

# Optimal Power Allocation for the Proposed Asymmetric Turbo Code for 3G Systems

K. Ramasamy, B. Balamuralithara, and Mohammad Umar Siddiqi

**Abstract**—We proposed a new class of asymmetric turbo encoder for 3G systems that performs well in both “water fall” and “error floor” regions in [7]. In this paper, a modified (optimal) power allocation scheme for the different bits of new class of asymmetric turbo encoder has been investigated to enhance the performance. The simulation results and performance bound for proposed asymmetric turbo code with modified Unequal Power Allocation (UPA) scheme for the frame length,  $N=400$ , code rate,  $r=1/3$  with Log-MAP decoder over Additive White Gaussian Noise (AWGN) channel are obtained and compared with the system with typical UPA and without UPA. The performance tests are extended over AWGN channel for different frame size to verify the possibility of implementation of the modified UPA scheme for the proposed asymmetric turbo code. From the performance results, it is observed that the proposed asymmetric turbo code with modified UPA performs better than the system without UPA and with typical UPA and it provides a coding gain of 0.4 to 0.52dB.

**Keywords**—Asymmetric turbo code, Generator polynomial, Interleaver, UPA, WCDMA, cdma2000.

## I. INTRODUCTION

TURBO codes have attracted attention since introduced in 1993 [1]. Since turbo codes are a parallel concatenation of two or more convolutional codes separated by pseudo-random interleaver, the characteristic of both constituent encoder as well as the interleaver is important in order to achieve good performance. The parallel concatenated version of turbo codes introduced by Berrou et al. assume identical component codes, hence known as symmetric turbo codes, have either a good “water-fall” bit error rate (BER) performance or a good “error-floor” BER performance, but not both [1]. Several new classes of asymmetric turbo codes are introduced which improve performance compared to the original turbo code over the entire range of signal-to-noise ratios (SNR). The simulation results of a new class of asymmetric turbo code for 3G systems, which consists of parallel concatenated convolutional codes with 8-state component codes (fixed constraint length), [13/11;13/9] have been presented in [7].

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The interleaver used is matched with the distance spectrum of the component code [7,8].

In symmetric turbo code system, both systematic and parity bits are allocated with equal power levels. This way of typical power allocation does not guarantee an optimal power allocation, which produce less optimum performance. Therefore, in order to overcome this problem, an UPA scheme was introduced by Mohammadi and Khandani in [2]. This method is used to improve the code performance by allocating different power level to systematic bits (d) and both parity bits (C1 and C2). Different strategies of UPA have been employed to investigate the effectiveness of UPA scheme on the turbo code performance. In paper [3], the authors look into the improvement of performance value at very low SNR (within 0.5dB of the Shannon limit). They concluded that by allocating more energy to the systematic bits, better performance can be achieved. In [4], the authors investigated optimizing the power allocation at higher SNR. They concluded that at higher SNR, the fraction of total energy allocated to systematic bits is usually lower than the parity bits for frame lengths of 1000 bits. In [5], the authors studied the enhancement of turbo code performance for short frame size (48 and 192 bits) at low and high SNR. They concluded that at low SNR, more power should be allocated for systematic bits, while, allocating less energy to the systematic bits at high SNR, which improves the performance. All the strategies discussed in [3,4,5] are for symmetrical turbo code system.

In this paper, we investigate the performance of proposed asymmetric turbo code that studied in [7] using UPA scheme. We consider the performance of proposed asymmetric turbo code for both low and high SNR with different frame size. The paper is organized as follows: In section 2, we present the proposed asymmetric turbo code. In section 3, we present a modified UPA scheme that suits with the proposed asymmetric turbo code system. A simulation study is conducted to choose the optimum power distribution for the systematic and both the parity bits. In section 4, we describe the performance bound for the proposed asymmetric turbo code using modified UPA scheme. In section 5, we present simulation results of proposed asymmetric turbo code system in 3G standards using modified UPA scheme. We conclude this paper in Section 6.

