(1 - \frac{2}{\sqrt{x^2 + y^2 + z^2}}\right)
\tag{17}
$$

$$
g_{11} = \frac{(1 + 2x^2)dx^2}{(x^2 + y^2 + z^2)^{3/2}}
\tag{18}
$$

$$
g_{22} = \frac{(1 + 2y^2)dx^2}{(x^2 + y^2 + z^2)^{3/2}}
\tag{19}
$$

$$g_{33} = \frac{(1+2z^2)dx^2}{(x^2+y^2+z^2)^{3/2}} \tag{20}$$

$$g_{12} = g_{21} = \frac{xy}{(x^2+y^2+z^2)^{3/2}} \tag{21}$$

$$g_{23} = g_{32} = \frac{yz}{(x^2+y^2+z^2)^{3/2}} \tag{22}$$

$$g_{31} = g_{13} = \frac{zx}{(x^2+y^2+z^2)^{3/2}} \tag{23}$$

other $g'_{ij}s$ being zero. The metric is first expressed in the cartesian system. It has the form

$$\begin{aligned} ds^2 &= g_{tt}dt^2 + g_{xx}dx^2 + g_{yy}dy^2 + g_{zz}dz^2 \\ &\quad + 2g_{xy}dxdy + 2g_{yz}dydz + 2g_{xz}dxdz \end{aligned} \tag{24}$$

where $g_{tt}, g_{xx}, g_{yy}, g_{zz}, g_{xy}, g_{yz}, g_{xz}$ are all functions of x, y, z and are independent of t. The Maxwell equations in such a metric have the form

$$(F^{\mu\nu}\sqrt{-g})_{,\nu} = 0 \tag{25}$$

and the gauge condition reads

$$(A^{\mu}\sqrt{-g})_{,\mu} = 0 \tag{26}$$

This gauge condition can be expressed as

$$A^{\mu}_{,\mu}\sqrt{-g} + A^{\mu}(\sqrt{-g})_{,\mu} = 0 \tag{27}$$

We have

$$F_{\mu\nu} = A_{\nu,\mu} - A_{\mu,\nu} \tag{28}$$

so that

$$F^{\mu\nu} = g^{\mu\alpha}g^{\nu\beta}F_{\alpha\beta} = g^{\mu\alpha}g^{\nu\beta}(A_{\beta,\alpha} - A_{\alpha,\beta}) \tag{29}$$

And

$$F^{\mu\nu}\sqrt{-g} = g^{\mu\alpha}g^{\nu\beta}\sqrt{-g}(A_{\beta,\alpha} - A_{\alpha,\beta}) \tag{30}$$

Now

$$\begin{aligned} (F^{\mu\nu}\sqrt{-g})_{,\nu} &= (g^{\mu\alpha}g^{\nu\beta}\sqrt{-g})_{,\nu}(A_{\beta,\alpha} - A_{\alpha,\beta}) \\ &\quad + g^{\mu\alpha}g^{\nu\beta}\sqrt{-g}(A_{\beta,\alpha} - A_{\alpha,\beta})_{,\nu} \end{aligned} \tag{31}$$
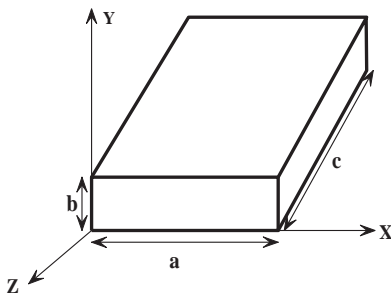

