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Trajectory Planning Design Equations and Control of a 4 - axes Stationary Robotic Arm

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Abstract **—** This paper features the trajectory planning design of a indigenously developed 4-Axis SCARA robot which is used for doing successful robotic manipulation task in the laboratory. Once, a trajectory is being designed and given as input to the robot, the robot's gripper tip moves along that specified trajectory. Trajectories have to be designed in the work space only. The main idea of this paper is to design a continuous path trajectory model for the indigenously developed SCARA robot arm during its maneuvering from one point to another point (during pick and place operations) in a workspace avoiding all the obstacles in its path of motion.

Keywords **—** SCARA, Trajectory, Planning.

I. Introduction

DIRECT kinematics gives the position and orientation of the robot arm in the 3DE space. Inverse Kinematics gives the sets of joint variables that will satisfy the same manipulator position and orientation. In between these two problems, robot motion comes into the picture. This robot motion consists of paths and trajectories. These paths and trajectories are nothing but the various possible routes that are taken by the robot to move from the source (pick point) to the goal / destination (place point) and traversed in a specified amount of time. We use this direct kinematics and inverse kinematics problem to solve a higher-level problem, i.e., to plan trajectories in the tool configuration space. Trajectory planning schemes helps us to interpolate / approximate between the points using a smooth motion.

Trajectory planning schemes generally interpolate or approximate the desired path by a class of polynomial functions and generate a sequence of time based control set points for the control of the manipulator from the initial location to the destination. Path end points can be specified either in joint coordinates or in Cartesian coordinates. However, they are usually specified in Cartesian coordinates, because it is easier to visualize the correct end effector configurations in Cartesian coordinates rather than in joint coordinates. Furthermore, joint coordinates are not suitable as a working coordinate system, because the joint axes of most of the robots are not orthogonal. Further, trajectory planning involves how to design and plan an obstacle collision free trajectory from the pick point to the place point in the work space of the robot. There exists an '**n**' number of trajectories between the two given end - points, i.e., the source and the destination (goal), since the space is 3D in nature.

The main goal of trajectory planning is to describe the motion of the robot arm as a time sequence of joint or end-effector locations and its derivatives of the joints or end-effector locations, which are generated by interpolating or approximating the desired path by polynomial functions and to achieve a smooth motion of the end-effector. If temporal information or time information is added to the tool path by specifying the times at where the tool / gripper should be at various points along the path; then, the path gets converted into a trajectory. Hence, a trajectory is defined as a path with time information or temporal information, i.e., trajectory is both spatial as well as temporal as shown in Fig. 1.

In this, work the mathematical modelling of the trajectory design is done for a unique 4 axes system, which was designed and fabricated with indigenous components starting from scratch. The kinematically modeled and designed robot is used for some PNP operations and was named as a Selective Compliance Assembly Robot Arm (SCARA). The primary motive behind this work was to develop a modular educational robotic system, the CRUST 2002 (Computerized Robotic Unit with Selective Tractability system) with the help of locally available components and sub-systems.

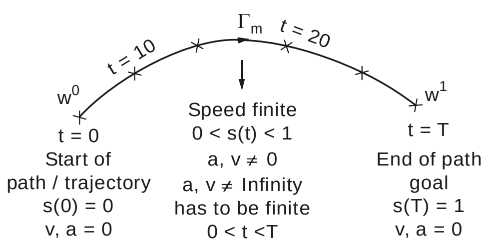

Fig. 1 Trajectory description

The paper is organized as follows. In Section 2, a brief introduction about the constructed robot is given. Next the mathematical model of the trajectory planning is presented. Simulations results are presented next followed by the conclusions and the references.

