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Dynamic optimization of industrial servomechanisms using motion laws based on Bezier curves

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Abstract—The motion planning procedure described in this paper has been developed in order to eliminate or reduce the residual vibrations of electromechanical positioning systems, without augmenting the motion time (usually imposed by production requirements), nor introducing overtime for vibration damping. The proposed technique is based on a suitable choice of the motion law assigned to the servomotor that drives the mechanism. The reference profile is defined by a Bezier curve, whose shape can be easily changed by modifying some numerical parameters. By means of an optimization technique these parameters can be modified without altering the continuity conditions imposed on the displacement and on its time derivatives at the initial and final time instants.

Keywords—Servomechanism, Residual vibrations, Motion optimization.

I. Introduction

THE compliance of the mechanical transmissions can generate vibratory effects inside many electro-mechanical systems. A particularly important problem is the one regarding the so-called *overshooting effect*, that is a residual vibration that appears at the end of a very fast motion cycle; in order to eliminate this type of oscillatory phenomenon it is needed to introduce supplementary stop intervals, so that the mechanical energy can be dissipated by the natural damping of the system. To avoid vibration, it is also possible to increase the motion time, with consequent reduction of the velocity and the acceleration of the moving parts of the machine. Nevertheless this action slows down the production process and, for this reason, it is not advantageous from the economical point of view.

The motion planning strategy here proposed has been studied to reduce the residual vibrations of an electro-mechanical device. To implement this technique the following steps are necessary:

- definition of a mathematical model that allows to simulate the actual behaviour of the servomechanism with good accuracy;
- definition of a parametric motion profile, whose shape (i.e. the displacement, velocity and acceleration profiles) can be easily modified by changing a set of numerical parameters;
- definition of a performance index, that allows a simple and practical evaluation of the mechanical energy of the system at the end of the motion interval;
- use of a numerical algorithm which is able to solve an optimization problem: in this way the performance index will be minimized by changing the parameters of the motion profile. When the solution corresponding to a local minimum is reached, the procedure is stopped and the optimal parameters are saved in the computer memory.

In order to demonstrate the effectiveness of this approach, this paper presents a practical example, where the previously described technique is employed to reduce the residual vibrations of a rotary platform driven by a servomotor through a non-rigid transmission.

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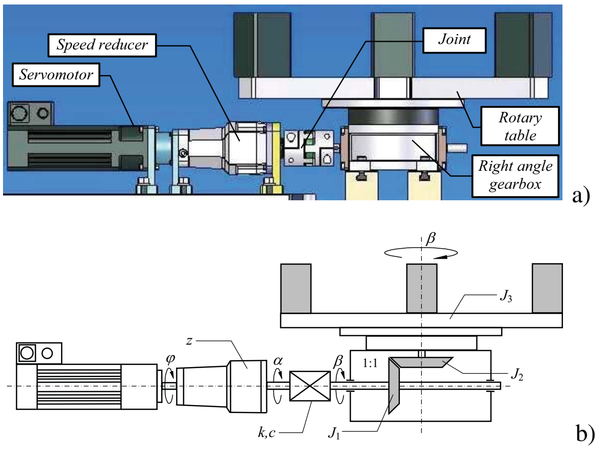


Fig. 1. a) Rotary platform driven by a servomotor through a speed reducer and a right angle gearbox. b) Schematic representation of the mechanism.

II. Mathematical modeling

Let us consider the electro-mechanical device represented in Fig. 1a, which consists of a rotary platform driven by a servomotor through a speed reducer and a right angle gearbox with 1:1 gear ratio. Fig. 1b shows a schematic representation of the mechanism. The system parameters and their corresponding symbols are listed in Table I. The output shaft of the speed reducer and the input shaft of the right angle gearbox are connected through a joint, which can be modeled by a torsional spring and a torsional viscous damper in parallel. If we suppose that the servomotor is able to correctly execute the motion profile assigned by the electronic control unit (this hypothesis is usually satisfied with good approximation, if a position and/or a velocity feed-back loop is implemented inside the motion controller), the rotation of the motor shaft $\varphi(t)$ and its time derivatives $\dot{\varphi}(t)$ and $\ddot{\varphi}(t)$ are known; through the gear ratio z it is immediate to calculate the angular displacement α at the output of the speed reducer. The rotation β of the platform differs from α owing to the elasticity of the joint.