Fig. 1. Cuboidal cavity resonator

II. METHOD OF MOMENTS FORMULATION TO A CAVITY RESONATOR

The term method of moments was first used in Western literature by R.F. Harrington. The Method of moments (MOM) has the advantage of being conceptually simple [23], [24]. Cavity resonator can be used as tuned circuits, UHF tubes, klystron amplifier and oscillators, cavity magnetron, and in duplex radars. Cuboidal cavity resonator is shown in figure 1. Eq. (31) can be brought to the form

$$\begin{aligned} (F^{\mu\nu}\sqrt{-g})_{,\nu} &= c(\mu,\nu,\alpha,\beta)A_{\nu,\alpha\beta} \\ &\quad + d(\mu,\nu,\alpha)A_{\nu,\alpha} + f(\mu,\alpha)A_{\alpha} \\ &= 0 \end{aligned} \tag{32}$$

We write the full expanded form of this system of equations and replace partial derivative with respect to time by multiplication by $j\omega$. In this way, we end up with a system of linear partial differential equations in the variables x, y, z alone [25]. We then expand the potentials as

$$A_{\mu} = \sum_{n,m,k=1}^{N} c(n,m,k)\sin\frac{n\pi x}{a}\sin\frac{m\pi x}{b}\sin\frac{k\pi x}{c} \tag{33}$$

where a, b, c are the lengths of the sides of the cuboidal cavity. Then take the inner product with the same functions to end up with a generalized eigenvalue equation for the frequencies of oscillation ω. To be more specific, we write the above equation as

$$\begin{aligned} &c(\mu,\nu,x,x)A_{\nu,xx} + c(\mu,\nu,y,y)A_{\nu,yy} + c(\mu,\nu,z,z) \\ &A_{\nu,zz} + 2c(\mu,\nu,x,y)A_{\nu,xy} + 2c(\mu,\nu,y,z)A_{\nu,yz} + 2c \\ &(\mu,\nu,x,z)A_{\nu,xz} - 2\omega^2 c(\mu,\nu,t,t)A_{\nu,tt} + d(\mu,\nu,x)A_{\nu,x} \\ &+ d(\mu,\nu,y)A_{\nu,y} + d(\mu,\nu,z)A_{\nu,z} + 2i\omega c(\mu,\nu,t,x)A_{\nu,x} \\ &+ 2i\omega c(\mu,\nu,t,y)A_{\nu,y} + 2i\omega c(\mu,\nu,t,z)A_{\nu,z} \\ &+ i\omega.d(\mu,\nu,t)A_{\nu} + f(\mu,\nu)A_{\nu} = 0 \end{aligned} \tag{34}$$

We then plug in the expressions

$$\begin{aligned} A_{\mu,x} &= \sum c(n,m,k)\left(\frac{n\pi}{a}\right)\cos\left(\frac{n\pi x}{a}\right)\sin\left(\frac{m\pi y}{b}\right) \\ &\quad \sin\left(\frac{k\pi z}{c}\right) \end{aligned} \tag{35}$$

$$\begin{aligned} A_{\mu,y} &= \sum c(n,m,k)\left(\frac{m\pi}{b}\right)\sin\left(\frac{n\pi x}{a}\right)cos\left(\frac{m\pi y}{b}\right) \\ &\quad \sin\left(\frac{k\pi z}{c}\right) \end{aligned} \tag{36}$$

$$\begin{aligned} A_{\mu,xx} &= -\sum c(n,m,k)\left(\frac{n\pi}{a}\right)^2\sin\left(\frac{n\pi x}{a}\right)sin\left(\frac{m\pi y}{b}\right) \\ &\quad \sin\left(\frac{k\pi z}{c}\right) \end{aligned} \tag{37}$$

$$\begin{aligned} A_{\mu,yy} &= -\sum c(n,m,k)\left(\frac{m\pi}{b}\right)^2\sin\left(\frac{n\pi x}{a}\right)sin\left(\frac{m\pi y}{b}\right) \\ &\quad \sin\left(\frac{k\pi z}{c}\right) \end{aligned} \tag{38}$$

$$A_{\mu,zz} = -\sum c(n,m,k)\left(\frac{k\pi}{c}\right)^2 \sin\left(\frac{n\pi x}{a}\right) sin\left(\frac{m\pi y}{b}\right)$$
$$\sin\left(\frac{k\pi z}{c}\right) \tag{39}$$