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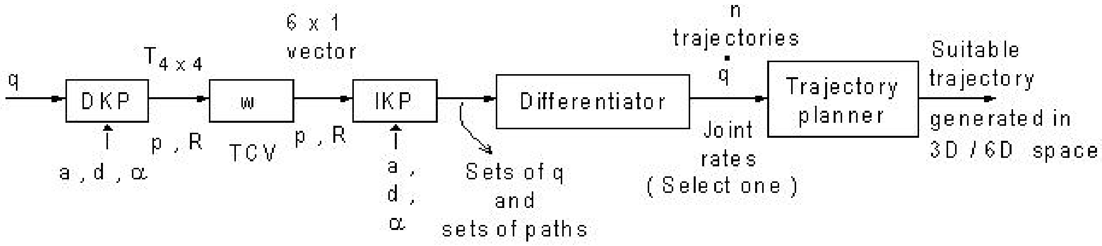

Fig. 2 Block diagram of the designed trajectory control

II. Hardware design of the robotic system

The robot is a 4 DOF stationary one having base, elbow, vertical extension and tool roll and consisting of both rotary and prismatic joints as shown in Fig. 2. There is no tool yaw and tool pitch (only tool roll) [1]. There are 4 joints, 4 axes (3 major axes - base, elbow, vertical extension and one minor axis - tool roll).

The 4 DOF's are given by Base, Elbow, Vertical Extension and Tool Roll (Fig. 3), i.e., there are 3 rotary joints and 1 prismatic joint. Since $n = 4$; 16 KP's are to be obtained and 5 RHOCF's are to be attached to the various joints [2] as shown in the LCD [1] in Fig. 6.

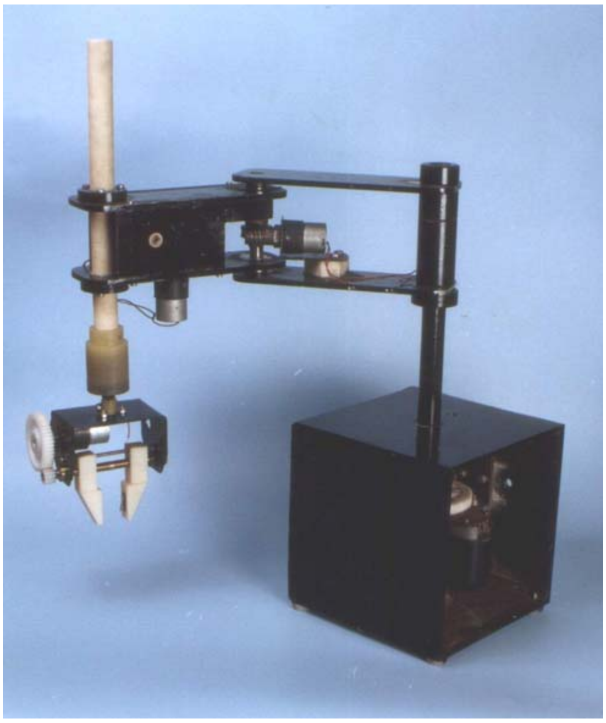

Fig. 3 The designed and fabricated robot used for trajectory control

The vector of joint variables is a combination of θ and d, i.e., $q = \{ \theta , d \}^T$. The joint variable vector q , vector of joint distances d , vector of link lengths a , vector of link twist angles α are

$$q = \{ \theta_1 , \theta_2 , d_3 , \theta_4 \}^T.$$

$$d = \{ d_1 , 0 , d_3 , d_4\}^T = \{500 , 0 , d_3 , 200\}^T \text{ mm}.$$

$$a = \{a_1 , a_2 , 0 , 0\}^T = \{300 , 250 , 0 , 0\}^T \text{ mm}.$$

$$\alpha = \{\alpha_1, \alpha_2, 0 , \alpha_4\}^T = \{\pm \pi , 0 , 0 , 0\}^T.$$

All the 4 joint axes are vertical in nature (all the z - axes can be pointing down or up) as shown in Fig. 3. The first 3 (B, E, VE) axes are called as the major axes and are used for positioning the wrist, while the last one, the minor axes (TR) is used to orient the tool in the direction of the object [13]. The first 3 major axes determine the shape and size of work envelope. It consists of an L shaped structure, to the end of which, the second link is attached. There are 2 links a_1 and a_2 which move parallel to work surface; the vertical extension d_3 is variable and moves in a direction $\perp^r$ to the work surface; length of the gripper / tool / EE is d_4 and its maximum protrusion is 300 mm [2].