## II. PROPOSED ASYMMETRIC TURBO CODE

In typical turbo code system, a turbo encoder consists of two identical Recursive Systematic Convolutional (RSC) encoders with a pseudorandom interleaver preceding the

second constituent encoder. The turbo decoder also consists of two identical component decoders. The performance of a turbo code may be affected by different parameters of the component codes, block size, interleaver design and weight spectrum. This typical system results into few low weight code words. However, we obtain more favorable distance spectrum by using a slightly different RSC encoder and a code-matched interleaver as shown in Fig. 1; the corresponding decoding scheme is shown in Fig. 2. In Fig. 1 and Fig. 2, “I” and “DI” denote “Interleaver” and “Deinterleaver”, respectively.

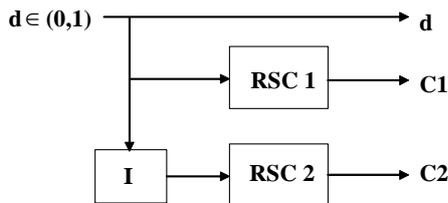


Fig. 1 Proposed asymmetric turbo encoder

It is shown in [7,8] that the generator polynomial [13/11;13/9] gives the best BER performance. The selection of generator polynomial is based on both better simulation results and improved weight spectrum as discussed in [7,8].

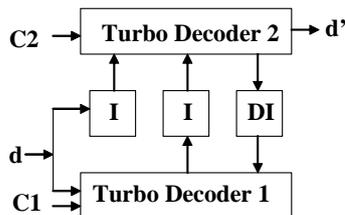


Fig. 2 Proposed asymmetric turbo decoder

The analysis of the distance spectrum of proposed asymmetric turbo code for its improved performance is presented separately in [8]. The design criteria of a code-matched interleaver used in proposed asymmetric code is provided in [7]. We eliminate low-weight code words with significant contributions to the error performance. The elimination of a specific code word can be done by breaking up the input pattern that generates that code word. The input information sequences with weights 2, 3 and 4 are considered in the interleaver design [7]. In [7], we noticed that the proposed asymmetric turbo code performs better than typical turbo code and the coding gain is from 0.5 to 0.8dB for different channel conditions.

### III. MODIFIED UPA SCHEME FOR THE PROPOSED ASYMMETRIC TURBO CODE

In this paper, we attempted to enhance the performance of the proposed asymmetric turbo code by optimizing the energy allocated to each bit stream. In standard turbo code system [1] with equal power distribution (code rate, R=1/3), the fractions

of power allocated to all transmitted bits are same. Let  $\alpha$ ,  $\beta$  and  $\chi$  represent the power allocated to systematic bits (d) and both parity bits (C1) and (C2), respectively. Then

$$\alpha = \beta = \chi = 1/3. \tag{1}$$

The optimal UPA scheme for systematic turbo code is to allocate a fraction of  $(1-\alpha)/2$  for each of the parity bits, because of the overall symmetry introduced by the random interleaver [9].

$$\beta = \chi = (1 - \alpha) / 2 \tag{2}$$

In order to implement UPA scheme for the proposed asymmetric turbo code system, we need to modify the existing UPA power allocation because the parity bits in this system are not symmetric. Therefore, the power distribution for modified UPA scheme to suit the proposed asymmetric turbo code system is given in Equation (3).

$$\alpha \neq \beta \neq \chi \tag{3}$$

To choose the optimal power distribution, simulations were carried out for frame length N=400 and code rate = 1/3. AWGN channel is assumed with Log-MAP decoder with maximum number of iterations as 6. The power fractions for the systematic bits, d ( $\alpha$ ), parity bits, C1 ( $\beta$ ) and parity bits, C2 ( $\chi$ ) were varied and based on exhaustive search method, the optimum power distribution which results better performance has been achieved. Fig. 6 shows the simulation results of proposed asymmetric turbo code with typical UPA scheme, modified UPA scheme and without any UPA scheme. The optimum power fractions for all transmitted bits in the proposed asymmetric turbo code with modified UPA scheme is presented in Table I.