$$A_{\mu,xy} = -\sum c(n,m,k)\left(\frac{n\pi}{a}\right)\left(\frac{m\pi}{b}\right) cos\left(\frac{n\pi x}{a}\right)$$
$$\cos\left(\frac{m\pi y}{b}\right)\sin\left(\frac{k\pi z}{c}\right) \tag{40}$$

$$A_{\mu,yz} = -\sum c(n,m,k)\left(\frac{m\pi}{b}\right)\left(\frac{k\pi}{c}\right) sin\left(\frac{n\pi x}{a}\right)$$
$$\cos\left(\frac{m\pi y}{b}\right)\cos\left(\frac{k\pi z}{c}\right) \tag{41}$$

$$A_{\mu,xz} = -\sum c(n,m,k)\left(\frac{n\pi}{a}\right)\left(\frac{k\pi}{c}\right) \cos\left(\frac{n\pi x}{a}\right)$$
$$\sin\left(\frac{m\pi y}{b}\right)\cos\left(\frac{k\pi z}{c}\right) \tag{42}$$

We assume that the range of the summation in all these expressions is $1 \leq n, m, k \leq N$. Plugging these expressions into the system of partial differential equations gives us an equation of the form $(L_0 + mL_1)(A^\mu) = 0$ where L_0 is the standard D'Alembertian wave operator and L_1 is the perturbation to this wave operator caused by the blackhole . m is the mass of the blackhole [26], [27]. We then assume sinusoidal dependence on time and reduce this wave equation to a modified Helmholtz equation which is an eigen equation for the frequencies [28].

III. CONCLUSION

A method of moments formulation for the computation of the propagation modes of the electromagnetic waveguide with gravitational effects has been presented and discussed. Vanishing boundary conditions for the potential on the faces of the box are assumed and then by applying the method of moments which involves expanding the potentials inside the box as linear combinations of test functions, we obtain a matrix generalized eigenvalue problem for the frequencies of oscillation. The solution for the eigen-frequencies can then be obtained using standard perturbation theory of matrix generalized eigenvalue problems. The method of perturbation theory required here involves the following: Let A_0, B_0, A_1, B_1 be square matrices of the same size and let ϵ be a small parameter. Then given the solutions to the generalized eigenvalue problem $(A_0 - \lambda.B_0)x = 0$, determine as a power series in ϵ the solutions to the perturbed generalized eigenvalue problem $(A_0 + \epsilon A_1 - \lambda(B_0 + \epsilon B_1))x = 0$. The generalized eigenvalues λ and eigenvectors x are to be expressed as power series in the parameter ϵ.