In order to analyze the mechanical vibrations of the system, it is necessary to determine the motion equation of the rotary platform and to solve it for a particular motion law of the motor. This can be achieved without difficulties through a Lagrangian approach. Using as variable the angular rotation β of the platform, the Lagrange equation for the system under consideration is:

$$\frac{d}{dt}\left(\frac{\partial E_k}{\partial \dot{\beta}}\right) - \frac{\partial E_k}{\partial \beta} + \frac{\partial D}{\partial \dot{\beta}} + \frac{\partial E_p}{\partial \beta} = 0 \quad (1)$$

where the symbols E_k and E_p indicate respectively the kinetic and the potential energy of the system, whereas the symbol D indicates the Rayleigh dissipation function, which considers the damping effects. Since the input and output shaft of the right-angle gearbox have the same angular velocity $\dot{\beta}$, the total kinetic energy is given by:

$$E_k = \frac{1}{2}\sum_{i=1}^{3} J_i \dot{\beta}^2 \quad (2)$$

The potential energy and the Rayleigh function assume the following expressions:

$$E_p = \frac{1}{2}k(\beta - z\varphi)^2 \qquad D = \frac{1}{2}c(\dot{\beta} - z\dot{\varphi})^2 \quad (3)$$

The substitution of Eqs. (2) and (3) into Eq. (1) gives the following result:

$$J_{eq}\ddot{\beta} + c(\dot{\beta} - z\dot{\varphi}) + k(\beta - z\varphi) = 0 \quad (4)$$

where $J_{eq} = J_1 + J_2 + J_3$. The rotation $\beta(t)$ of the platform can be calculated by solving Eq. (4), if the motion law $\varphi(t)$ of the servomotor is known.

At this point it is convenient to introduce the variable $\psi = \beta - z\varphi$, which represents the difference between the actual position β of the platform and its theoretical position $\beta^* = z\varphi$, corresponding to a perfectly rigid behaviour of the joint. Using this new variable we obtain from Eq. (4):

$$J_{eq}(\ddot{\psi} + z\ddot{\varphi}) + c\dot{\psi} + k\psi = 0 \quad (5)$$

Introducing now the natural angular frequency of the system $\omega_n = \sqrt{k/J_{eq}}$ and the non-dimensional damping ratio $\xi = c/2J_{eq}\omega_n$, Eq. (5) can be rearranged as follows:

$$\ddot{\psi} + 2\xi\omega_n\dot{\psi} + \omega_n^2\psi = -z\ddot{\varphi} \quad (6)$$

Knowing the analytical expression of the motor angular acceleration $\ddot{\varphi}$ and starting from null initial conditions (that is $\psi(0) = 0$, $\dot{\psi}(0) = 0$), the solution of the differential equation (6) can be calculated through the convolution integral [1]. If the system is underdamped ($\xi < 1$) we have:

$$\psi(t) = -\frac{z}{\omega_d}\int_0^t f(t,\tau)\,d\tau \quad (7)$$

where $\omega_d = \omega_n\sqrt{1-\xi^2}$ is the damped natural frequency of the system and $f(t,\tau)$ is defined as:

$$f(t,\tau) = \ddot{\varphi}(\tau)e^{-\xi\omega_n(t-\tau)}\sin[\omega_d(t-\tau)] \quad (8)$$

The angular position of the rotary platform can be now easily calculated through the following relationship:

$$\beta(t) = \psi(t) + z\varphi(t) \quad (9)$$

The residual vibration of the platform may be eliminated or reduced in amplitude through a proper choice of the acceleration profile of the motor $\ddot{\varphi}(t)$, without changing the motion time and the rotation to be done, that is maintaining the same average value of angular velocity. The motion law of the motor is generated through a Bezier curve and it is successively optimized, in order to obtain the desired result.

Section III provides some mathematical details to define a motion law by means of a Bezier curve, whereas Section IV illustrates a criterion to eliminate the residual vibration.

TABLE I
PARAMETERS AND VARIABLES OF THE SYSTEM IN FIG. 1

Symb.	Description
J_1, J_2, J_3	Mass moments of inertia
k	Torsional stiffness of the joint
c	Damping constant of the joint
φ	Motor shaft rotation
α	Rotation of the output shaft of the reducer
β	Rotation of the platform
$z = \alpha/\varphi$	Gear ratio of the speed reducer

III. MOTION LAWS BASED ON BEZIER CURVES

The Bezier curves have been widely used in many fields of engineering; their denomination derives from the surname of their creator, the French mathematician and engineer Pierre Bezier, who implemented them in his software UNISURF, a CAD system purposely developed to design the body components of many Renault cars.

Nowadays such curves are used for computer graphics applications [2] [3] and to design the laws of motion of cam mechanism or servo-controlled devices [4] [5] [6].

An interesting feature of the Bezier curves is that their shape can be easily modified by changing the values of some numerical coefficients, which define the so called *control polygon*; this feature will be here exploited to define the reference motion profile $\varphi(t)$ of the servomotor that drives the mechanical system. The definition of a Bezier curve through the use of the Bernstein polynomials is given below.