TABLE I  
OPTIMUM POWER ALLOCATION FOR THE PROPOSED ASYMMETRIC TURBO CODE SYSTEM OVER AWGN CHANNEL, N = 400 BITS, R = 1/3

$E_b/N_o$ (dB)	$\alpha$	$\beta$	$\chi$
0	0.90	0.09	0.01
0.5	0.70	0.20	0.10
1.0	0.60	0.25	0.15
1.5	0.50	0.30	0.20
2.0	0.40	0.35	0.25

From the simulation results shown in Fig. 3, we noticed that the proposed asymmetric turbo code with modified UPA scheme performs better compared to using typical UPA and without UPA schemes, and the coding gain at BER of  $10^{-5}$  is 0.4 dB.

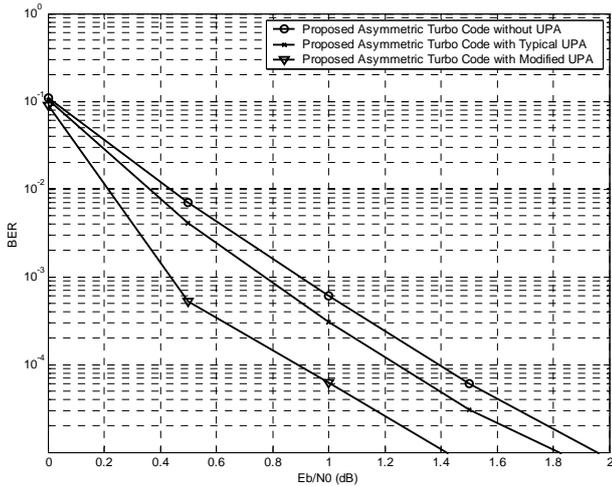


Fig. 3 Simulation results for proposed asymmetric turbo code with and without UPA over AWGN channel,  $N=400$ ,  $r=1/3$

#### IV. PERFORMANCE BOUND FOR THE PROPOSED ASYMMETRICAL TURBO CODE USING MODIFIED UPA SCHEME

The union bound for turbo code system assuming BPSK modulation and AWGN channel is given by [10]

$$P_{bit} \leq \sum_j \sum_w \frac{w}{N} A_w^{TC}(Z) P_2(j) \quad (4)$$

where  $N$  is the frame length and  $A_w^{TC}$  represents the number of code words produced in turbo code with weight ( $j$ ) generated by a word of information with weight,  $w$ ,  $d_1$  and  $d_2$  corresponding to systematic and both the parity bits, respectively. Therefore, the total weight,  $j = w + d_1 + d_2$ .  $P_2(j)$  is the pair-wise error probability between the all-zero codeword and the codeword with weight,  $j$  and is given by :

$$P_2(j) = Q \left[ \sqrt{2jr \frac{Eb}{No}} \right] \quad (5)$$

where  $r$  is the code rate of the code and  $Q[*]$  is the tail integral of Gaussian density with zero mean and unit variance. We define a uniform interleaver as a statistical device which maps a given input sequence of length,  $N$  and weight  $w$  into all distinct  $N_{cw}$  permutation of it with equal probability  $1/N_{cw}$ . Making use of the properties of a uniform interleaver, the average Conditional Weight Enumerate Function (CWEF) of all possible turbo codes with respect to the whole class of interleavers can be evaluated as given in equation (6) [6]:

$$A_w^{TC}(Z) = \frac{A_w^{C_1}(Z) \cdot A_w^{C_2}(Z)}{N_{cw}} \quad (6)$$

where  $N_{cw} = \binom{N}{w} = \frac{N!}{(N-w)!w!}$ ,  $A^{C_1}$  and  $A^{C_2}$  are the

weight enumerating functions for RSC1 and RSC2 encoders respectively. Equation (6) represents an average turbo code with given constituent codes and block size,  $N$  over all possible interleavers. Here code words produced by both

encoders are independent of each other, because  $A^{C_1}$  and  $A^{C_2}$  are assumed as individual components [10].