The tool / gripper / EE is permanently pointing down as shown in Fig. 5 and can rotate in a plane $\perp^r$ to the work surface plane $x^0 y^0$. The approach vector r^3 is fixed, i.e., $r^3 \perp^r x^0 y^0$ (work surface) plane ; $r^3 = - z^0$. Because of this reason, the designed and kinematically modeled SCARA robot can do robotic manipulation directly from above the object when exact perpendicularity is required. The SCARA robot is a minimal representation of any robot [3] as shown in Fig. 4.

Fig. 4 The overall robotic system with the computer

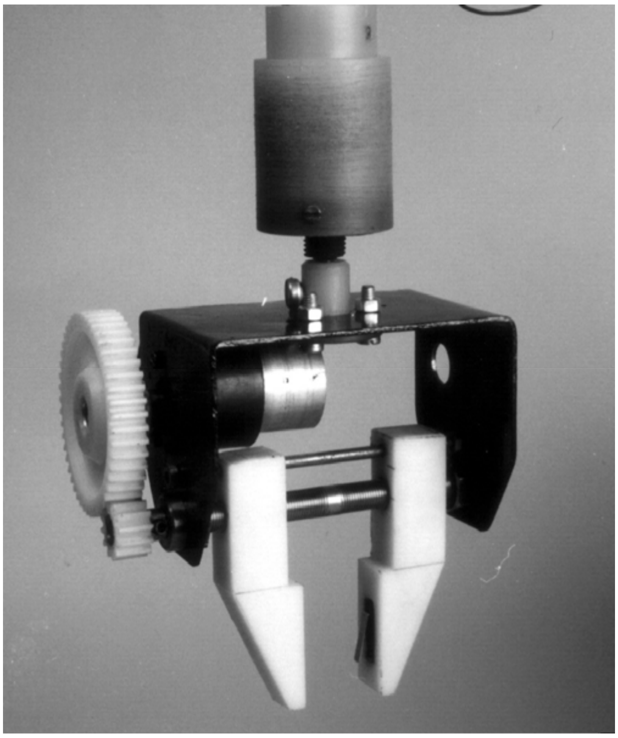

Fig. 5 Robot's gripper with tool roll as degree of freedom

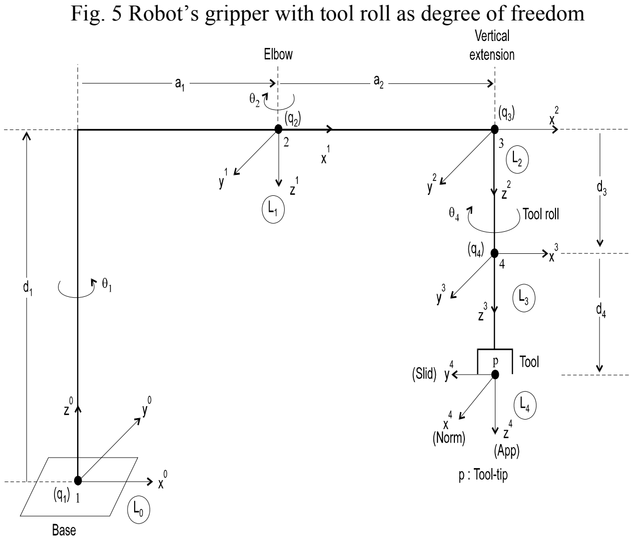

Fig. 6 The link coordinate diagram (LCD) of the SCARA robot

Our SCARA robot is a special type of polar / spherical coordinate robot in which the major axes are R R P [14].