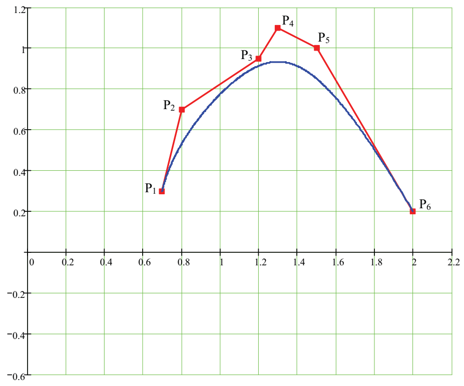
Let $P_0, P_1, \ldots P_n$ be $(n+1)$ points of the plane, defined by their Cartesian coordinates (x_i, y_i). The Bezier curve $\Gamma(\lambda)$ corresponding to these points is defined by the following parametric equations:

$$\Gamma(\lambda) = \begin{Bmatrix} x(\lambda) \\ y(\lambda) \end{Bmatrix} = \sum_{i=0}^{n} \begin{Bmatrix} x_i \\ y_i \end{Bmatrix} B(n,i,\lambda) \quad \lambda \in [0,1] \quad (10)$$

where

$$B(n,i,\lambda) = \binom{n}{i}\lambda^i(1-\lambda)^{n-1} \quad (11)$$

is the i^{th} Bernstein polynomial and λ is the parameter. The points P_i ($i = 0,\ldots,n$) are called *knots* and they are used to build up the control polygon of the curve; as an example, Fig. 2a shows a Bezier curve and its control polygon consisting of six points with non-uniform spacing on the abscissa.



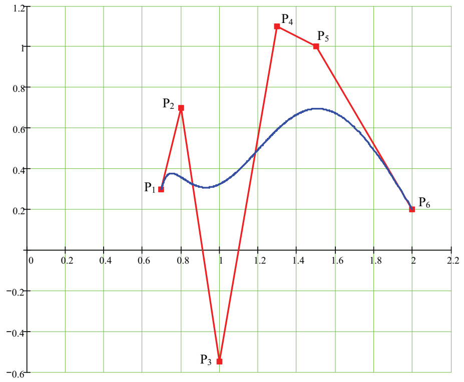


Fig. 2. a) An example of Bezier curve with control polygon consisting of six points. b) Modified curve, obtained by changing the position of point P_3.

A Bezier curve satisfies the following properties:

- the first point P_0 and the last point P_n of the control polygon belong to the curve, whereas the intermediate points act as attractors and generally they do not belong to the curve;
- the first and the last side of the polygon are tangent to the curve, at the initial and final point respectively;
- it can be shown that the curvature at point P_0 depends on the first three points of the control polygon and the curvature at point P_n depends on the last three points of the polygon;
- the curve can be locally deformed by changing the position of a point of the control polygon; the shape modification is evident only in the region near the modified point (see Fig. 2b).

If the operations of derivation and integration are carried out on a Bezier curve (with respect to the coordinate x), the derivative and integral curves are generally not Bezier curves. However it is still possible to obtain a Bezier curve if the points P_i are equally spaced along the abscissa; in this case the mathematical relationship between the abscissa x and the parameter λ is:

$$x(\lambda) = x_0 + \lambda(x_n - x_0) \tag{12}$$

where x_0 and x_n are respectively the abscissas of the points P_0 and P_n.

We report here some useful properties, which can be used to calculate the derivative and the integral curve.

1) The x-derivative of a Bezier curve defined by a control polygon with $(n+1)$ equally spaced knots is again a Bezier curve and its polygon has n equally spaced knots; the ordinates y_i^* of these knots are given by:

$$y_i^* = \frac{n}{x_n - x_0}(y_{i+1} - y_i) \quad i = 0, \ldots, n-1 \tag{13}$$

2) The integral (with respect to the x variable) of a Bezier curve defined by a control polygon with $(n+1)$ equally spaced knots is again a Bezier curve and its polygon has $(n+2)$ equally spaced knots; the ordinates $\widetilde{y}_i$ of these knots are given by:

$$\widetilde{y}_i = \frac{x_n - x_0}{n+1} y_i + \widetilde{y}_{i-1} + \widetilde{y}_0 \quad i = 1, \ldots, n+1 \tag{14}$$

3) If $\mathcal{F}(x)$ is a Bezier curve defined by a control polygon having $(n+1)$ knots, its definite integral on the interval $[x_0 \; x_n]$ can be calculated through the following relationship:

$$\int_{x_0}^{x_n} \mathcal{F}(x)dx = \frac{x_n - x_0}{n+1} \sum_{i=0}^{n} y_i \tag{15}$$

We now apply this procedure to define the displacement profile $\varphi(t)$ for the motor that drives the system. This function is defined in the time interval $[0, T]$, where T is the motion time: therefore we have $t_0 = 0$ and $t_n = T$.

