

# A Fuzzy Predictive Filter for Sinusoidal Signals with Time-Varying Frequencies

X. Z. Gao, S. J. Ovaska, and X. Wang

**Abstract**—Prediction of sinusoidal signals with time-varying frequencies has been an important research topic in power electronics systems. To solve this problem, we propose a new fuzzy predictive filtering scheme, which is based on a Finite Impulse Response (FIR) filter bank. Fuzzy logic is introduced here to provide appropriate interpolation of individual filter outputs. Therefore, instead of regular ‘hard’ switching, our method has the advantageous ‘soft’ switching among different filters. Simulation comparisons between the fuzzy predictive filtering and conventional filter bank-based approach are made to demonstrate that the new scheme can achieve an enhanced prediction performance for slowly changing sinusoidal input signals.

**Keywords**—Predictive filtering, fuzzy logic, sinusoidal signals, time-varying frequencies.

## I. INTRODUCTION

During recent years, sinusoidal predictive filters have been studied to deal with primary sinusoidal waveforms in electric power systems [1]. As we know, the frequencies of these signals can somewhat deviate from the nominal values 50 or 60 Hz. Typically, a  $\pm 2\%$  frequency variation exists in the Western European power networks. For example, the frequency of a sinusoidal voltage signal could increase from 49 to 51 Hz, and then decrease to 49 Hz within a lengthy time period. Unfortunately, *fixed* sinusoidal predictive filters are not robust to these frequency variations [2]. In other words, a filter designed for a specific frequency may not give satisfactory performance when facing different frequencies. Therefore, it is important but challenging to develop other filtering strategies that can provide efficient prediction of sinusoidal signals with time-varying frequencies. In the current paper, based on the fusion of fuzzy logic and FIR filters, we propose an alternative fuzzy predictive filtering approach, which is demonstrated to out-

perform the conventional methods concerning their prediction capabilities.

This paper is organized as follows. In Section II, a brief introduction of sinusoidal predictive filters is given. We also discuss the conventional filter bank-based solution to the aforementioned time-varying frequency problem. The new predictive fuzzy filtering scheme is presented in Section III. In the following section, we make performance comparisons between the filter bank-based method and our new method using computer simulations. Finally, some conclusions are drawn in Section V.

## II. PREDICTIVE FILTERS FOR SINUSOIDAL SIGNALS WITH TIME-VARYING FREQUENCIES

### A. Sinusoidal Predictive Filters

A discrete sinusoidal signal  $x(n)$ , free of noise, can be represented as follows:

$$x(n) = \sin[2\pi fn + \phi], \quad (1)$$

where  $f$  is the nominal frequency, and  $\phi$  is an arbitrary phase shift. Our goal here is to design an FIR filter that can give a  $p$ -step ahead prediction of  $x(n+p)$ , based on only the  $N$  currently available samples:  $x(n), x(n-1), \dots, x(n-N+1)$ , where  $N$  is the filter length. Thus, the output of this sinusoidal predictive filter  $y(n)$  is written:

$$y(n) = \sum_{k=0}^{N-1} h(k)x(n-k) = \sum_{k=0}^{N-1} h(k) \sin[2\pi f(n-k) + \phi], \quad (2)$$

in which  $h(k)$  ( $k = 0, 1, \dots, N-1$ ) are the filter coefficients. To make an exact prediction of  $x(n+p)$  for any  $n$ , the equation below must hold:

$$\sin[2\pi f(n+p) + \phi] = \sum_{k=0}^{N-1} h(k) \sin[2\pi f(n-k) + \phi]. \quad (3)$$

With (3) and other necessary constraints, such as removal of the dc component in practical  $x(n)$ , under consideration, the well-known method of Lagrange multipliers [3] can be used to obtain the optimal  $h(0), h(1), \dots, h(N-1)$ . Nevertheless, detailed derivation is beyond the discussion scope of our paper. Readers are referred to [2] for further information, where the filter parameters with  $p=2$  and  $N=22$  were drawn. Note that the nominal frequency  $f$  plays a crucial role in the optimization of  $h(0), h(1), \dots, h(N-1)$ . This makes the filter prediction accuracy

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sensitive to its value. Unfortunately, in practice,  $f$  can slowly change with time. Hence, the performance of *fixed-parameter* sinusoidal predictive filters may deteriorate under the circumstance of input signals with time-varying frequencies.