In typical turbo codes, the squared Euclidean distance between all-zero-codeword and the codeword with weight  $j$  is  $rjE_b$ . Implementing UPA scheme in turbo code system, it can be viewed as changing the SNR of the transmitted bits while keeping the average SNR constant. In order to implement UPA scheme, we need to modify the Euclidean distance. In each frame, we have systematic bits,  $w$ , parity bits  $d_1$  and  $d_2$  with power fractions  $\alpha$ ,  $\beta$  and  $\chi$ , respectively. Therefore, we can rewrite the modified squared Euclidean distance as  $[\alpha w + \beta d_1 + \chi d_2] E_b$ . Thus by substituting the modified squared Euclidean distance in Equations (5) and (4), we get the modified performance bound as below:

$$P_{bit} \leq \sum_j \sum_w \frac{w}{N} A_w^{TC}(Z) Q \left[ \sqrt{\frac{2(\alpha w + \beta d_1 + \chi d_2) E_b}{No}} \right] \quad (7)$$

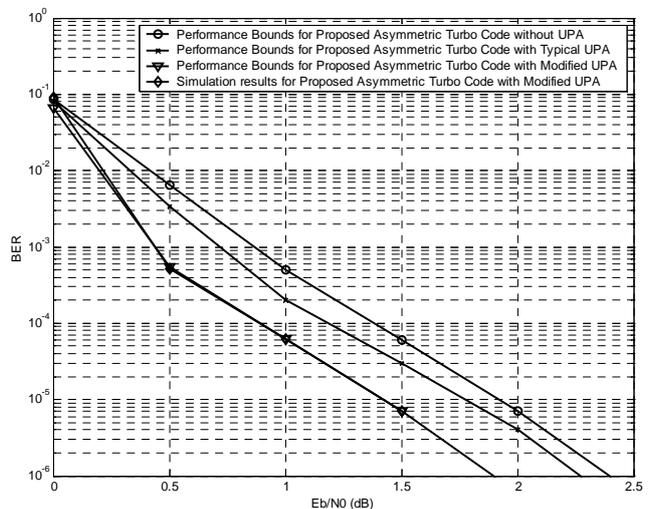


Fig. 4 Performance bound for proposed asymmetric turbo code with and without UPA over AWGN channel,  $N=400$ ,  $r=1/3$

Fig. 4 shows the performance bounds for the proposed asymmetric turbo code system without UPA, with typical UPA and with modified UPA schemes. The power distribution for the modified UPA scheme is provided in Table III. We notice that the proposed asymmetric turbo code with modified UPA scheme performs better than the system without UPA scheme and typical UPA scheme and the coding gain ranges from 0.4-0.5 dB at BER of  $10^{-6}$ .

#### V. 3G PERFORMANCE OF PROPOSED ASYMMETRICAL TURBO CODE SYSTEM USING MODIFIED UPA SCHEME

In this section, to verify the possibility of practical implementation of the proposed asymmetric turbo code system using modified UPA scheme, we simulate the performance of typical and proposed asymmetric turbo code system using 3G Wireless Communication standards. There are two main families of 3G standards: WCDMA (or UMTS) and cdma2000. WCDMA standard (3GPP TS 25.212/3G)

allows a range of frame size from 40 to 5114bits with code rates of 1/3 and 1/2. For typical turbo code system, WCDMA turbo encoder uses 8 states identical RSC encoders with an internal interleaver [9]. Simulations were carried out for the information block length,  $N=5114$ , code rate,  $r=1/3$  using log-MAP decoder with maximum number of decoding iterations as 8 over AWGN channel. The power distribution for the modified UPA scheme is shown in Table II and the simulation result is presented in Fig. 5.