To determine the number of points of the control polygon, it is necessary to know: 1) the maximum order of the derivative on which we must guarantee the continuity; 2) the number of interior points of the control polygon, whose ordinates can be modified in order to perform the optimization.

Indeed it can be shown [7] that if we use a control polygon (for the displacement function) similar to the one depicted in Fig. 3, the resulting Bezier function satisfies the following properties:

$$\begin{array}{ll} \varphi(0) = 0 & \varphi(T) = \Phi \\ \dot{\varphi}(0) = 0 & \dot{\varphi}(T) = 0 \\ \vdots & \vdots \\ \varphi^{(m)}(0) = 0 & \varphi^{(m)}(T) = 0 \end{array} \tag{16}$$

where Φ is the maximum rotation of the motor (final value). The particularity of this polygon is to have its first $(m+1)$ points with null ordinate and its final $(m+1)$ points with ordinate equal to Φ. Inserting additional p internal points, we obtain a polygon with $2(m+1)+p$ points; through an automatic procedure, the ordinates of these internal points can be modified in order to optimize the motion profile, following the criteria described in Section IV. As an example, Fig. 4 shows the displacement, velocity and acceleration profiles obtained by imposing the continuity condition on the acceleration $(m = 2)$ and inserting three internal points $(p = 3)$, whose ordinates are fixed by the user. The final result is a control polygon (for the displacement curve) consisting of 9 equally spaced points (Fig. 4a); the corresponding polygons for the velocity and acceleration curves have respectively 8 and 7 equally spaced points (see Figs. 4b and c).

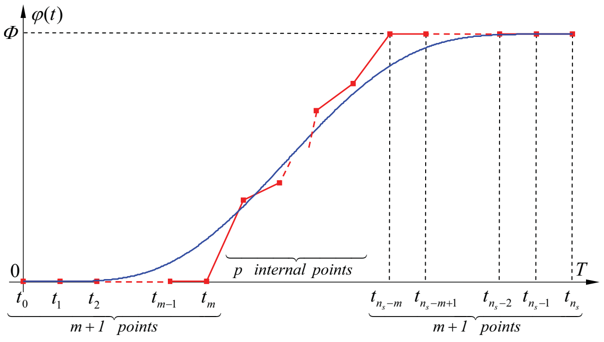


Fig. 3. Control polygon for the displacement function $\varphi(t)$.

Let us denote with the symbol S_i the ordinates of the control polygon for the displacement function $\varphi(t)$ (motor rotation); the corresponding Bezier curve, obtained from Eq. (10), is:

$$\varphi(t) = \sum_{i=0}^{n_s} S_i B(n_s, i, \frac{t}{T}) \tag{17}$$

where $n_s = 2m + p + 1$ and B is the Bernstein polynomial, defined as function of the normalized time $\lambda = t/T$ (see Eq. (11)). Through a double application of Eq. (13) we can calculate the ordinates V_i of the control polygon for the velocity function $\dot{\varphi}(t)$ and the ordinates A_i of the control polygon for the acceleration function $\ddot{\varphi}(t)$: defining for simplicity $n_v = n_s - 1$ and $n_a = n_v - 1$, we obtain:

$$V_i = \frac{n_s}{T}(S_{i+1} - S_i) \quad i = 0, \ldots, n_v \tag{18}$$

$$A_i = \frac{n_v}{T}(V_{i+1} - V_i) \quad i = 0, \ldots, n_a \tag{19}$$

The corresponding Bezier curves are:

$$\dot{\varphi}(t) = \sum_{i=0}^{n_v} V_i B(n_v, i, \frac{t}{T}) \tag{20}$$

$$\ddot{\varphi}(t) = \sum_{i=0}^{n_a} A_i B(n_a, i, \frac{t}{T}) \tag{21}$$

IV. Elimination of the Overshooting Effect

As mentioned above, the overshooting effect is a free vibration that appears at the end of the motion interval, that is for $t > T$; in this situation the shaft of the servomotor is kept locked on the final position, whereas the rotary platform oscillates due to the elasticity of the transmission joint. The vibration is possible because some energy is still present in the mechanical device at the final time instant $t = T$; therefore, to eliminate the vibration of the platform, it is necessary to set at zero the mechanical energy of the system, corresponding to the final time instant. If this is not possible, we can impose that the value of this energy is reduced to a minimum; in this case the residual oscillations will be in any case strongly reduced, even if they will not be completely eliminated. To achieve these results an accurate motion planning of the mechanical system is necessary. Using the Bezier curves described in Section III, this can be implemented without difficulties, because the