*B. Sinusoidal Predictive Filter Banks*

To cope with the above problem, Vainio and Ovaska have proposed an efficient filter bank-based solution [4], whose principal structure is shown in Fig. 1. In Fig. 1,  $x(n)$  is the sinusoidal signal with a *varying* frequency  $f(n)$ :

$$x(n) = \sin[2\pi f(n)n + \phi]. \tag{4}$$

In case  $f(n)$  is unknown beforehand,  $\hat{f}(n)$  can be the *instantaneous* estimate from  $x(n)$ . There are totally  $M$  sinusoidal predictive filters in this filter bank. Filter 1, Filter 2, ..., Filter  $M$  are the filters designed at targeted frequencies of  $f_1, f_2, \dots, f_M$ , respectively. Derivation of the coefficients for these filters follows the same principles mentioned earlier. Normally,  $f_1, f_2, \dots, f_M$  should cover the actual variation range of  $f(n)$ .  $y(n)$  is a multi-step ahead prediction of  $x(n)$  from the filter bank output.

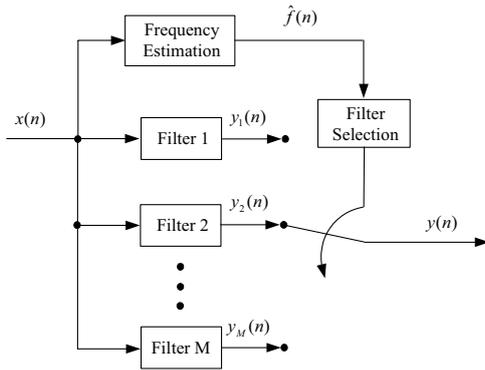


Fig. 1. Sinusoidal predictive filter bank.

Based on the estimated input frequency  $\hat{f}(n)$ ,  $y(n)$  is switched among different predictive filters as follows:

$$\text{IF } \frac{f_{i-1} + f_i}{2} \leq \hat{f}(n) < \frac{f_i + f_{i+1}}{2} \tag{5}$$

THEN  $y(n) = y_i(n)$ ,

where  $i = 2, 3, \dots, M - 1$ . Particularly,

$$\text{IF } \hat{f}(n) < \frac{f_1 + f_2}{2} \tag{6}$$

THEN  $y(n) = y_1(n)$ ,

and

$$\text{IF } \hat{f}(n) \geq \frac{f_{M-1} + f_M}{2}$$

$$\text{THEN } y(n) = y_M(n). \tag{7}$$

In [3], the authors design such a filter bank that consists of nine sinusoidal predictive filters, and the frequency coverage is [49 Hz, 51 Hz] with a 2% variation of the nominal frequency 50 Hz. It has been successfully employed in the application of line frequency zero-crossing detection.

Although the idea of this filter bank-oriented approach is simple and straightforward, it has some obvious drawbacks. For instance, at each sampling point, only one filter in the whole filter bank is activated, while the contributions from other filters are all neglected. It could be expected that an improved performance is acquired using an appropriate *interpolation* among the outputs of relevant filters. Moreover, the filter bank size grows significantly with the desired prediction accuracy, and can, therefore, suffer from the ‘curse of dimensionality’. In the next section, we propose a fuzzy predictive filtering scheme to handle these difficulties.

III. PREDICTIVE FUZZY FILTERING

During the past decade, fuzzy logic has found numerous successful applications in the area of signal processing [5] [6]. Compared with the conventional FIR and IIR filters, fuzzy filters indeed have a few unique characteristics, such as adaptation and prediction [7]. In this section, we introduce a fuzzy logic-based predictive filtering scheme, as illustrated in Fig. 2. Filter 1, Filter 2, ..., Filter  $M$  are the regular sinusoidal predictive filters at frequencies  $f_1, f_2, \dots, f_M$ . However, our method utilizes fuzzy inference to produce *interpolated* prediction from some simultaneously activated filters. More precisely, we first define  $M$  fuzzy membership functions  $\mu_1, \mu_2, \dots, \mu_M$  for individual frequencies of  $f_1, f_2, \dots, f_M$ , respectively.  $\mu_1, \mu_2, \dots, \mu_M$  need to not only cover the frequency variation range but also overlap with each other. Fig. 3 shows an example of the five Gaussian membership functions case:  $\mu_1, \mu_2, \dots, \mu_5$  ( $M = 5$ ), and the frequency range is 49-51 Hz, i.e.,  $f_1 = 49$  Hz,  $f_2 = 49.5$  Hz,  $f_3 = 50$  Hz,  $f_4 = 50.5$  Hz, and  $f_5 = 51$  Hz.

