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# Controller Design of Discrete Systems by Order Reduction Technique Employing Differential Evolution Optimization Algorithm

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***Abstract*—**One of the main objectives of order reduction is to design a controller of lower order which can effectively control the original high order system so that the overall system is of lower order and easy to understand. In this paper, a simple method is presented for controller design of a higher order discrete system. First the original higher order discrete system in reduced to a lower order model. Then a Proportional Integral Derivative (PID) controller is designed for lower order model. An error minimization technique is employed for both order reduction and controller design. For the error minimization purpose, Differential Evolution (DE) optimization algorithm has been employed. DE method is based on the minimization of the Integral Squared Error (ISE) between the desired response and actual response pertaining to a unit step input. Finally the designed PID controller is connected to the original higher order discrete system to get the desired specification. The validity of the proposed method is illustrated through a numerical example.

***Keywords*—**Discrete System, Model Order Reduction, PID Controller, Integral Squared Error, Differential Evolution.

## I. INTRODUCTION

THE mathematical procedure of system modeling often leads to detailed description of a process in the form of high order differential equations. These equations in the frequency domain lead to a high order transfer function. Therefore, it is desirable to reduce higher order transfer functions to lower order systems for analysis and design purposes. Reduction of high order systems to lower order models has also been an important subject area in control engineering for many years [1,2]. One of the main objectives of order reduction is to design a controller of lower order which can effectively control the original high order system.

The conventional methods of reduction, developed so far, are mostly available in continuous domain. However, the high order systems can be reduced in continuous as well as in discrete domain [3-5]. There are two approaches for the reduction of discrete system, namely the indirect method and direct method. The indirect method uses some transformation and then reduction is carried out in the transformed domain.

First the z- domain transfer functions are converted into s-domain by the bilinear transformation and then after reducing them in s-domain, suitably, they are converted back into z-domain. In the direct method the higher order z-domain transfer functions are reduced to a lower order transfer function in the same domain without any transformation [6].

There are two common approaches for controller design. First approach is to obtain the controller on the basis of reduced order model called process reduction [7]. In the second approach, the controller is designed for the original higher order system and then the closed loop response of higher order controller with original system is reduced pertaining to unity feedback called controller reduction [8]. Both the approaches have their own advantages and disadvantages. The process reduction approach is computationally simpler as it deals with lower order models and controller but at the same time errors are introduced in the design process as the reduction is carried out at the early stages of design. In the controller reduction approach error propagation is minimized as the design process is carried out at the final stages of reduction but the approaches deals with higher order models and thus introduces computational complexity.

In recent years, one of the most promising research fields has been “Evolutionary Techniques”, an area utilizing analogies with nature or social systems. Evolutionary techniques are finding popularity within research community as design tools and problem solvers because of their versatility and ability to optimize in complex multimodal search spaces applied to non-differentiable objective functions. Differential evolution (DE) is a branch of evolutionary algorithms developed by Rainer Stron and Kenneth Price in 1995 for optimization problems [9]. It is a population-based direct search algorithm for global optimization capable of handling non-differentiable, non-linear and multi-modal objective functions, with few, easily chosen, control parameters. It has demonstrated its usefulness and robustness in a variety of applications such as, Neural

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network learning, Filter design and the optimization of aerodynamics shapes. DE differs from other evolutionary algorithms (EA) in the mutation and recombination phases. DE uses weighted differences between solution vectors to change the population whereas in other stochastic techniques such as genetic algorithm (GA) and expert systems (ES), perturbation occurs in accordance with a random quantity. DE employs a greedy selection process with inherent elitist features. Also it has a minimum number of EA control parameters, which can be tuned effectively [10]. In view of the above, this paper proposes to use DE optimization technique for both model reduction and controller design.