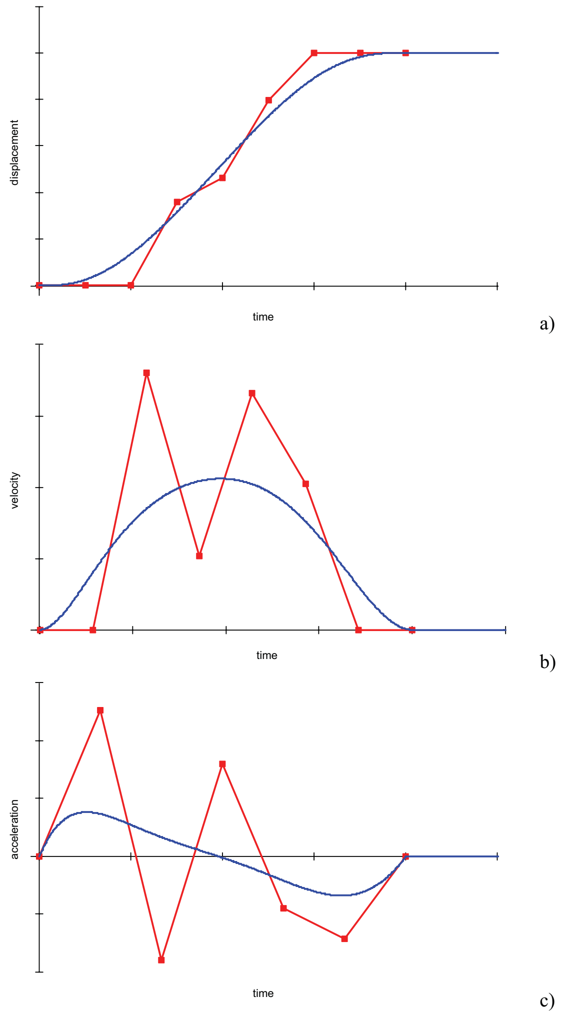


Fig. 4. Displacement, velocity and acceleration profiles obtained for $m = 2$ and $p = 3$.

reference profiles can be quickly modified by changing the ordinates of the intermediate points of the control polygon. Since this procedure can not be manually carried out (for example through a *trial and error* approach), we must use an automatic procedure, that can determine the optimal motion profile.

Starting from these preliminary remarks, the problem under consideration can be studied as an optimization problem, where we must seek the conditions for which the mechanical energy of the system at the time instant $t = T$ is minimum. Therefore this energy plays the role of a target function, whose value can be set to zero (or minimized), simply acting on the intermediate points of the control polygon. In particular, we can consider the ordinates of the intermediate points as variables of the optimization process.

For the 1-DOF system described by Eq. (4) the total mechanical energy E_{tot}, at the generic time instant t, can be easily calculated by adding the kinetic energy of the system to the potential energy due to the deformation of the elastic joint; using the expressions of E_k and E_p given in Section II, we have:

$$E_{tot} = E_k + E_p = \frac{1}{2}[J_{eq}\dot{\beta}^2 + k(\beta - z\varphi)^2] \tag{22}$$

If we introduce the variable ψ, Eq. (22) can be rewritten as:

$$E_{tot} = \frac{1}{2}J_{eq}[(\dot{\psi} + z\dot{\varphi})^2 + \omega_n^2\psi^2] \tag{23}$$

where $\omega_n^2 = k/J_{eq}$. At the final time instant $t = T$ the angular velocity of the motor is null ($\dot{\varphi}(T) = 0$), as stated by Eq. (16) (2^{nd} row, right column); therefore we obtain from Eq. (23):

$$E_{tot}(T) = \frac{1}{2}J_{eq}[\dot{\psi}^2(T) + \omega_n^2\psi^2(T)] \tag{24}$$