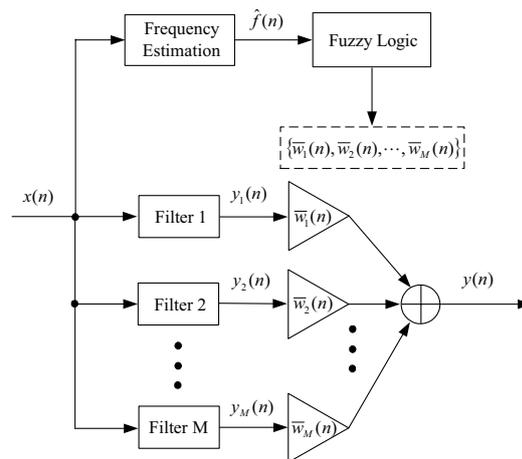


Fig. 2. Fuzzy logic-based sinusoidal predictive filtering.

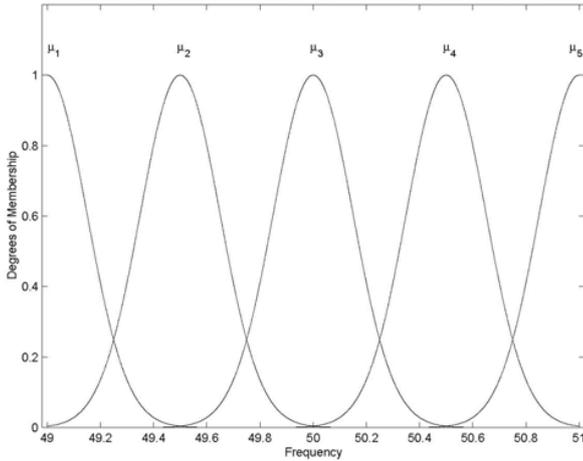


Fig. 3. Fuzzy membership functions for frequencies of sinusoidal input signal.

In Fig. 2, similarly with a Sugeno fuzzy model [8], the normalized inference output  $\bar{w}_i(n)$  for the estimated frequency  $\hat{f}(n)$  is calculated separately:

$$w_i(n) = \mu_i[\hat{f}(n)], \quad (8)$$

and

$$\bar{w}_i(n) = \frac{w_i(n)}{\sum_{j=1}^M w_j(n)}, \quad (9)$$

where  $i=1,2,\dots,M$ . Therefore, at sampling point  $n$ , the output  $y_i(n)$  of each filter is weighted by corresponding  $\bar{w}_i(n)$  in the final prediction of our fuzzy filtering  $y(n)$ :

$$y(n) = \sum_{i=1}^M \bar{w}_i(n) y_i(n). \quad (10)$$

From the above descriptions, it is apparent that with the deployment of fuzzy membership functions for input frequencies, our scheme has the distinguished feature of ‘soft’ instead of ‘hard’ switching among available filters. In other words, depending on the grade of membership, each filter plays its partial role in the system prediction output, which is different from the principles of the aforementioned filter bank-based approach. Taking full advantage of all the predictive filters, this strategy can not only effectively enhance the prediction accuracy of an existing filter bank, but also reduce the bank size, while still maintaining an acceptable performance. Actually, membership functions  $\mu_1, \mu_2, \dots, \mu_M$  provide greater flexibility for designing our predictive filtering system. In addition, parameters of  $\mu_1, \mu_2, \dots, \mu_M$ , such as centers and widths, could be adaptively trained, based on some Back-Propagation (BP) learning algorithm [9]. This would result in the remarkable capability of tracking even rapidly changing frequencies. Simulations are made to demonstrate the efficiency of our method in Section IV.

#### IV. SIMULATIONS

In the simulations, we verify the effectiveness of the proposed fuzzy predictive filtering scheme, and further make comparisons with the conventional filter banks. The sinusoidal input signal with time-varying frequencies  $x(n)$  is sampled at 1.67 kHz. The nominal frequencies are in the range of  $50 \text{ Hz} \pm 1\%$ . More specifically, in our case,  $f(n)$  grows from 49.5 Hz to 50.5 Hz during a period of 2 seconds, as shown in Fig. 4.

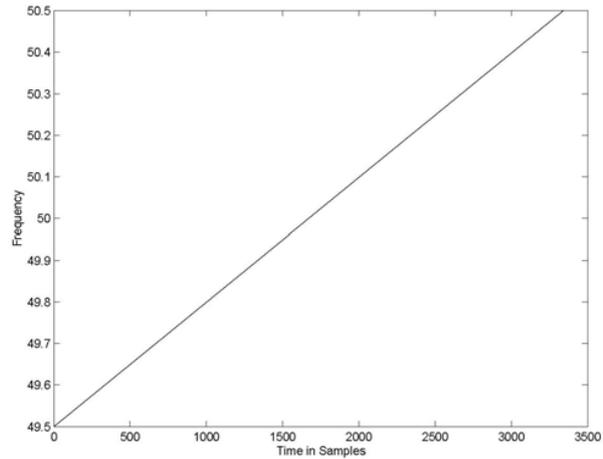


Fig. 4. Time-varying  $f(n)$  of sinusoidal input signal  $x(n)$ .

