International Journal of Engineering, Mathematical and Physical Sciences
ISSN: 2517-9934
Vol:5, No:8, 2011

Confidence intervals for the difference of two normal population variances

Suparat Niwitpong

Abstract—Motivated by the recent work of Herbert, Hayen, Macaskill and Walter [Interval estimation for the difference of two independent variances. Communications in Statistics, Simulation and Computation, 40: 744-758, 2011.], we investigate, in this paper, new confidence intervals for the difference between two normal population variances based on the generalized confidence interval of Weerahandi [Generalized Confidence Intervals. Journal of the American Statistical Association, 88(423): 899-905, 1993.] and the closed form method of variance estimation of Zou, Huo and Taleban [Simple confidence intervals for lognormal means and their differences with environmental applications. Environmetrics 20: 172-180, 2009]. Monte Carlo simulation results indicate that our proposed confidence intervals give a better coverage probability than that of the existing confidence interval. Also two new confidence intervals perform similarly based on their coverage probabilities and their average length widths.

Keywords—confidence interval, generalized confidence interval, the closed form method of variance estimation, variance.

I. Introduction

Recent paper of Herbert et al. [9] have argued that analytical methods for interval estimation of the difference between variances have not been described. They proposed a simple analytical method to construct confidence interval for the difference between variances of two independent samples. It is shown that their proposed confidence interval generated coverage probability close to nominal level even when sample sizes are small and unequal. Their method also works well when observations are highly skewed and leptokurtic. Cojbasica and Tomovica [12] has shown that the methods based on t-statistics combined with bootstrap techniques for determination of nonparametric confidence interval of the population variance of two-sample problems work very well when data are from exponential families. Related works of the confidence interval for the difference between two variances, the reader is referred to the references cited in the above two papers. In this paper, we emphasize only data is from a normal distribution and we propose two new confidence intervals for the difference of two normal population variances, one is based on the generalized confidence interval, Weerahandi [10] and another one is based on the closed form method of variance estimation(CMVE), Zou et al. [2]. Generalized confidence interval has been widely used for statistical estimation when there is no explicit pivotal statistic to construct the confidence interval see e.g. Krishnamoorthy and Mathew [6], Krishnamoorthy et al. [7], Tian [8] and the references cited in these papers. The closed form method of variance estimation has been used to construct the confidence interval for the parameters in the forms of $\theta_1 + \theta_2$ and $\theta_1 \setminus \theta_2$ see e.g. Zou et al. [2], Zou and Donner [4], Zou et al. [5] and the latest paper of Donner and Zou [1]. Zou et al. [3] have argued that, in terms of the coverage probability, the confidence interval constructed from the method of variance recovery estimation method performs as well as the confidence interval from GCI method but in some situations the average length width from this method is shorter than that of the confidence interval constructed from GCI method. In this paper, it is therefore of interest to construct the confidence interval for the difference of normal population variances using the generalized confidence interval and the closed form of variance method of variance estimation compared to the existing method see e.g. Cojbasica and Tomovica [12]. The paper is organized as follows. Section 2 presents new confidence intervals for difference of two normal population means. Simulation design to study coverage probabilities and average length widths for each interval is outlined in Section 3. Section 4 gives simulation results of coverage probabilities and ratio of expected lengths of confidence intervals for difference of two normal population variances for selected sample sizes with a range of values of ratio of variances. Section 5 contains a discussion of the results and conclusions.

Suparat Niwitpong is with the Department of Applied Statistics, King Mongkut's University of Technology North Bangkok, Bangkok, THAILAND, 10800, e-mail: suparatn@kmutnb.ac.th

II. Confidence intervals for the difference of two normal population variances

Let $X_1, \dots, X_n$ and $Y_1, \dots, Y_m$ be random samples from two independent normal distributions with means μ_x, μ_y and standard deviations σ_x and σ_y, respectively. The sample means and variances for X and Y are, respectively, denoted as $\bar{X}$, $\bar{Y}$, S_x^2 and S_y^2 when $\bar{X} = n^{-1}\sum_{i=1}^{n} X_i$, $\bar{Y} = m^{-1}\sum_{i=1}^{m} Y_i$, $S_x^2 = (n-1)^{-1}\sum_{i=1}^{n}\left(X_i - \bar{X}\right)^2$ and $S_y^2 = (m-1)^{-1}\sum_{i=1}^{m}\left(Y_i - \bar{Y}\right)^2$. We are interested in 100(1-α)% confidence interval for $\theta = \sigma_x^2 - \sigma_y^2$.