In this paper, controller design of a higher order discrete system is presented employing process reduction approach. The original higher order discrete system in reduced to a lower order model employing DE technique. DE technique is based on the minimization of the Integral Squared Error (ISE) between the transient responses of original higher order model and the reduced order model pertaining to a unit step input. Then a Proportional Integral Derivative (PID) controller is designed for lower order model. The parameters of the PID controller are tuned by using the same error minimization technique employing DE. The performance of the designed PID controller is verified by connecting the designed PID controller with the original higher order discrete system to get the desired specification.

Despite significant strides in the development of advanced control schemes over the past two decades, the conventional lead-lag (LL) structure controller as well as the classical proportional-integral-derivative (PID) controller and its variants, remain the controllers of choice in many industrial applications. These controller structures remain an engineer's preferred choice because of their structural simplicity, reliability, and the favorable ratio between performance and cost. Beyond these benefits, these controllers also offer simplified dynamic modeling, lower user-skill requirements, and minimal development effort, which are issues of substantial importance to engineering practice.

## II. Problem Statement

### A. Model order reduction

Please Given a high order discrete time stable system of order '$n$' that is described by the $z$-transfer function:

$$G_O(z) = \frac{N(z)}{D(z)} = \frac{a_0 + a_1 z + ......... + a_{n-1} z^{n-1}}{b_0 + b_1 z + ......... + b_{n-1} z^{n-1} + b_n z^n} \tag{1}$$

The objective is to find a reduced $r^{th}$ order model that has a transfer function ($r < n$):

$$R(z) = \frac{N_r(z)}{D_r(z)} = \frac{c_0 + c_1 z + ......... + c_{r-1} z^{r-1}}{d_0 + d_1 z + ......... + d_{r-1} z^{r-1} + d_r z^r} \tag{2}$$

The polynomial $D(z)$ is stable, that is all its zeros reside inside the unit circle $|z|$=1. Where, $a_i\,(0 \le i \le n-1)$, $b_i\,(0 \le i \le n)$, $c_i\,(0 \le i \le r-1)$ and $d_i\,(0 \le i \le r)$ are scalar constants.

The numerator order is given as being one less than that of the denominator, as for the original system. The $R(z)$ approximates $G_0(z)$ in some sense and retains the important characteristics of $G_0(z)$ and the transient responses of $R(z)$ should be as close as possible to that of $G_0(z)$ for similar inputs.

### B. Controller design

All The proposed method of design of a controller by process reduction technique involves the following steps:

*Step-1*

Reduce the given higher order discrete system to a lower order model by error minimization technique.

The objective function *J* is defined as an integral squared error of difference between the responses given by the expression:

$$J = \int_0^{t_\infty} [y_0(t) - y_r(t)]^2\, dt \tag{3}$$

Where $y_0(t)$ and $y_r(t)$ are the unit step responses of original and reduced order systems.

*Step-2*

Design a PID controller for the reduced order system. The parameters of the PID controller are optimized using the same error same error minimization technique employing DE.

*Step-3*

Test the designed PID controller for the reduced order model for which the PID controller has been designed.

*Step-4*

Test the designed PID controller for the original higher order model.

## III. Proportional Integral Derivative (PID) Controller

PID controller is basic type of feedback controller. The basic structure of conventional feedback control systems is shown in Fig. 1, using a block diagram representation. In this figure, the process is the object to be controlled. In this figure, the object to be controlled is the process. To make the process variable *y* follow the set-point value *r* is the main objective of control. To achieve this purpose, the manipulated variable *u* is changed at the authority of the controller. The "disturbance *d*" is any factor, other than the manipulated variable, that influences the process variable. Fig.1 assumes that only one disturbance is added to the manipulated variable. In some applications, however, a major disturbance enters the process in a different way, or plural disturbances need to be considered. The error $e$ is defined by $e = r - y$.