TABLE II  
OPTIMUM POWER ALLOCATION FOR THE PROPOSED ASYMMETRIC TURBO CODE SYSTEM OVER AWGN CHANNEL,  $N = 5114$  BITS,  $R = 1/3$

$E_b/N_o$ (dB)	$\alpha$	$\beta$	$\chi$
0	0.8	0.15	0.05
0.5	0.55	0.35	0.1
1.0	0.35	0.45	0.2
1.5	0.15	0.55	0.3

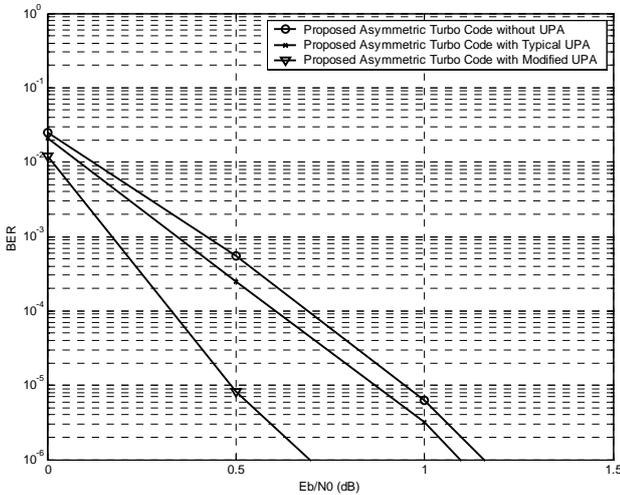


Fig. 5 Simulation results for proposed asymmetric turbo code with and without UPA over AWGN channel,  $N=5114$ ,  $r=1/3$

From Fig. 5, we observe that the proposed asymmetric turbo code with modified UPA scheme results better performance and the coding gain ranges from 0.49 – 0.52 dB at BER of  $10^{-6}$ .

Cdma2000 turbo encoder consists of a pair of rate 1/3 systematic convolutional encoders with constraint length,  $K=4$  separated by an interleaver (3GPP TS 125.212). Typical cdma2000 system uses RSC encoders with generator polynomial (13/11). The frame size must be one of the following specific values: 378, 570, 762, 1146, 1530, 2398, 3066, 4602, 6138, 9210, 12282 or 20730 bits with code rates 1/2, 1/3 and 1/4. The simulations were carried out for the information block length,  $N=6138$ , code rate,  $r=1/3$  using Log-MAP decoder over AWGN channel with 9 iterations and the results are shown in Fig. 6. We notice that the proposed asymmetric turbo code with modified UPA scheme performs better and the coding gain at BER of  $10^{-6}$  is 0.48 to 0.52dB. The power fractions for modified UPA scheme are shown in Table III.

TABLE III  
OPTIMUM POWER ALLOCATION FOR THE PROPOSED ASYMMETRIC TURBO CODE SYSTEM OVER AWGN CHANNEL,  $N = 6138$  BITS,  $R = 1/3$

$E_b/N_o$ (dB)	$\alpha$	$\beta$	$\chi$
0	0.7	0.2	0.1
0.5	0.35	0.4	0.25
1.0	0.15	0.55	0.3

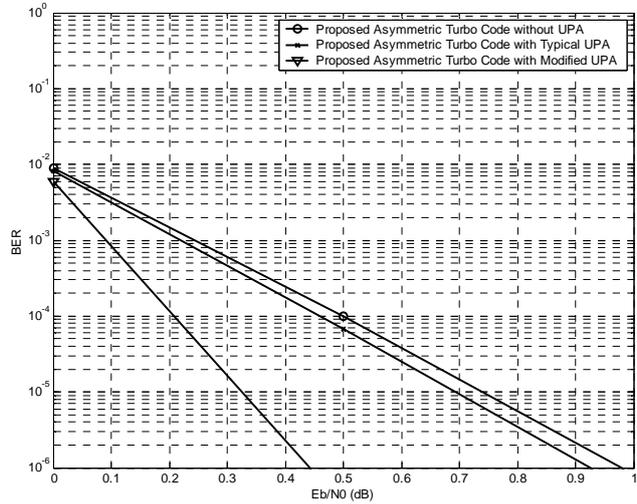


Fig. 6 Simulation results for proposed asymmetric turbo code with and without UPA over AWGN channel,  $N=6138$ ,  $r=1/3$