1) Standard confidence interval for θ: It is well known that statistic $(n-1)S_x^2/\sigma_x^2$ has Chi-square distribution (χ^2_{n-1}) with $n-1$ degrees of freedom and hence we have the following results, $E(\chi^2_{n-1}) = n-1$ and $Var(\chi^2_{n-1}) = 2(n-1)$. Define

$$Z = \frac{(n-1)S_x^2/\sigma_x^2 - (n-1)}{\sqrt{2(n-1)}} = \frac{S_x^2 - \sigma_x^2}{\sqrt{Var(S_x^2)}}$$

when $Var(S_x^2) = \sigma_x^4/2(n-1)$ and the test statistic, Z is the standard normal distribution. As a results, we have

$$Z = \frac{(S_x^2 - S_y^2) - \theta}{\sqrt{Var(S_x^2) + Var(S_y^2)}} = \frac{(S_x^2 - S_y^2) - \theta}{\sqrt{\frac{2\sigma_x^4}{(n-1)} + \frac{2\sigma_y^4}{(m-1)}}}.$$

In general, σ_x^2 and σ_y^2 are unknown parameters and the unbiased estimators for these parameters are respectively S_x^2 and S_y^2. It is easily seen that, using Z as a pivotal statistic, the 100(1-α)% confidence interval for θ is

$$CI_s = \left[(S_x^2 - S_y^2) \mp z_{1-\alpha/2}\sqrt{\frac{2S_x^4}{(n-1)} + \frac{2S_y^4}{(m-1)}}\right]$$

where $z_{1-\alpha/2}$ is an upper $1-\alpha/2$ quantile of the standard normal distribution.

2) Generalized confidence interval for θ: In this section, we present the generalized confidence interval (GCI), proposed by Weerahandi [10], for θ. We now give a brief introduction to the GCI idea based on Weerahandi [10]. The method of GCI is easy to use based on computational approach. To introduce the concept,

1.Let X and Y be random variables with probability distribution $f(X, Y, \theta, \varsigma)$, where θ is the parameter of interest and ς is a set of nuisance parameters.

2. Let (x, y) denote the observed value of (X, Y). To obtain a generalized confidence interval for θ, we start from the generalized pivotal quantity $T(X, Y; x, y, \theta, \varsigma)$, which is a function of the random variable (X, Y), its observed value (x, y) and the parameters θ and ς.

3. Also, $T(X, Y; x, y, \theta, \varsigma)$ is required to satisfy the following conditions:

(3.1) For a fixed (x, y) the probability distribution of $T(X, Y; x, y, \theta, \varsigma)$ is free of unknown parameters

(3.2) The observed value of $T(X, Y; x, y, \theta, \varsigma)$, namely $T(x, y; x, y, \theta, \varsigma)$ is simply θ.

A $100(1-\alpha)\%$ generalized confidence interval for θ is therefore given by

$$\left[T(\alpha/2), T(1-\alpha/2)\right].$$

where $T(\alpha/2)$, $T(1-\alpha/2)$ are respectively the $\alpha/2$ and $1-\alpha/2$ of $T(x, y; x, y, \theta, \varsigma)$.