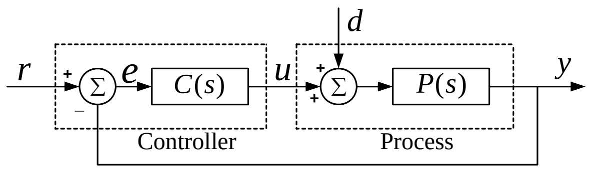


Fig. 1. Block diagram of basic feedback controller

PID control is the method of feedback control that uses the PID controller as the main tool. PID controller is most widely used in industrial control applications because of its structural simplicity, reliability, and the favorable ratio between performance and cost. Beyond these benefits, these controllers also offer simplified dynamic modeling, lower user-skill requirements, and minimal development effort, which are issues of substantial importance to engineering practice. A PID controller calculates an error value as the difference between a measured process variable and a desired set point. The controller attempts to minimize the error by adjusting the process control inputs. In the absence of knowledge of the underlying process, PID controllers are the best controllers. However, for best performance, the PID parameters used in the calculation must be according to the nature of the system – while the design is generic, the parameters depend on the specific system. The structure of a PID controller is shown in Fig. 2.

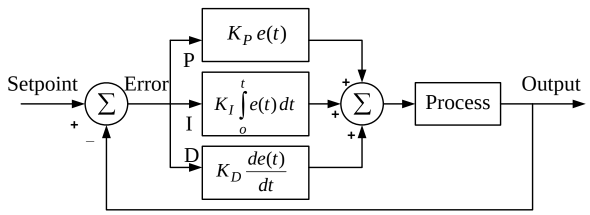


Fig. 2. Structure of PID controller

The PID controller involves three separate parameters, and is accordingly sometimes called three-term control: the Proportionality, the integral and derivative values, denoted by P, I, and D. The proportional value determines the reaction to the current error, the integral value determines the reaction based on the sum of recent errors, and the derivative value determines the reaction based on the rate at which the error has been changing. The weighted sum of these three actions is used to adjust the process via a control element. Heuristically, these values can be interpreted in terms of time: P depends on the present error, I on the accumulation of past errors, and D is a prediction of future errors, based on current rate of change.

By tuning the three constants in the PID controller algorithm, the controller can provide control action designed for specific process requirements. The response of the controller can be described in terms of the responsiveness of the controller to an error, the degree to which the controller overshoot signal overshoots the set point and the degree of system oscillation.

Some applications may require using only one or two modes to provide the appropriate system control. This is achieved by setting the gain of undesired control outputs to zero. A PID controller will be called a PI, PD, P or I controller in the absence of the respective control actions. PI controllers are fairly common, since derivative action is sensitive to measurement noise, whereas the absence of an integral value may prevent the system from reaching its target value due to the control action.

In application, engineers have independence of implementing the three functional elements (P, I, and D) of the PID controller in whatsoever grouping they consider most suitable for their problems. The combination of element(s) used is called the action mode of the PID controller. Tuning a control loop is the adjustment of its control parameters (gain/proportional band, integral gain/reset, derivative gain/rate) to the optimum values for the desired control response. Stability (bounded oscillation) is a basic requirement, but beyond that, different systems have different behavior, different applications have different requirements, and some desiderata conflict. Further, some processes have a degree of non-linearity and so parameters that work well at full-load conditions don't work when the process is starting up from no-load; this can be corrected by gain scheduling (using different parameters in different operating regions). PID controllers often provide acceptable control even in the absence of tuning, but performance can generally be improved by careful tuning, and performance may be unacceptable with poor tuning.