III. Introduction to Mathematical Modelling of the Trajecory Designing

The tool configuration vector of the robot [1] is given by

$$w(q) = \begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ --- \\ w_4 \\ w_5 \\ w_6 \end{bmatrix},$$

$$= \begin{bmatrix} p_1 \\ p_2 \\ p_3 \\ \cdots\cdots \\ \left\{\exp\left(\frac{q_4}{\pi}\right)\right\} R_{13} \\ \left\{\exp\left(\frac{q_4}{\pi}\right)\right\} R_{23} \\ \left\{\exp\left(\frac{q_4}{\pi}\right)\right\} R_{33} \end{bmatrix}, \qquad (1)$$

$$= \begin{bmatrix} a_1 C_1 + a_2 C_{1-2} \\ a_1 S_1 + a_2 S_{1-2} \\ d_1 - q_3 - d_4 \\ -------- \\ 0 \\ 0 \\ -\exp\left(\frac{q_4}{\pi}\right) \end{bmatrix},$$

$$= \begin{bmatrix} p_1 \\ p_2 \\ p_3 \\ --- \\ R_4 \\ R_5 \\ R_6 \end{bmatrix}. \qquad (2)$$

The output of inverse kinematics problem for the four axes Adept - 1 SCARA robot is given by the following four inverse kinematic equations [1]:

The base angle (rotary in nature) is given by

$$q_1 = a\tan 2\,(b_2\,, b_1)\,, \qquad (3)$$

$$= \tan^{-1}\left\{\frac{(a_1 + a_2 C_2) w_2 + (a_2 S_2) w_1}{(a_1 + a_2 C_2) w_1 - (a_2 S_2) w_2}\right\}, \qquad (4)$$

The elbow angle (rotary in nature) is given by

$$q_2 = \pm \cos^{-1}\left\{\frac{w_1^2 + w_2^2 - a_1^2 - a_2^2}{2a_1 a_2}\right\}. \quad (5)$$

The vertical extension (prismatic in nature) is given by

$$q_3 = d_1 - d_4 - w_3. \quad (6)$$

The tool roll angle (rotary in nature) is given by

$$q_4 = \pi \ ln \ |w_6|. \quad (7)$$

Consider two intermediate variables b_1 and b_2. These intermediate variables [9] are used to remove the coupling between the joints and to extract the joint rates, i.e., the rate at which a particular joint is driven or moved [1].

$b_1, b_2 \rightarrow$ Obtained from q_2 and $w(q)$ as follows :

$$b_1 = (a_1 + a_2 C_2) w_1 - (a_2 S_2) w_2 \quad (8)$$

$$b_2 = (a_1 + a_2 C_2) w_2 + (a_2 S_2) w_1 \quad (9)$$

From the TCV, we get,

$$w_1 = a_1 C_1 + a_2 C_{1-2} = a_1 C_1 + a_2 (C_1 C_2 + S_1 S_2)$$

$$= (a_1 + a_2 C_2) C_1 + (a_2 S_2) S_1 \quad (10)$$

$$w_2 = a_1 S_1 + a_2 S_{1-2} = a_1 S_1 + a_2 (S_1 C_2 - C_1 S_2)$$

$$= (-a_2 S_2) C_1 + (a_1 + a_2 C_2) S_1 \quad (11)$$

$$w_3 = d_1 - q_3 - d_4 \quad (12)$$

$$w_4 = w_5 = 0 \quad (13)$$

$$w_6 = -\exp\left(\frac{q_4}{\pi}\right) \quad (14)$$

The rate at which the joints are driven can be found out by differentiating the expressions for joint angles from equations (3) to (7).