To obtain GCI for our θ, consider

$$\sigma_x^2 - \sigma_y^2 = \frac{(n-1)S_x^2}{U} - \frac{(m-1)S_y^2}{V}, U \sim \chi^2_{n-1}, V \sim \chi^2_{m-1}.$$

We therefore define $T(X, Y; x, y, \theta, \varsigma) = \frac{(n-1)S_x^2}{U} - \frac{(m-1)S_y^2}{V}$. It is easy to see that $T(x, y; x, y, \theta, \varsigma)$ satisfies (3.1) and (3.2). As a result, a $100(1-\alpha)\%$ generalized confidence interval for θ is given by

$$CI_g = \left[T(\alpha/2), T(1-\alpha/2)\right].$$

3) The closed form method of variance estimation for θ: The idea of estimating confidence limits for lognormal data by the closed form method of variance estimation was presented by Zou et al. [2]. Their strategy is to recover variance estimates from these limits and then to form approximate confidence intervals for functions of the parameters. Using the central limit theorem, a general approach to setting two-sided $(1-\alpha)100\%$ confidence limits for $\theta_1 + \theta_2$ is given by

$$(\hat{\theta}_1 + \hat{\theta}_2) \mp z_{(1-\alpha/2)}\sqrt{var(\hat{\theta}_1) + var(\hat{\theta}_2)}$$

where the estimators $\hat{\theta}_i$ $(i = 1, 2)$ and $var(\hat{\theta}_i)$ are statistically independent of each other. Since $var(\hat{\theta}_i)$ is unknown, Zou et al. [2] argued that the method to improve of estimating variance of $(\hat{\theta}_1 + \hat{\theta}_2)$ in the neighborhood of the limits (L, U) for $\theta_1 + \theta_2$ was estimated in the setting of $\min(\theta_1 + \theta_2)$ for L and that of $\max(\theta_1 + \theta_2)$ for U. Suppose that a $(1-\alpha)100\%$ two-sided confidence interval for θ_i is given by (l_i, u_i). Using the central limit theorem, we have

$$\frac{(\hat{\theta}_i - l_i)^2}{var(\hat{\theta}_i)} \approx z^2_{1-\alpha/2}$$

which outcomes in the estimated variance $\widehat{var}(\hat{\theta}_i)$ under the condition $\theta_i = l_i$ of

$$\widehat{var_l}(\hat{\theta}_i) \approx \frac{(\hat{\theta}_i - l_i)^2}{z^2_{1-\alpha/2}}.$$

Similarly, the estimated variance $\widehat{var}(\hat{\theta}_i)$ under the condition $\theta_i = u_i$ of

$$\widehat{var_u}(\hat{\theta}_i) \approx \frac{(u_i - \hat{\theta}_i)^2}{z^2_{1-\alpha/2}}.$$

Therefore, the confidence limit $[L, U]$ is given by

$$\begin{aligned}
[L, U] &= \left[(\hat{\theta}_1 + \hat{\theta}_2) \mp z_{1-\alpha/2}\sqrt{\widehat{var}(\hat{\theta}_1) + \widehat{var}(\hat{\theta}_2)}\right] \\
&= \left[(\hat{\theta}_1 + \hat{\theta}_2) - z_{1-\alpha/2}\sqrt{\frac{(\hat{\theta}_1 - l_1)^2}{z^2_{1-\alpha/2}} + \frac{(\hat{\theta}_2 - l_2)^2}{z^2_{1-\alpha/2}}},\right. \\
&\qquad \left.(\hat{\theta}_1 + \hat{\theta}_2) + z_{1-\alpha/2}\sqrt{\frac{(u_1 - \hat{\theta}_1)^2}{z^2_{1-\alpha/2}} + \frac{(u_2 - \hat{\theta}_2)^2}{z^2_{1-\alpha/2}}}\right] \\
&= \left[(\hat{\theta}_1 + \hat{\theta}_2) - \sqrt{(\hat{\theta}_1 - l_1)^2 + (\hat{\theta}_2 - l_2)^2},\right. \\
&\qquad \left.(\hat{\theta}_1 + \hat{\theta}_2) + \sqrt{(u_1 - \hat{\theta}_1)^2 + (u_2 - \hat{\theta}_2)^2}\right].
\end{aligned}$$