The analysis for designing a digital implementation of a PID controller requires the standard form of the PID controller to be discretised. Approximations for first-order derivatives are made by backward finite differences. The integral term is discretised, with a sampling $\Delta t$ time, as follows:

$$\int_0^t e(\tau)d\tau = \sum_{i=1}^{k} e(t_i)\Delta t \tag{4}$$

The derivative term is approximated as,

$$\frac{de(t_k)}{d(t)} = \frac{e(t_k) - e(t_{k-1})}{\Delta t} \tag{5}$$

## IV. Differential Evolution

Differential Evolution (DE) algorithm is a stochastic, population-based optimization algorithm recently introduced [9]. DE works with two populations; old generation and new generation of the same population. The size of the population is adjusted by the parameter $N_P$. The population consists of real valued vectors with dimension $D$ that equals the number of design parameters/control variables. The population is randomly initialized within the initial parameter bounds. The optimization process is conducted by means of three main operations: mutation, crossover and selection. In each generation, individuals of the current population become target vectors. For each target vector, the mutation operation produces a mutant vector, by adding the weighted difference between two randomly chosen vectors to a third vector. The crossover operation generates a new vector, called trial vector, by mixing the parameters of the mutant vector with those of the target vector. If the trial vector obtains a better fitness value than the target vector, then the trial vector replaces the target vector in the next generation. The evolutionary operators are described below [9, 10].

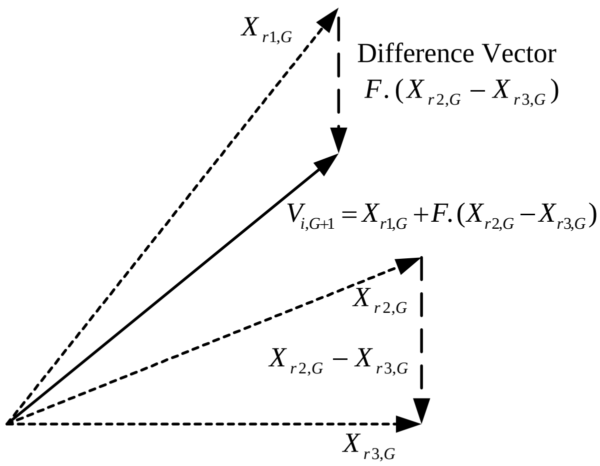


Fig. 3 Vector addition and subtraction in differential evolution

### A. Initialization

For each parameter $j$ with lower bound $X_j^L$ and upper bound $X_j^U$, initial parameter values are usually randomly selected uniformly in the interval [ $X_j^L, X_j^U$ ].

### B. Mutation

For a given parameter vector $X_{i,G}$, three vectors ( $X_{r1,G}$ $X_{r2,G}$ $X_{r3,G}$ ) are randomly selected such that the indices *i, r1, r2* and *r3* are distinct. A donor vector $V_{i,G+1}$ is created by adding the weighted difference between the two vectors to the third vector as:

$$V_{i,G+1} = X_{r1,G} + F.(X_{r2,G} - X_{r3,G}) \quad (6)$$

Where $F$ is a constant from (0, 2).

### C. Crossover

Three parents are selected for crossover and the child is a perturbation of one of them. The trial vector $U_{i,G+1}$ is developed from the elements of the target vector ( $X_{i,G}$ ) and the elements of the donor vector ( $X_{i,G}$ ).Elements of the donor vector enter the trial vector with probability *CR* as:

$$U_{j,i,G+1} = \begin{cases} V_{j,i,G+1} & if \quad rand_{j,i} \le CR \quad or \quad j = I_{rand} \\ X_{j,i,G+1} & if \quad rand_{j,i} > CR \quad or \quad j \ne I_{rand} \end{cases} \quad (7)$$

With $rand_{j,i}$ ~ $U$ (0,1), $I_{rand}$ is a random integer from (1,2,….D) where $D$ is the solution's dimension i.e number of control variables. $I_{rand}$ ensures that $V_{i,G+1} \ne X_{i,G}$.

### D. Selectionn

The target vector $X_{i,G}$ is compared with the trial vector $V_{i,G+1}$ and the one with the better fitness value is admitted to the next generation. The selection operation in DE can be represented by the following equation:

$$X_{i,G+1} = \begin{cases} U_{i,G+1} & if \ f(U_{i,G+1}) < f(X_{i,G}) \\ X_{i,G} & otherwise. \end{cases} \quad (8)$$

where $i \in [1, N_P]$.