REFERENCES

[1] J. A. Kong, *Electromagnetic Wave Theory*, 2nd Ed. New York: Wiley, 1990.
[2] K. Murawski, *Analytical and Numerical Methods for wave propagation in fluid media*, World Scientific Publishing Co. Pte. Ltd., Vol. 7, series A, 2002.
[3] P.A.M. Dirac,*General Theory of Relativity*, second edition, Princeton University Press, 1996.
[4] Geroch and Robert, *General Relativity from A to B*, University of Chicago Press, Chicago, 1981.
[5] Thorne, Kip S.; Misner, Charles and Wheeler, John (1973), *Gravitation* , W. H. Freeman and Company, ISBN 0-7167-0344-0.
[6] Wald, Robert M. (1992), *Space, Time, and Gravity: The Theory of the Big Bang and Black Holes*, University of Chicago Press, ISBN 0-226-87029-4.
[7] D. Finkelstein (1958), *Past-Future Asymmetry of the Gravitational Field of a Point Particle*, Phys. Rev. 110: 965-967.
[8] Lewis G. F. and Kwan, J. *No Way Back: Maximizing Survival Time Below the Schwarzschild Event Horizon*, Publications of the Astronomical Society of Australia 24 (2): 46-52(2007).
[9] R. P. Kerr, *Gravitational field of a spinning mass as an example of algebraically special metrics* , Phys. Rev. Lett. 11, 237 (1963).
[10] Carr B.J. , *Primordial Black Holes: Do They Exist and Are They Useful?*, Proceedings of "Inflating Horizon of Particle Astrophysics and Cosmology", Universal Academy Press Inc and Yamada Science Foundation, http://arxiv.org/abs/astro-ph/0511743 2005.
[11] McClintock, Jeffrey E.; Shafee, Rebecca; Narayan, Ramesh; Remillard, Ronald A.; Davis, Shane W. and Li, Li-Xin (2006), *The Spin of the Near-Extreme Kerr Black Hole GRS 1915+105* , Astrophys.J. 652: 518-539, ¡http://arxiv.org/abs/astro-ph/0606076¿.
[12] Page, Ron N. , *Hawking Radiation and Black Hole Thermodynamics*, New.J.Phys. 7(203), (2005).
[13] Bloom, J.S., Kulkarni, S. R., Djorgovski, S. G., *The Observed Offset Distribution of Gamma-Ray Bursts from Their Host Galaxies: A Robust Clue to the Nature of the Progenitors*, Astronomical Journal 123: 1111-1148(2002).
[14] Winter, L.M., Mushotzky, R.F. and Reynolds, C.S., *XMM-Newton Archival Study of the ULX Population in Nearby Galaxies*, Astrophysical Journal 649: 730, 2006.
[15] Munyaneza, F.; R.D. Viollier (2001), *The motion of stars near the Galactic center: A comparison of the black hole and fermion ball scenarios* , Retrieved on 2006-03-25.
[16] Tsiklauri, David; Raoul D. Viollier (1998), *Dark matter concentration in the galactic center*, Retrieved on 2006-03-25.
[17] Heusler M, *Stationary Black Holes: Uniqueness and beyond* , Living Rev.Relativity1(6), 1998.
[18] Hawking S.W. and Penrose R. *The Singularities of Gravitational Collapse and Cosmology* Proc.Roy.Soc.Lon 314(1519): 529548,(1970) http://www.jstor.org/stable/2416467
[19] Hawking and Stephen, *Black Hole Explosions* Nature 248(1974): pp. 3031. doi:10.1038/248030a0.
[20] Celotti A., Miller J.C. and Sciama D.W., *Astrophysical evidence for the existence of black holes* Class. Quant. Grav. 16 (1999), http://arxiv.org/abs/astro-ph/9912186.
[21] Chandrasekhar, Subrahmanyan , *Mathematical Theory of Black Holes* , Oxford University Press,1999, ISBN 0-19-850370-9.
[22] Heusler M, *Stationary Black Holes: Uniqueness and beyond* , Living Rev.Relativity1(6), 1998.
[23] R. F. Harrington, *Field Computation by Moment Methods*, New York: Macmillan, 1968.
[24] M.N.O.Sadiku, *Elements of electromagnetics* , Third Edition, Oxford University Press, New York.
[25] Arti Vaish and H. Parthasarathy, *Modal Analysis of Waveguide using Method of Moment*, Accepted for publication in HAIT Journal of Science and Engineering-B, 2007.
[26] Taylor, Edwin F. and Wheeler, John Archibald (2000), *Exploring Black Holes*, Addison Wesley Longman, ISBN 0-201-38423-X.
[27] Orosz, J.A et al (2007), *A 15.65 solar mass black hole in an eclipsing binary in the nearby spiral galaxy Messier 33*, Nature 449: 872-875. doi:10.1038/nature06218.
[28] Arti Vaish and H. Parthasarathy, *Finite Element Analysis of Propagation Modes in a Waveguide: Effect of Gravitational Field*, Accepted for publication in HAIT Journal of Science and Engineering-B, 2007.

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