Confidence limits for $\theta_1 - \theta_2$ can be constructed by replacing $\theta_1 - \theta_2$ in the form $\theta_1 + (-\theta_2)$ and replacing the confidence limits for $-\theta_2$ which is $(-u_2, -l_2)$. We therefore construct the confidence interval for θ as

$$\begin{aligned}
\left[L_1, U_1\right] &= \left[(\hat{\theta}_1 - \hat{\theta}_2) - \sqrt{(\hat{\theta}_1 - l_1)^2 + (u_2 - \hat{\theta}_2)^2},\right. && (1) \\
&\qquad \left.(\hat{\theta}_1 - \hat{\theta}_2) + \sqrt{(u_1 - \hat{\theta}_1)^2 + (\hat{\theta}_2 - l_2)^2}\right]. && (2)
\end{aligned}$$

We now propose the confidence interval for θ. Define $\theta_1 = \sigma_x^2$, $\theta_2 = \sigma_y^2$,$\hat{\theta}_1 = S_x^2$ and $\hat{\theta}_2 = S_y^2$. Also, confidence limits for $\theta_i, i = 1, 2$, is (l_i, u_i) where

$$[l_1, u_1] = \left[\frac{(n-1)S_x^2}{\chi^2_{1-\alpha/2,n-1}}, \frac{(n-1)S_x^2}{\chi^2_{\alpha/2,n-1}}\right]$$

and

$$[l_2, u_2] = \left[\frac{(n-1)S_y^2}{\chi^2_{1-\alpha/2,m-1}}, \frac{(n-1)S_y^2}{\chi^2_{\alpha/2,m-1}}.\right]$$

Replacing $\hat{\theta}_1 = S_x^2$,$\hat{\theta}_2 = S_y^2$, l_i and u_i in the above equations into (1) and (2), we then obtain a new confidence interval for θ which is called CI_r. In the next section, we evaluate these

International Journal of Engineering, Mathematical and Physical Sciences
ISSN: 2517-9934
Vol:5, No:8, 2011

three confidence intervals based on their coverage probabilities and average length widths Typically, we prefer a confidence interval whose coverage probability is at least the nominal level $1 - \alpha$ and has a shorter length. Monte Carlo simulation will be used to assess these criteria.

III. Simulation framework

In this section, we use Monte Carlo simulation to assess three confidence intervals notified in the previous section: CI_s, CI_g and CI_r based on their coverage probabilities and average length widths. We design a simulation, without losing generality, by setting $\mu_1 = \mu_2 = 1$, a ratio of variances $\sigma_1^2/\sigma_2^2 = 0.01, 0.25, 0.5, 0.8, 1, 2, 4, 8, 10, 20, 50, 100$ and the samples sizes $(n = m = 5)$, $(n = 10, m = 5)$, $(n = m = 10)$, $(n = 20, m = 10)$, $(n = m = 20)$, $(n = 40, m = 20)$ and $(n = m = 40)$. We wrote function in R program [11] to generate the data which is normally distributed with means and variances which are mentioned previously to construct three confidence intervals, i.e. CI_s, CI_g and CI_r and then compute coverage probability and average length width of each confidence interval. All results are illustrated in Tables I and II with a number of simulation run, M=5000 and the nominal level $(1 - \alpha = 0.95)$.

IV. Simulation results

From Tables I and II, we found that the coverage probability of the CI_s confidence intervals is generally below the nominal level for all sample sizes. As a result its average length width is shorter than other confidence intervals. We therefore do not recommend to use this confidence interval for small and moderate sample sizes. The coverage probabilities of the CI_g and CI_r confidence intervals are very close to the nominal level 0.95 for all ratio of variances and sample sizes. Tables I and II also show that average length widths of these confidence intervals, CI_g and CI_r, are not different. We therefore conclude that the performance of confidence interval CI_g and CI_r, based on the mentioned criteria, is not difference but the confidence interval CI_r is easy to use while the confidence interval CI_g relies completely on computational approach.