Computation of Elbow Rate $\dot{q}_2$

It is the rate at which the elbow joint is driven and depends on b_1, b_2 and its derivatives and is directly α^{al} to the output speed of elbow motor [1].

$$q_2 = \pm \cos^{-1}\left\{\frac{w_1^2 + w_2^2 - a_1^2 - a_2^2}{2a_1 a_2}\right\}. \quad (15)$$

Differentiating the expression for q_2 expression w.r.t. w, we get,

$$\dot{q}_2 = \frac{d}{dw}(q_2) = \frac{1}{\sqrt{1 - \left(\frac{w_1^2 + w_2^2 - a_1^2 - a_2^2}{2a_1 a_2}\right)^2}} \times \frac{d}{dw}\left(\frac{w_1^2 + w_2^2 - a_1^2 - a_2^2}{2a_1 a_2}\right) \quad (16)$$

$$\dot{q}_2 = \frac{1}{\sqrt{\frac{(2a_1 a_2)^2 - (w_1^2 + w_2^2 - a_1^2 - a_2^2)^2}{(2a_1 a_2)^2}}} \times \frac{2a_1 a_2 (2w_1 \dot{w}_1 + 2w_2 \dot{w}_2) - (w_1^2 + w_2^2 - a_1^2 - a_2^2)(0)}{(2a_1 a_2)^2} \quad (17)$$

$$\dot{q}_2 = \frac{1}{\frac{1}{2a_1 a_2}\sqrt{(2a_1 a_2)^2 - (w_1^2 + w_2^2 - a_1^2 - a_2^2)^2}} \times \frac{2w_1 \dot{w}_1 + 2w_2 \dot{w}_2}{2a_1 a_2}. \quad (18)$$

$$\dot{q}_2 = \frac{2(w_1 \dot{w}_1 + w_2 \dot{w}_2)}{\left[(2a_1 a_2)^2 - (w_1^2 + w_2^2 - a_1^2 - a_2^2)^2\right]^{1/2}}. \quad (19)$$

$$\dot{q}_2 = \mp \frac{2(w_1 \dot{w}_1 + w_2 \dot{w}_2)}{\sqrt{(2a_1 a_2)^2 - (w_1^2 + w_2^2 - a_1^2 - a_2^2)^2}}. \quad (20)$$

Computation of Base Rate $\dot{q}_1$

It is the rate at which the base joint is driven or moves and is directly α^{al} to the output speed of base motor [12]. It involves more number of computations and depends on intermediate variables b_1 and b_2. Note that in this paper, the rate at which the base joint should be driven is expressed in terms of b_1, b_2 and its derivatives [1].

$$q_1 = \tan^{-1}\left\{\frac{(a_1 + a_2 C_2)w_2 + (a_2 S_2)w_1}{(a_1 + a_2 C_2)w_1 - (a_2 S_2)w_2}\right\}$$

$$= \tan^{-1}\left\{\frac{b_2}{b_1}\right\}; \quad (21)$$

Differentiating this equation q_2 w.r.t. b, we get,

$$b_1 = (a_1 + a_2 C_2) w_1 - (a_2 S_2) w_2 \quad (22)$$

$$\dot{b}_1 = (a_1 + a_2 C_2)\dot{w}_1 + w_1(-a_2 S_2 \dot{q}_2) - a_2 S_2 \dot{w}_2 - a_2 w_2 C_2 \dot{q}_2$$

$$= (a_1 + a_2 C_2)\ \dot{w}_1 - a_2 S_2 \dot{w}_2 - a_2 (S_2 w_1 + C_2 w_2)\ \dot{q}_2 \quad (23)$$

$$b_2 = (a_1 + a_2 C_2) w_2 + (a_2 S_2) w_1 \quad (24)$$

$$\dot{b}_2 = (a_1 + a_2 C_2)\ \dot{w}_2 - a_2 S_2 w_2 \dot{q}_2 + a_2 S_2 \dot{w}_1 + a_2 C_2 w_1 \dot{q}_2$$

$$= (a_1 + a_2 C_2)\ \dot{w}_2 + a_2 S_2 \dot{w}_1 + a_2 (C_2 w_1 - S_2 w_2)\ \dot{q}_2 \quad (25)$$