The terms $\psi(T)$ and $\dot{\psi}(T)$ that appear at the right-hand side of Eq. (24) can be calculated through Eq. (7); in particular, the velocity term (for a generic time instant t) is given by the following relationship:

$$\dot{\psi}(t) = -\frac{z}{\omega_d}\frac{d}{dt}\int_0^t f(t,\tau)\,d\tau \tag{25}$$

which requires differentiation under the integral sign[1]. Since $f(t,\tau)|_{\tau=t} = 0$, the formula indicated in the footnote gives the following result:

$$\dot{\psi}(t) = -\frac{z}{\omega_d}\int_0^t g(t,\tau)\,d\tau \tag{26}$$

where $g(t,\tau) = \frac{\partial}{\partial t}f(t,\tau)$. From Eq. (8) we obtain the analytical expression of the g function:

$$g(t,\tau) = \omega_n\ddot{\varphi}(\tau)e^{-\xi\omega_n(t-\tau)}\cos[\omega_d(t-\tau) + \delta] \tag{27}$$

where $\tan\delta = \xi/\sqrt{1-\xi^2}$. If now we introduce the following definitions:

$$F = \int_0^T f(T,\tau)\,d\tau \qquad G = \int_0^T g(T,\tau)\,d\tau \tag{28}$$

Eq. (24) assume the form:

$$E_{tot}(T) = \frac{1}{2}J_{eq}\left(\frac{z}{\omega_d}\right)^2[G^2 + \omega_n^2F^2] = \eta Q \tag{29}$$

where $\eta = \frac{1}{2}J_{eq}\left(\frac{z}{\omega_d}\right)^2$ and $Q = G^2 + \omega_n^2F^2$.

For the mechanical system under consideration, the coefficient η is a constant, whereas Q is a function depending on the motor acceleration $\ddot{\varphi}$, through the functions $f(T,\tau)$ and $g(T,\tau)$ and therefore it depends on the values assigned to the ordinates of the intermediate points of the control polygon.

[1]We report here the general formula for differentiation under the integral sign:

$$\frac{d}{dt}\int_{a(t)}^{b(t)} f(t,\tau)d\tau = \\ = \int_{a(t)}^{b(t)} \frac{\partial}{\partial t}f(t,\tau)d\tau + f(t,b(t))b'(t) - f(t,a(t))a'(t)$$

If $a(t) = 0$ e $b(t) = t$ the above-mentioned formula can be simplified as follows:

$$\frac{d}{dt}\int_0^t f(t,\tau)d\tau = \int_0^t \frac{\partial}{\partial t}f(t,\tau)d\tau + f(t,\tau)|_{\tau=t}$$

It follows that Q is a function of p variables, where p is the number of the intermediate points, as described in Section III. Hence, with mathematical formalism, we can write:

$$\frac{E_{tot}(T)}{\eta} = Q(\boldsymbol{\gamma}) \tag{30}$$

where $\boldsymbol{\gamma} = \{\gamma_1, \gamma_2, \ldots, \gamma_p\}^T$ is a p-dimensional vector, containing the numerical values of the ordinates of the intermediate points.

The values γ_i ($i = 1, \ldots, p$) can be automatically selected by a numerical procedure, (see [8] for details) in order to optimize the target function Q. Using this approach, it is also possible to introduce of some algebraic constraints, to reduce the computational time and to drive the optimization process towards a satisfactory solution.

V. Motion optimization: a numerical example

This paragraph presents a numerical example which illustrates the results of the motion optimization procedure based on the Bezier curves. The calculation have been performed for the mechanical device represented in Fig. 1, whose parameters are listed in Table II. Using these data, the resulting natural frequency and damping factor are $\omega_n = 53.3\,\mathrm{rad/s}$ and $\xi = 11.7\%$ respectively.

The angular displacement of the motor was set to 20 revolutions, corresponding to a single revolution of the platform; the total motion time was set to $T = 1\,\mathrm{s}$. The motion command for the servomotor was generated by means of Eqs. (17), (20) and (21).

In Figs. 5 and 6 we can see the results obtained by computer simulation before and after the optimization process. The motion commands (position, velocity and acceleration) used to drive the rotary platform are shown on the left columns together with their control polygons; the right columns show the motion of the platform, which is evidently influenced by the elastic behaviour of the transmission joint.

The comparison between the acceleration diagrams in Fig. 5f and 6f shows that the residual vibration disappears, when an optimized motion profile is used to drive the system.

VI. Conclusions

A method for reducing the overshooting effect of electromechanical systems has been presented in the paper. The calculation procedure employs a mathematical model of the system, a class of parametric functions and an optimization algorithm, that minimizes the total mechanical energy of the system at the final time instant. Through computer simulation, the technique has been successfully tested on a 1-DOF vibrating system,

TABLE II
Numerical values of the system parameters

Symb.	Val.	Unit	Symb.	Val.	Unit
J_1	5×10^{-3}	$\mathrm{kg\,m^2}$	k	8000	Nm/rad
J_2	5×10^{-3}	$\mathrm{kg\,m^2}$	c	35	Nms/rad
J_3	2.8	$\mathrm{kg\,m^2}$	z	1/20	-