V. Conclusion

In this paper we proposed two new confidence intervals for the difference normal population variances. Both methods: GCI and CMVE perform well in terms of coverage probabilities and average length widths. For all most cases, we found that coverage probabilities of these two intervals, CI_g and CI_r, for all sample sizes and ratio of variances, are very close to the nominal level 0.95. The average length widths of these confidences are not different. There are only cases when the ratio of variance ranges between 0.5–4, the confidence interval from CMVE method is slightly better than the GCI method. We also do not recommend to use the standard confidence interval (CI_s) for small sample sizes since its coverage probability is far below the nominal level 0.95. Further research is to investigate the confidence intervals for the ratio of variances using the generalized confidence interval and the closed form method of variance estimation.

TABLE I
The coverage probabilities of CI_s, CI_g and CI_r and their ratio of average length widths when the number of simulation runs = 5,000, E_s, E_g and E_r are respectively the average length widths of CI_s, CI_g and CI_r.

σ_1/σ_2	CI_s	CI_r	CI_g	E_s/E_r	E_s/E_g	E_r/E_g
			$n=5$	$m=5$		
0.01	0.8065	0.9550	0.9520	0.3505	0.3382	0.9649
0.25	0.8395	0.9595	0.9530	0.3176	0.3156	0.9938
0.50	0.9280	0.9605	0.9505	0.3000	0.3022	1.0072
0.80	0.8970	0.9635	0.9515	0.2921	0.2958	1.0128
1.00	0.9023	0.9575	0.9475	0.2921	0.2958	1.0128
2.00	0.9350	0.9650	0.9575	0.2994	0.3025	1.0103
4.00	0.8420	0.9645	0.9595	0.3175	0.3167	0.9975
6.00	0.8010	0.9580	0.9505	0.3276	0.3267	0.9973
8.00	0.7900	0.9540	0.9475	0.3340	0.3283	0.9829
10.0	0.8140	0.9540	0.9495	0.3372	0.3308	0.9810
20.0	0.9775	0.9420	0.9465	0.3456	0.3365	0.9737
50.0	0.7880	0.9450	0.9440	0.3496	0.3403	0.9734
100	0.7995	0.9530	0.9485	0.3505	0.3420	0.9756
			$n=10$	$m=5$		
0.01	0.7960	0.9555	0.9580	0.3509	0.3428	0.9769
0.25	0.7985	0.9455	0.9475	0.3477	0.3406	0.9796
0.50	0.8565	0.9615	0.9545	0.3485	0.3465	0.9938
0.80	0.9375	0.9585	0.9490	0.3553	0.3568	1.0040
1.00	0.8985	0.9600	0.9510	0.3617	0.3646	1.0077
2.00	0.8940	0.9640	0.9495	0.4075	0.4143	1.0168
4.00	0.9090	0.9630	09575	0.4755	0.4892	1.0289
6.00	0.8960	0.9635	0.9605	0.5229	0.5345	1.0222
8.00	0.8655	0.9530	0.9500	0.5540	0.5647	1.0192
10.0	0.8725	0.9630	0.9595	0.5711	0.5779	1.0118
20.0	0.8600	0.9485	0.9460	0.6174	0.6170	0.9993
50.0	0.8625	0.9455	0.9440	0.6391	0.6343	0.9925
100	0.8740	0.9495	0.9475	0.6443	0.6383	0.9906
			$n=10$	$m=10$		
0.01	0.8585	0.9520	0.9510	0.9460	0.6396	0.9901
0.25	0.8955	0.9590	0.9545	0.6105	0.6160	1.0090
0.50	0.9320	0.9560	0.9540	0.5811	0.5969	1.0270
0.80	0.8975	0.9615	0.9570	0.5669	0.5832	1.0287
1.00	0.8999	0.9590	0.9500	0.5657	0.5825	1.0298
2.00	0.9300	0.9575	0.9535	0.5808	0.5983	1.0301
4.00	0.8860	0.9595	0.9515	0.6110	0.6165	1.0090
6.00	0.8720	0.9565	0.9495	0.6246	0.6253	1.0010
8.00	0.8565	0.9490	0.9480	0.6318	0.6283	0.9945
10.0	0.8640	0.9485	0.9425	0.6358	0.6328	0.9951
20.0	0.8610	0.9460	0.9445	0.6433	0.6384	0.9924
50.0	0.8675	0.9475	0.9445	0.6456	0.6398	0.9910
100	0.8580	0.9445	0.9490	0.6460	0.6395	0.9899
			$n=20$	$m=10$		
0.01	0.8640	0.9495	0.9465	0.6461	0.6370	0.9858
0.25	0.8700	0.9470	0.9430	0.6389	0.6327	0.9902
0.50	0.8880	0.9535	0.9490	0.6307	0.6345	1.0060
0.80	0.9430	0.9530	0.9495	0.6295	0.6398	1.0162
1.00	0.9630	0.9525	0.9455	0.6318	0.6433	1.0181
2.00	0.9765	0.9560	0.9525	0.6673	0.6861	1.0281
4.00	0.9270	0.9605	0.9565	0.7300	0.7475	1.0239
6.00	0.9125	0.9560	0.9500	0.7619	0.7733	1.0149
8.00	0.9105	0.9610	0.9560	0.7807	0.7858	1.0065
10.0	0.9090	0.9560	0.9510	0.7908	0.7962	1.0067
20.0	0.9085	0.9470	0.9475	0.8098	0.8080	0.9978
50.0	0.9030	0.9505	0.9460	0.8157	0.8110	0.9942
100	0.9050	0.9510	0.9490	0.8175	0.8121	0.9933