$$q_1 = \tan^{-1}\left\{\frac{b_2}{b_1}\right\} \quad (26)$$

$$\dot{q}_1 = \frac{d}{db}\left(\tan^{-1}\left\{\frac{b_2}{b_1}\right\}\right) \quad (27)$$

$$= \frac{1}{1 + \left(\frac{b_2}{b_1}\right)^2} \times \frac{d}{db}\left(\frac{b_2}{b_1}\right) \quad (28)$$

$$= \frac{1}{\frac{b_1^2 + b_2^2}{b_1^2}} \times \left(\frac{b_1 \dot{b}_2 - b_2 \dot{b}_1}{b_1^2}\right) \quad (29)$$

$$= \frac{b_1 \dot{b}_2 - b_2 \dot{b}_1}{b_1^{\ 2} + b_2^{\ 2}} \tag{30}$$

Substitute the value of $\dot{b}_1$ and $\dot{b}_2$ from Eqs. (23) and (25) in (30) and compute $\dot{q}_1$. Note that b_1 and b_2 are the intermediate variables and nothing but the derivatives of b_1 and b_2.

Computation of Vertical Extension Rate $\dot{q}_3$

It is the rate at which the vertical extension moves along the vertical extension / tool- roll axis (perpendicular to work surface, x^0 y^0 plane) and is directly proportional to the output speed of the vertical extension motor.

It is not coupled with other three joints and is easiest to recover, since prismatic in nature [1].

$$q_3 = d_1 - d_4 - w_3 \tag{31}$$

$$\dot{q}_3 = - \dot{w}_3 \tag{32}$$

Computation of Tool Roll Rate $\dot{q}_4$

It is the rate at which tool roll joint is varied and is proportional to the output speed of the tool roll motor and is obtained from last component [1] of TCV, w(q).

$$q_4 = \pi \, ln \, |w_6| \tag{33}$$

$$\dot{q}_4 = \pi \left(\frac{\dot{w}_6}{w_6} \right) \tag{34}$$

The rate at which the 4 joints of the robot rotate is given below in Eqs. (35) - (38). By giving all these four equations as the input to the trajectory planner block diagram as designed in Fig. 2, a suitable trajectory is defined in the 3 dimensional euclidean space [10] and the robot moves along that designed trajectory.

$$\dot{q}_1 = \frac{b_1 \dot{b}_2 - b_2 \dot{b}_1}{b_1^{\ 2} + b_2^{\ 2}} \tag{35}$$

$$\dot{q}_2 = \mp \frac{2\left(w_1 \dot{w}_1 + w_2 \dot{w}_2\right)}{\sqrt{\left(2 a_1 a_2\right)^2 - \left(w_1^{\ 2} + w_2^{\ 2} - a_1^{\ 2} - a_2^{\ 2}\right)^2}}. \tag{36}$$

$$\dot{q}_3 = - \dot{w}_3 \tag{37}$$

$$\dot{q}_4 = \pi \left(\frac{\dot{w}_6}{w_6} \right) \tag{38}$$

Note that the robot in our case is controlled at the joint level rather than at the gripper tip level. This is because; there is no direct control over the gripper tip. The only thing we can control in a robot is the speed of the motor, which is connected to the joint.

Hence, by controlling the joint, i.e., by varying the speed of the actuators, the robot tool tip is made to move along a particular path / trajectory. If q_k is the k^{th} joint variable , then $\dot{q}_k$ will be the k^{th} joint velocity / speed or the rate at which the k^{th} joint will move and is directly $\propto^{al}$ to the output speed of the k^{th} motor , where $1 \le k \le n$.

Note that a trajectory in our robot context is defined as, “if temporal information or time information is added to the gripper tip path by specifying the times at where the tool / gripper should be at various points along the path; then, the path gets converted into a trajectory.