We first choose two different filter banks, one with five and the other with three two-step ahead sinusoidal predictive filters. For the five-filter bank, filters are designed at frequencies of 49.5 Hz, 49.75 Hz, 50 Hz, 50.25 Hz, and 50.5 Hz, respectively. These targeted frequencies can fully cover the above variation range, and filter coefficients are quoted from [3]. On the other hand, only the three filters associated with the frequencies of 49.75 Hz, 50 Hz, and 50.25 Hz are selected in our three-filter bank. It should be pointed out that in order to simplify the presentation, we assume that input frequency  $f(n)$  is exactly known in advance without any estimation. That is, switching among filters is based on  $f(n)$  instead of  $\hat{f}(n)$ . To put it into more details, the switching policy of the five-filter bank is as follows:

IF  $f(n) < 49.625 \text{ Hz}$  THEN switch to filter designed at 49.5 Hz,

IF  $49.625 \text{ Hz} \leq f(n) < 49.875 \text{ Hz}$  THEN switch to filter designed at 49.75 Hz,

IF  $49.875 \text{ Hz} \leq f(n) < 50.125 \text{ Hz}$  THEN switch to filter designed at 50 Hz,

IF  $50.125 \text{ Hz} \leq f(n) < 50.375 \text{ Hz}$  THEN switch to filter designed at 50.25 Hz,

IF  $f(n) \geq 50.375 \text{ Hz}$  THEN switch to filter designed at 50.5 Hz.

Similarly, the three-filter bank behaves like:

IF  $f(n) < 49.875 \text{ Hz}$  THEN switch to filter designed at 49.75 Hz,

IF  $49.875 \text{ Hz} \leq f(n) < 50.125 \text{ Hz}$  THEN switch to filter designed at 50 Hz,

IF  $f(n) \geq 50.125 \text{ Hz}$  THEN switch to filter designed at 50.25 Hz.

The Sum Squared Prediction Error (SSPE) is used to evaluate the predictive filtering performances:

$$\text{SSPE} = \sum_{n=22}^L [x(n+2) - y(n)]^2, \quad (11)$$

where  $L$  is the number of samples ( $L = 1,670 \times 2 = 3,340$  here). Figs. 5 and 6 illustrate the prediction errors of the five-filter and three-filter banks, respectively. It is clearly visible that prediction error of the former is smaller than that of the later. Hence, a higher prediction accuracy could be reasonably acquired with more such filters involved.

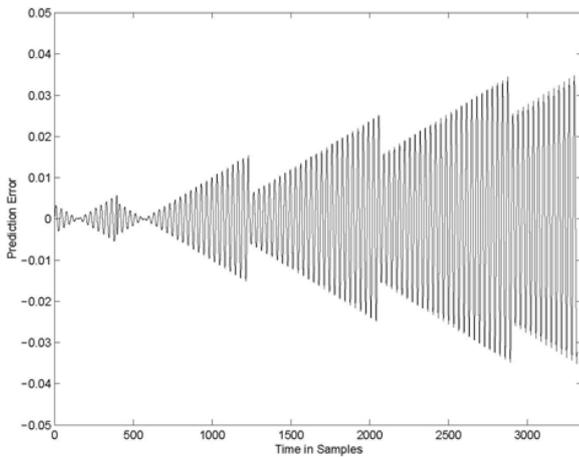


Fig. 5. Prediction error of five-filter bank.

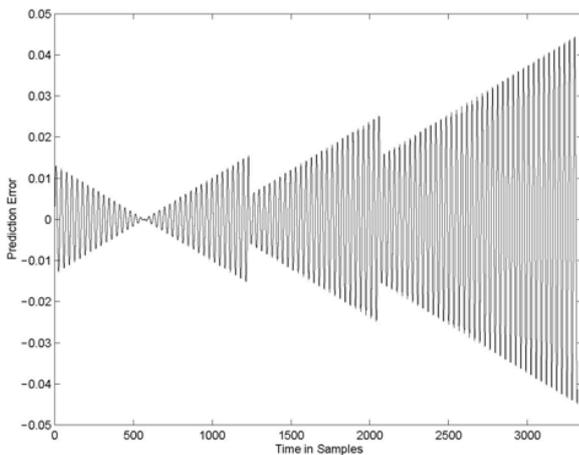


Fig. 6. Prediction error of three-filter bank.