TABLE II
THE COVERAGE PROBABILITIES OF CI_s, CI_g AND CI_r AND THEIR RATIO OF AVERAGE LENGTH WIDTHS WHEN THE NUMBER OF SIMULATION RUNS = 5,000, E_s, E_g AND E_r ARE RESPECTIVELY THE AVERAGE LENGTH WIDTHS OF CI_s, CI_g AND CI_r.

σ_1/σ_2	CI_s	CI_r	CI_g	E_s/E_r	E_s/E_g	E_r/E_g
			$n=20$	$m=20$		
0.01	0.9085	0.9525	0.9465	0.8178	0.8142	0.9955
0.25	0.9065	0.9525	0.9450	0.7955	0.8001	1.0057
0.50	0.9425	0.9560	0.9490	0.7682	0.7787	1.0135
0.80	0.9745	0.9525	0.9450	0.7551	0.7733	1.0240
1.00	0.9735	0.9465	0.9380	0.7539	0.7703	1.0217
2.00	0.9405	0.9540	0.9450	0.7693	0.7829	1.0176
4.00	0.9085	0.9575	0.9550	0.7955	0.8015	1.0075
6.00	0.9080	0.9460	0.9450	0.8059	0.8046	1.9983
8.00	0.9100	0.9445	0.9430	0.8106	0.8078	0.9966
10.0	0.9070	0.9625	0.9550	0.8130	0.8091	0.9951
20.0	0.9080	0.9505	0.9475	0.8166	0.8126	0.9950
50.0	0.8930	0.9510	0.9485	0.8177	0.8129	0.9942
100	0.9060	0.9540	0.9545	0.8178	0.8114	0.9921
			$n=40$	$m=20$		
0.01	0.8990	0.9445	0.9455	0.8179	0.8135	0.9946
0.25	0.9130	0.9555	0.9575	0.8126	0.8118	0.9990
0.50	0.9270	0.9530	0.9510	0.8039	0.8090	1.0062
0.80	0.9505	0.9470	0.9435	0.7991	0.8091	1.0125
1.00	0.9590	0.9440	0.9390	0.7992	0.8102	1.0137
2.00	0.9570	0.9465	0.9425	0.8220	0.8356	1.0165
4.00	0.9370	.9450	0.9425	0.8636	0.8742	1.0122
6.00	0.9250	0.9570	0.9525	0.8825	0.8877	1.0058
8.00	0.9220	0.9495	0.9475	0.8918	0.8945	1.0030
10.0	0.9320	0.9520	0.9505	0.8972	0.8967	0.9994
20.0	0.9250	0.9505	0.9485	0.9050	0.9016	0.9962
50.0	0.9290	0.9530	0.9500	0.9074	0.9046	0.9968
100	0.9100	0.9520	0.9485	0.9077	0.9043	0.9962
			$n=40$	$m=40$		
0.01	0.9275	0.9535	0.9560	0.9078	0.9045	0.9963
0.25	0.9330	0.9490	0.9430	0.8958	0.8981	1.0025
0.50	0.9340	0.9485	0.9405	0.8781	0.8863	1.0092
0.80	0.9655	0.9575	.9560	0.8677	0.8780	1.0119
1.00	0.9620	0.9500	0.9480	0.8664	0.8805	1.0163
2.00	0.9480	0.9590	0.9575	0.8782	0.8846	1.0073
4.00	0.9245	0.9430	0.9445	0.8958	0.8968	1.0010
6.00	0.9180	0.9545	0.9505	0.9019	0.8998	0.9976
8.00	0.9335	0.9455	0.9410	0.9044	0.9018	0.9971
10.0	0.9440	0.9570	0.9540	0.9056	0.9037	0.9978
20.0	0.9350	0.9395	0.9385	0.9073	0.9015	0.9936
50.0	0.9185	0.9475	0.9405	0.9078	0.9039	0.9957
100	0.9190	0.9495	0.9485	0.9078	0.9015	0.9930