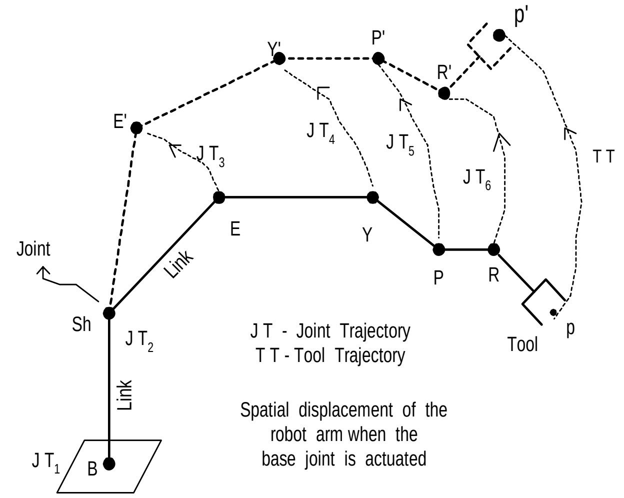

Fig. 7 Joint trajectories and tool trajectory

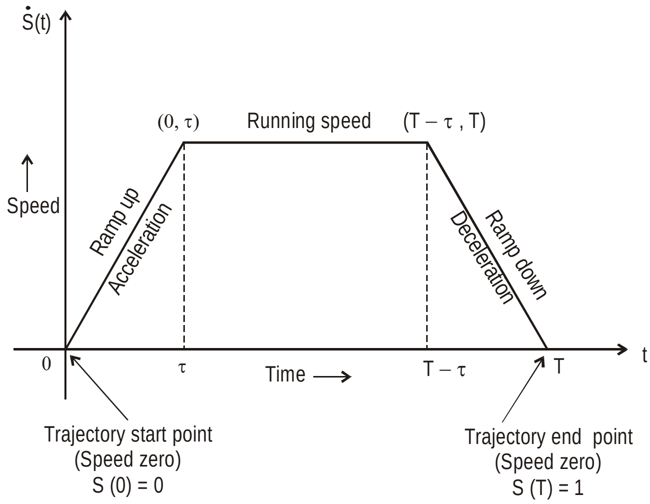

Fig. 8 A typical trapezoidal speed profile curve

Hence, a trajectory is defined as a path with time / temporal information, i.e., a trajectory is both spatial as well as temporal [11]. The joint trajectories, the gripper tip trajectory and the trajectory speed profile curve is shown in Figs. 7 and 7 respectively. The simulation results of the position, velocity and the acceleration curves are shown in the Figs. 9 - 12 respectively.

IV. Conclusions

A brief investigation into the trajectory design of the manipulator arm was discussed. The mathematical analysis of the trajectory design is also presented in this paper along with the block-diagrammatic representation.

A numerical analysis is also done. This trajectory is inputted into the computer which is interfaced with the robotic system and the robot's tip of the gripper moves along the desired / specified / designed trajectory. Here in this paper, the design of a continuous path trajectory is presented for the robot arm. It is also called as controlled path motion trajectory.

Once the user specifies the path, the robot moves continuously along the specified path. Hence the name continuous path motion and the corresponding trajectory traced by the gripper tip are known as a continuous path motion trajectory. The user explicitly specifies the path, the robot moves continuously along the specified path.

When the robot is moving continuously along the path, care has been taken to see that all the joint angles are controlled properly, otherwise it will not move along the specified path. Hence the name controlled path motion and the corresponding trajectory traced by the tool tip is known as a controlled path motion trajectory.