From Tables I, II, and III, we observe that for all frame lengths and for every value of  $E_b/N_o$ ,  $\beta$  is greater than  $\chi$ . As  $E_b/N_o$  increases,  $\alpha$  decreases and  $(\beta+\chi)$  increases. We also notice that for each frame length, for lower values of  $E_b/N_o$ ,  $\alpha > (\beta+\chi)$  and beyond a particular value of  $E_b/N_o$ ,  $\alpha < (\beta+\chi)$ .

VI. CONCLUSION

In this paper, we have proposed an optimal power allocation scheme for the new class of asymmetric turbo code system proposed in [7]. Although the search procedure of optimal power allocation for the systematic and both the parity bits for different frame size at low and high SNR is quite exhaustive, the modified UPA scheme in turbo encoder really contribute performance improvements in the proposed asymmetric turbo code system. The simulation results and the performance bound indicate that the performance of proposed asymmetric turbo code with modified UPA scheme is superior to the performance of turbo code without UPA and with typical UPA and the coding gain ranges from 0.4 - 0.52dB at BER of  $10^{-6}$  for different frame size. The future work involves investigating the performance in fading channel.

REFERENCES

[1] Berrrou, C., Glavieux, A., Thitimajshima, O., "Near Shannon Limit Error-Correcting Coding and Decoding: Turbo Codes", Proceedings of International Conference on Communication, ICC'93, Geneva, Switzerland, (1993), 1064-1070.  
 [2] Mohammadi, A.H.S. and Khandani, A.K., "Effect of the labeling of the constellation points in a combined Turbo Code and Modulation" in

- Proc.1996 Conf. of Information Science and Systems (CISS'96), (Mar.1996), pp 67-69.
- [3] Hokfelt, J and Maseng, T. , "Optimizing the energy of Different bitstream of Turbo Codes", Turbo Coding Seminar Proc., Lund, Sweden, (Aug.1996), pp.59-63.
- [4] Duman, T.M. and Salehi, M., "On Optimal Power Allocation for Turbo Codes", in Proc. of IEEE International Symposium on Information Theory, Germany, pp.104 (1997).
- [5] Salah, M.M, Raines, R.A., Temple, M.A., and Bailey, T.G., "Energy Allocation Strategies for Turbo Codes with Short Frames", Proc. of Information Tech: Coding and Computing (2000), pp. 408-411.
- [6] Benedetto, Montorsi, G., "Unveiling turbo codes: Some results on parallel concatenated coding schemes", IEEE Transactions on Information Theory, 42, No. 2, March, (1996), 409-428.
- [7] Ramasamy, K., Balamuralithara, B., and Siddiqi, M.U., "A new class of asymmetric turbo code for 3G systems", International journal of Electronics and Communications, Elsevier publications, vol: 60/6 pp. 447-458.
- [8] Ramasamy, K., and Siddiqi, M.U., "Weight distribution analysis of proposed asymmetric turbo code for improved performance", International journal of Electronics and Communications, Elsevier publications, vol: 60/7 pp. 479-493.
- [9] Mohammadi, A.H.S. and Khandani, A.K., "Unequal Power Allocation to the Turbo Encoder Output Bits with application to CDMA system", IEEE. Trans. on Communication, Vol.47.No.11.(1999), pp.1609-1610.
- [10] Divsalar, D., Dolinar, S., Pollara, F., McEliece, R. J., "Transfer function bounds on the performance of turbo codes", TDA progress Report 42-122, Jet Propulsion Lab., Pasadena, CA, August, 15, (1995), pp. 44-55.