ACKNOWLEDGMENT

The author is grateful to King Mongkut's University of Technology North Bangkok for their partial financial support.

REFERENCES

[1] A. Donner, GY. Zou, Closed-form confidence intervals for function of the normal mean and standard deviation, Statistical Methods in Medical Research, (2010) 1–12.
[2] GY. Zou, CY. Huo, J. Taleban, Simple confidence intervals for lognormal means and their differences with environmental applications, Environmetrics 20 (2009) 172–180.
[3] GY. Zou, J. Taleban, C. Huo, Confidence interval estima tion for lognormal data with application to health economics, Computational Statistics and Data Analysis, 53 (2009) 3755–3764.
[4] GY. Zou, A. Donner, A simple alternative confidence interval for the difference between two proportions, Controlled Clinical Trials 25 (2004) 3–12.
[5] GY. Zou, W. Huang, X. Zhang, A note on confidence interval estimation for a linear function of binomial proportions, Computational Statistics and Data Analysis, 53 (2009) 1080–1085.
[6] K. Krishnamoorthy, T. Mathew, Inferences on the means of lognormal distributions using generalized p-values and generalized confidence intervals, Journal of Statistical Planning and Inference, 115 (2003)103–120.
[7] K. Krishnamoorthya, Y. Lina, Y. Xia, Confidence limits and prediction limits for a Weibull distribution based on the generalized variable approach, Journal of Statistical Planning and Inference, 139 (2009) 2675 – 2684
[8] L. Tian, Interval estimation and hypothesis testing of intraclass correlation coeffcients: the generalized variable approach, Statistical in medicine, 24 (2005) 1745–1753.
[9] RD. Herbert, A. Hayen, P. Macaskill, SD. Walter, Interval estimation for the difference of two independent variances, Communications in StatisticsSimulation and Computation, 40 (2011) 744–758.
[10] S. Weerahandi, Generalized confidence intervals, The Journal of American Statistical Association, 88 (1993) 899-905.
[11] The R Development Core Team, An Introduction to R, Vienna: R Foundation for Statistical Computing, http://www.R-project.org, 2010.
[12] V. Cojbasica, A. Tomovica, Nonparametric confidence intervals for population variance of one sample and the diffeence of variances of two samples, Computational Statistics & Data Analysis, 51 (2007) 5562-5578.