V. Simulation results

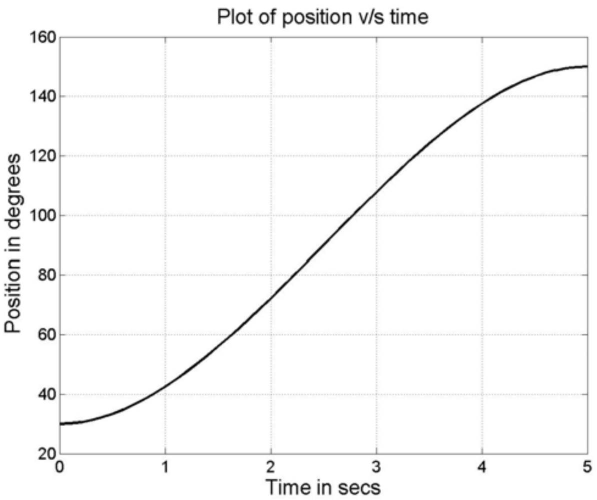

Fig. 9 Plot of position versus time (Matlab output)

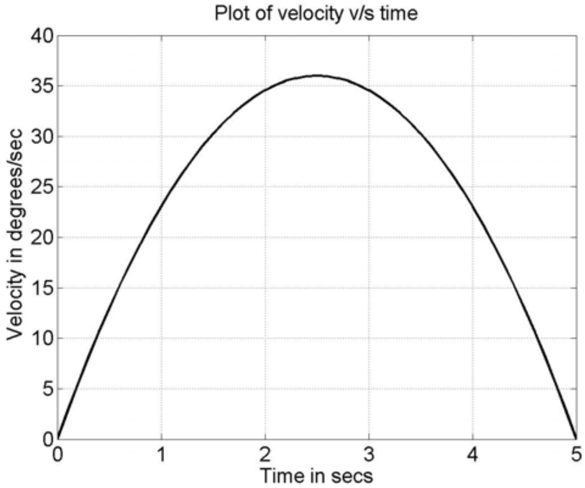

Fig. 10 Plot of velocity versus time (Matlab output)

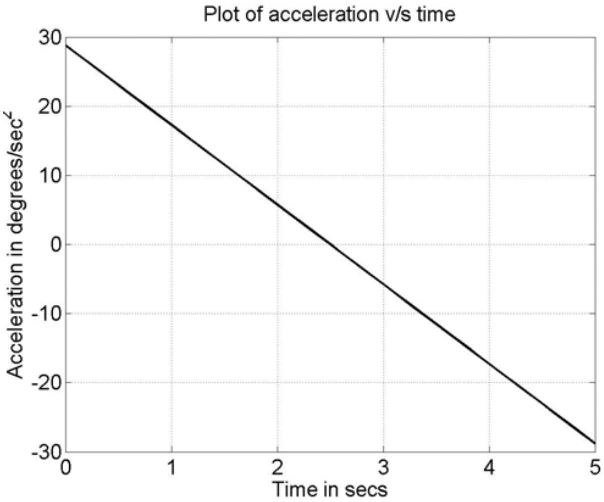

Fig. 11 Plot of acceleration versus time (Matlab output)

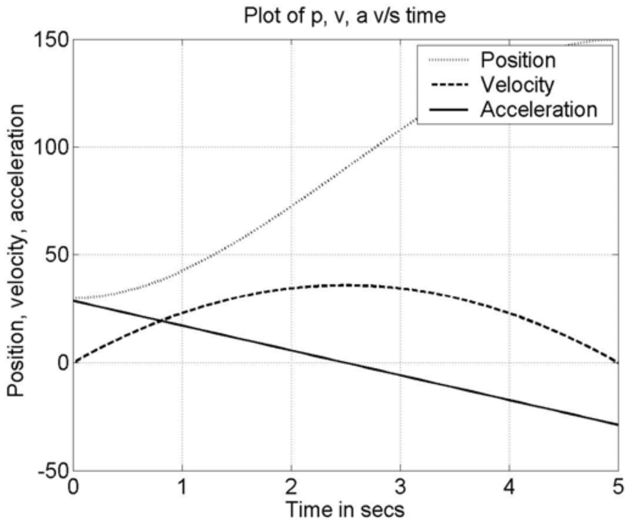

Fig. 12 Plot of position / velocity / acceleration versus time (Matlab output)

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