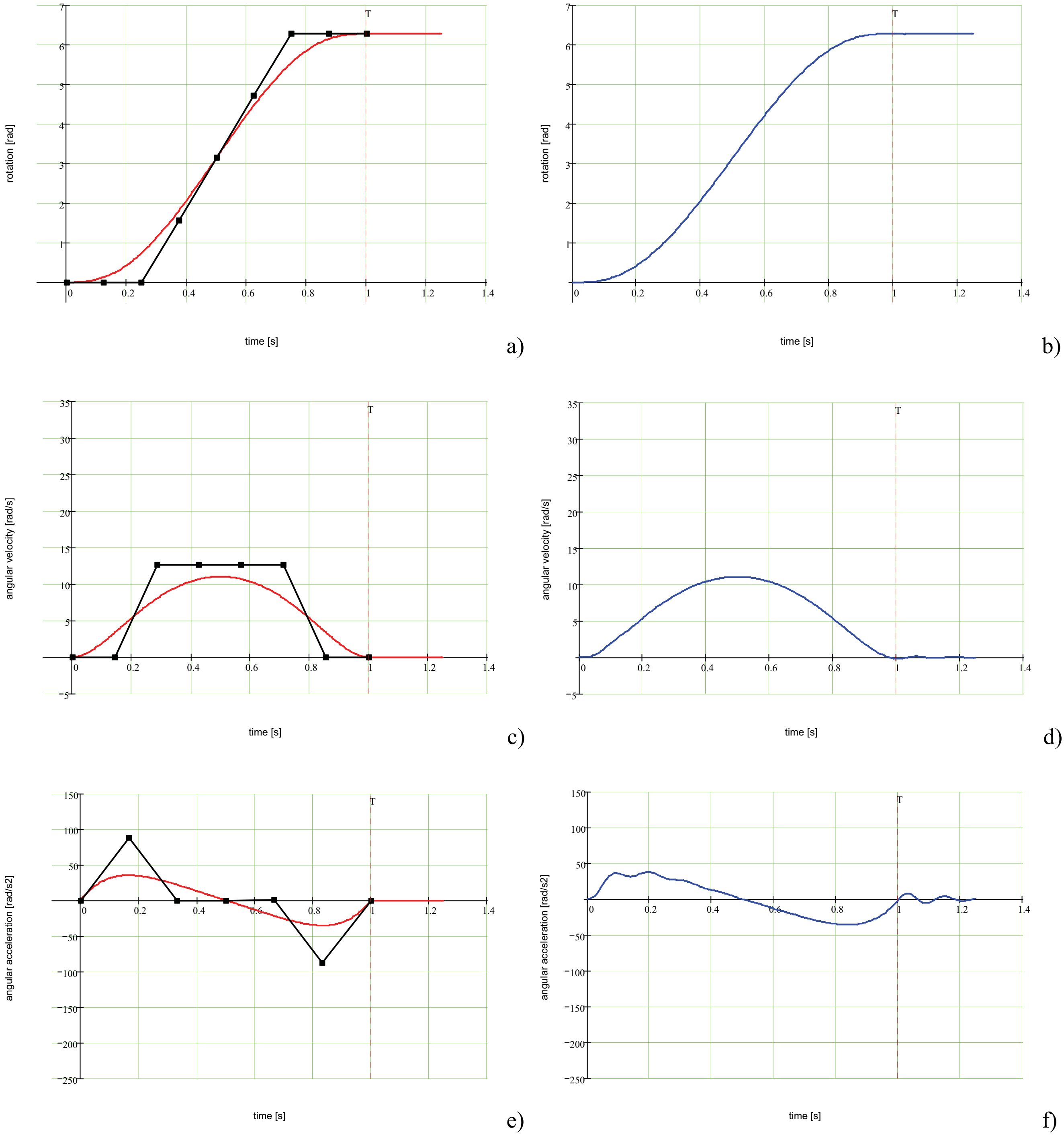


Fig. 5. Motion simulation for a non-optimized motion command: **a)** position command $\alpha(t)$; **b)** rotation of the platform $\beta(t)$; **c)** velocity command $\dot{\alpha}(t)$; **d)** angular velocity of the platform $\dot{\beta}(t)$; **e)** acceleration command $\ddot{\alpha}(t)$; **f)** angular acceleration of the platform $\ddot{\beta}(t)$.