Next, our fuzzy predictive filtering scheme is examined based on the same sinusoidal input signal. Those filters in the three-filter bank are used again, and three membership functions,  $\mu_1$ ,  $\mu_2$ , and  $\mu_3$ , are defined at the filter frequencies of 49.75 Hz, 50 Hz, and 50.25 Hz, as depicted in

Fig. 7. We would like to emphasize that the membership function parameters are manually chosen and fine-tuned, which result in the prediction error in Fig. 8. Comparing Fig. 6 with Fig. 8, we can find out the prediction improvement of our fuzzy filtering method mainly lies in the 'mid-frequency' band, from around 49.8 Hz to 50.1 Hz. This is exactly the same overlapping area among the three frequency membership functions  $\mu_1$ ,  $\mu_2$ , and  $\mu_3$ , refer to Fig. 7. As a matter of fact, the advantage of the proposed filtering scheme is due to the unique interpolation of fuzzy membership functions that provides appropriate weightages for individual predictive filter outputs. However, the add-on membership functions also lead to an increased computation complexity.

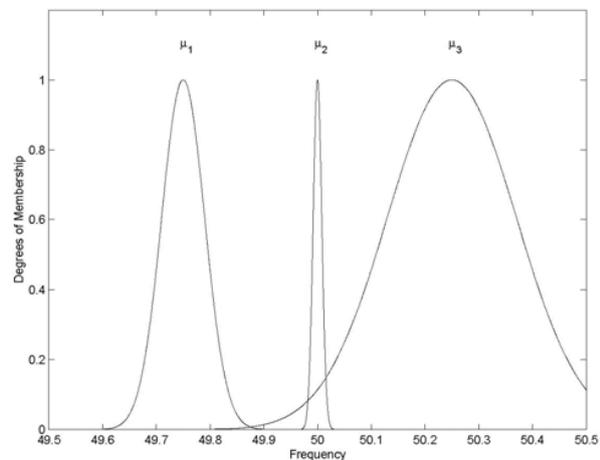


Fig. 7. Fuzzy membership functions for filter frequencies.

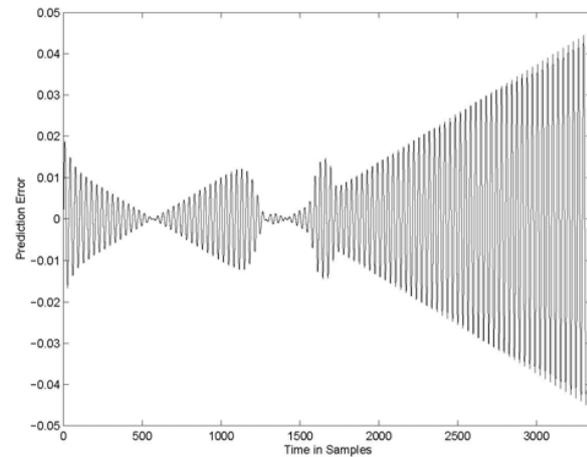


Fig. 8. Prediction error of fuzzy filtering scheme.

The SSPEs of the five-filter bank, three-filter bank, and our fuzzy filtering scheme are given in Table I. Table I shows introduction of fuzzy logic in the three-filter bank effectively enhances its prediction accuracy (about 10%). Thus, it can be concluded that fuzzy filtering is indeed *complementing* rather than competitive with the existing filter banks-based approaches. Additionally, adaptation methods for the parameters of those membership functions, such as the well-known Least Square Estima-

tion (LSE) [10] and Evolutionary Programming (EP) [11], could be promising to further improve the prediction performance as well as on-line tracking of sinusoidal input signals with fast time-varying frequencies.

Table I. SSPEs of five-filter bank, three-filter bank, and fuzzy filtering scheme.

Filtering Systems	Five-Filter Bank	Three-Filter Bank	Fuzzy Filtering
SSPE	0.6153	0.7723	0.6919

#### IV. CONCLUSIONS

In this paper, we proposed a fuzzy predictive filtering scheme for the prediction of sinusoidal signals with time-varying frequencies. Our approach has the remarkable features of intuitive principle and simple structure. Simulations demonstrate that better prediction results can be achieved by employing fuzzy logic in the conventional filter banks. Adaptive learning algorithms for this method [12] to track fast changing input signals are currently under development, and results will be reported in the future.

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