## V. Numerical Example

Consider the transfer function of the plant from references [11, 12] as:

$$G(z) = \frac{N(z)}{D(z)} = \frac{(0.1625z^7 + 0.125z^6 - 0.0025z^5 + 0.00525z^4 - 0.02263z^3 - 0.00088z^2 + 0.003z - 0.000413)}{(z^8 - 0.6307z^7 - 0.4185z^6 + 0.078z^5 - 0.057z^4 - 0.1935z^3 + 0.09825z^2 - 0.0165z + 0.00225)} \quad (9)$$

For which a controller is to be designed to get the desired output.

### A. Application of DE for Model Order Reduction

To reduce the higher order model in to a lower order model DE is employed. The objective function $J$ defined as an integral squared error of difference between the responses given by the equation (3) is minimized by DE. Implementation of DE requires the determination of six fundamental issues: DE step size function, crossover probability, the number of population, initialization, termination and evaluation function. Generally DE step size (F) varies in the interval (0, 2). A good initial guess to F is in the interval (0.5, 1). Crossover probability (CR) constants are generally chosen from the interval (0.5, 1). If the parameter is co-related, then high value of CR work better, the reverse is true for no correlation [10]. In the present study, a population size of $N_P$=20, generation number $G$=200, step size F=0.8 and crossover probability of CR =0.8 have been used. Optimization is terminated by the pre-specified number of generations for DE. One more important factor that affects the optimal solution more or less is the range for unknowns. For the very first execution of the program, a wider solution space can be given and after getting the solution one can shorten the solution space nearer to the values obtained in the previous iteration. The flow chart of the DE algorithm employed in the present study is given in Fig. 4. One more important point that affects the optimal solution more or less is the range for unknowns. For the very first execution of the program, more wide solution space can be given and after getting the solution one can shorten the solution space nearer to the values obtained in the previous iteration. Optimization was performed with the total number of generations set to 100. Simulations were conducted on a Pentium 4, 3 GHz, 504 MB RAM computer, in the MATLAB 7.0.1 environment. A typical convergence

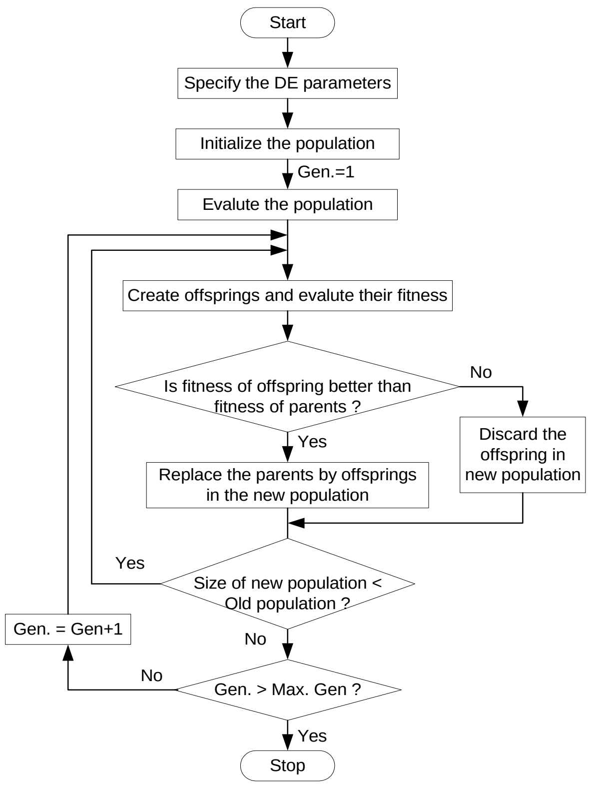


Fig. 4 Flow chart of proposed DE optimization approach

of objective function with the number of generation is shown in Fig.5. The optimization processes is run 20 times and best among the 20 runs are taken as the final result.