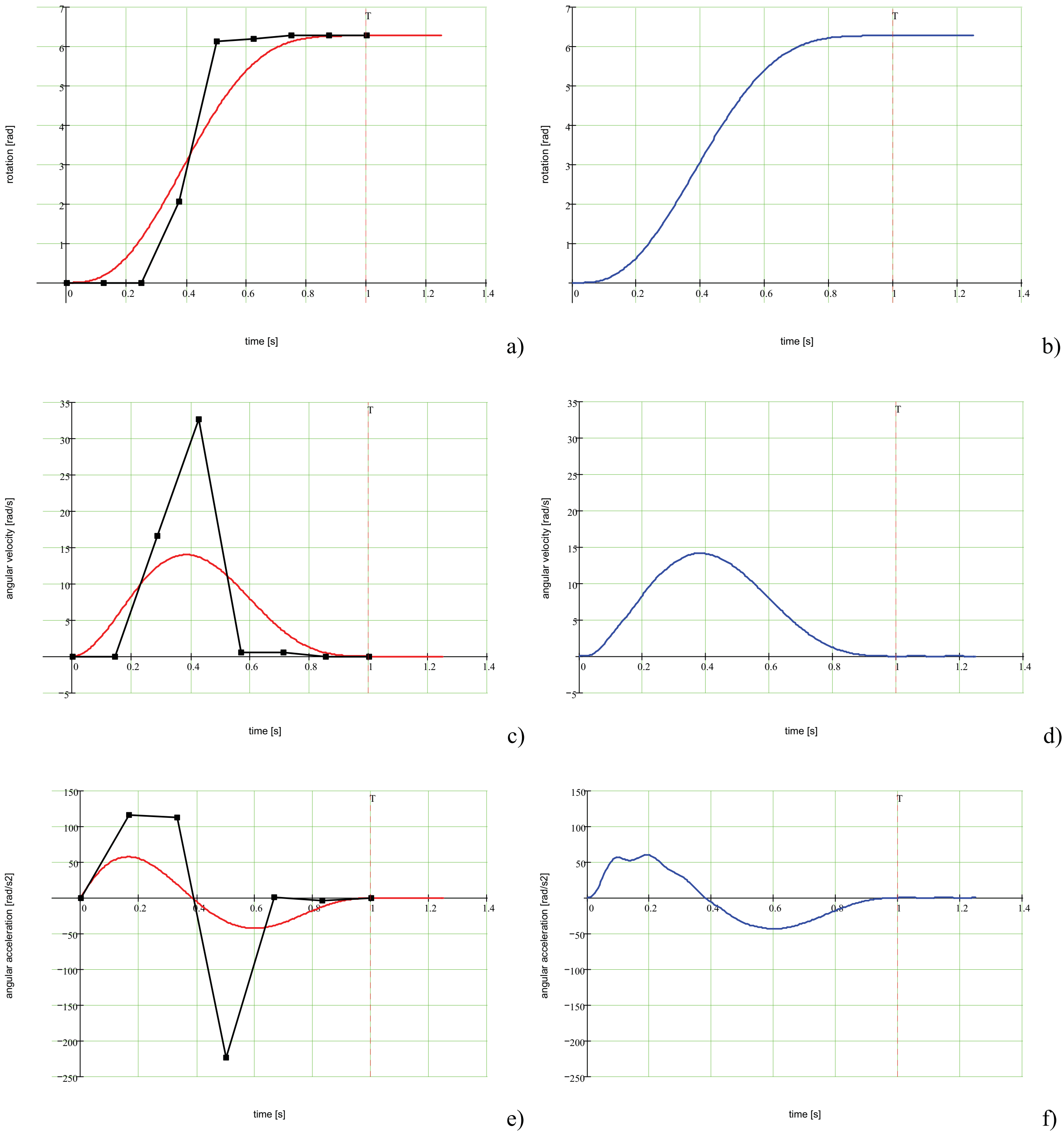


Fig. 6. Motion simulation for an optimized motion command: **a)** position command $\alpha(t)$; **b)** rotation of the platform $\beta(t)$; **c)** velocity command $\dot{\alpha}(t)$; **d)** angular velocity of the platform $\dot{\beta}(t)$; **e)** acceleration command $\ddot{\alpha}(t)$; **f)** angular acceleration of the platform $\ddot{\beta}(t)$.

consisting of a rotary platform driven by an electric servomotor coupled with an elastic transmission. The numerical results indicate that the proposed approach seems to be able to reduce the residual vibrations, without changing the motion time, nor altering the boundary conditions of the motion command.

The proposed method does not require complex control algorithms for the servomotor or additional feedback sensors to measure the vibration amplitude and, for these reasons, it can be implemented on an actual machine with very low costs: in fact it is just sufficient a modification of the reference motion profile memorized in the motion controller.

Since a mathematical model of the actual device is employed for motion optimization, it is necessary an accurate identification of the mechanical parameters, in particular as regards the equivalent damping coefficient. For this reason, in the future the technique will be implemented on an experimental test-bed, in order to validate the theoretical results here presented.

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