The reduced 2nd order model employing DE technique is obtained as given in equation (15):

$$R_2(z) = \frac{0.004821z - 0.002508}{0.030465z^2 - 0.053953z + 0.025634} \qquad (15)$$

The unit step responses of original and reduced systems are shown in Fig. 6. It can be seen that the steady state responses of proposed reduced order models is exactly matching with that of the original model. Also, the transient response of proposed reduced model by DE is very close to that of original model. It can be seen from Fig. 6 that both the original model and the reduced model settle at a value of 1.07 for a input of 1.0 (unit step input). Now, to get the desired out put i.e. 1.0, a PID controller is designed.

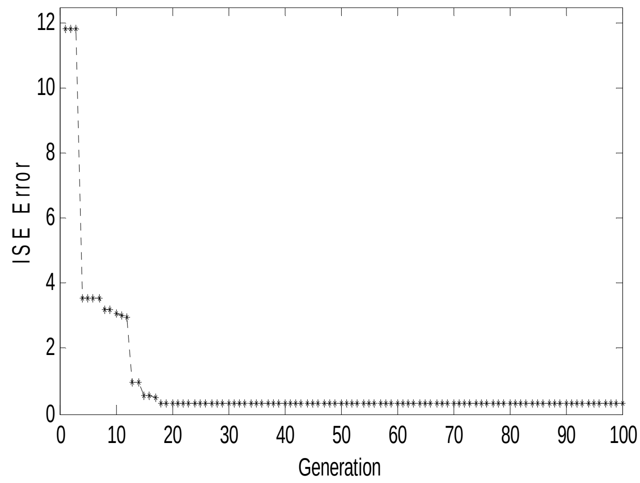


Fig. 5. Convergence of fitness function

### B. *Application of DE for PID Controller Design*

In this study, the PID controller has been designed employing process reduction approach. The original higher order discrete system given by equation (14) is reduced to a lower order model employing DE technique given by equation (15). Then the PID controller is designed for lower order model. The parameters of the PID controller are tuned by using the same error minimization technique employing DE as explained in section 5.1. The optimized PID controller parameters are:

$$K_P = 6.7105,\ K_I = 14.9726,\ K_D = 0.5089$$

The unit step response of the reduced system with DE optimized PID controller and original system with DE optimized PID controller are shown in Fig. 7 and 8. It is clear from Fig. 8 that the design of PID controller using the proposed DE optimization technique helps to obtain the designer's specifications in transient as well as in steady state responses for the original system.

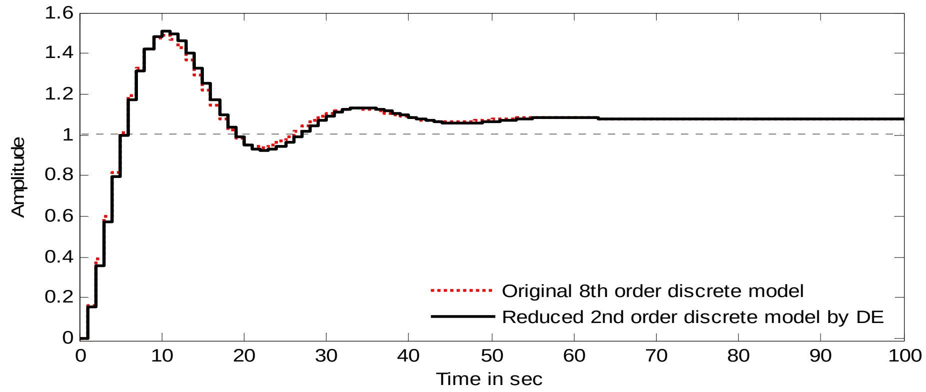


Fig. 6. Step Responses of original system and reduced model

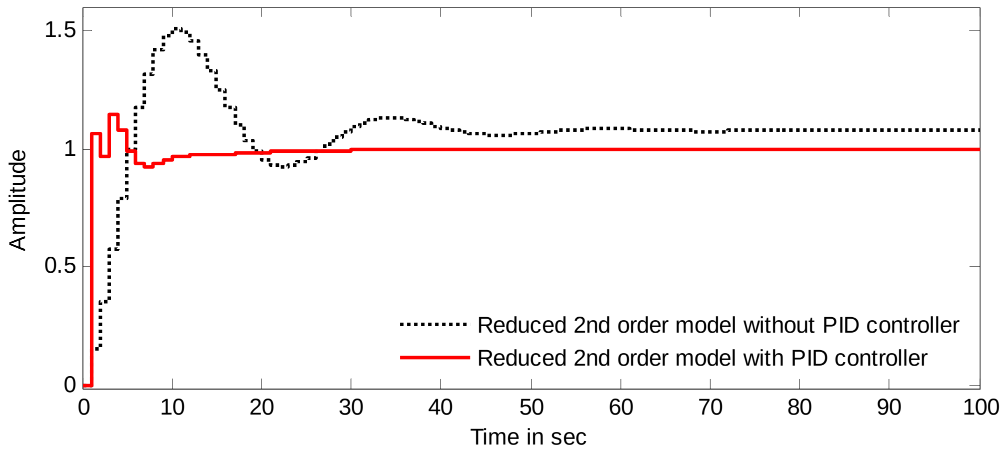


Fig. 7. Step response of reduced model with PID Controller

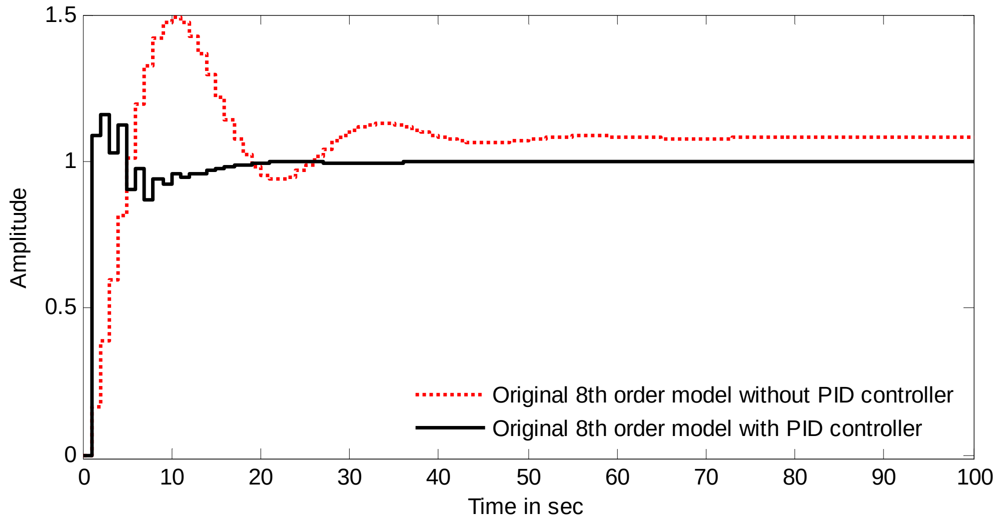


Fig. 8. Step response of original model with PID Controller

## VI. CONCLUSION

The proposed model reduction method uses the modern heuristic optimization technique in its procedure to derive the stable reduced order model for the discrete system. The algorithm has also been extended to the design of controller for the original discrete system. The algorithm is simple to implement and computer oriented. The matching of the step response is assured reasonably well in this proposed method. Algorithm preserves more stability and avoids any error between the initial or final values of the responses of original and reduced model. This approach minimizes the complexity involved in direct design of PID Controller. The values for PID Controller are optimized using the reduced model and to meet the required performance specifications. The tuned values of the PID controller parameters are tested with the original system and its closed loop response for a unit step input is found to be satisfactory with the response of reduced order model.

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