International Journal of Business, Human and Social Sciences
ISSN: 2517-9411
Vol:3, No:7, 2009

A Method under Uncertain Information for the Selection of Students in Interdisciplinary Studies

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***Abstract*—**We present a method for the selection of students in interdisciplinary studies based on the hybrid averaging operator. We assume that the available information given in the problem is uncertain so it is necessary to use interval numbers. Therefore, we suggest a new type of hybrid aggregation called uncertain induced generalized hybrid averaging (UIGHA) operator. It is an aggregation operator that considers the weighted average (WA) and the ordered weighted averaging (OWA) operator in the same formulation. Therefore, we are able to consider the degree of optimism of the decision maker and grades of importance in the same approach. By using interval numbers, we are able to represent the information considering the best and worst possible results so the decision maker gets a more complete view of the decision problem. We develop an illustrative example of the proposed scheme in the selection of students in interdisciplinary studies. We see that with the use of the UIGHA operator we get a more complete representation of the selection problem. Then, the decision maker is able to consider a wide range of alternatives depending on his interests. We also show other potential applications that could be used by using the UIGHA operator in educational problems about selection of different types of resources such as students, professors, etc.

***Keywords*—**Decision making, Selection of students, Uncertainty, Aggregation operators.

I. Introduction

A very typical problem in educational management is the selection of students for interdisciplinary studies. In these situations, the students are required to have knowledge on different fields in order to be accepted in the program. The selection process is complex because it is not easy to evaluate the students when their backgrounds may be different. Therefore, it is necessary to establish a decision making method for the selection process. In the literature, there are a wide range of methods for decision making such as [1-9]. Some of them carry out an aggregation process of the information in order to obtain a result that permits to take a decision.

Today, there exist a wide range of aggregation operators for aggregating the information such as the weighted average (WA) and the ordered weighted averaging (OWA) operator. The WA is very useful when we want to consider the degree of importance of the characteristics and sometimes it can be seen as the subjective probability of the problem. The OWA operator [10] is very useful for situations where we want to consider the degree of optimism (or attitudinal character) of the decision maker and it provides a parameterized family of operators between the minimum and the maximum.

Recently, Xu and Da [11] have suggested a new aggregation operator that uses the WA and he OWA at the same time. They called it the hybrid averaging (HA) operator. It is very useful for situations where we want to consider the degree of importance of the characteristics and the attitudinal character of the decision maker at the same time. Both the OWA and the HA operators can be extended for more complex situations where the attitudinal character of the decision maker includes other factors apart from the degree of optimism such as the interaction between different persons that condition the decision, etc. For doing so, we will use induced aggregation operators that where initially introduced by Yager and Filev [12] with the induced OWA (IOWA) operator and later applied for the HA operator with the induced HA (IHA) operator [4].

Sometimes, the available information is not clear and can not be assessed with exact numbers. Thus, it is necessary to use another approach such as the use of interval numbers. The main advantage of using interval numbers is that we can at least consider the best and worst result that may happen in the problem. Moreover, it is also possible to consider the most possible result between the minimum and the maximum. In these cases, it is also possible to extend the OWA and the HA operator obtaining the uncertain OWA (UOWA) [13] and the uncertain HA (UHA) operators [5]. Furthermore, it is also

Manuscript received April 05, 2009.

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possible to extend the IOWA and the IHA operators obtaining the UIOWA and the UIHA operators.

Another interesting problem in the analysis of the aggregation process is to establish a general formulation that includes a wide range of cases. The best way for doing so is by using generalized and quasi-arithmetic means. Thus, if we extend the OWA and the HA for this formulation, we get the generalized OWA (GOWA) [14-15] and the generalized HA (GHA) [2] operators and other similar extensions. Note that by using quasi-arithmetic means we get the Quasi-OWA and the Quasi-HA operator. For further reading on the OWA and its extensions, see [16-30].

In this paper, we present a new generalization of the previous types of aggregation operators. We call it the uncertain induced generalized hybrid averaging (UIGHA) operator. It is an aggregation operator that uses generalized means, order inducing variables and uncertain information represented in the form of interval numbers. Moreover, it includes the WA and the OWA as special cases of this formulation. The main advantage of the UIGHA is that it provides a more complete formulation that includes a wide range of aggregation operators including all the previous ones commented in this section. We study some of its main properties and different particular cases very useful in the aggregation process. We also present a further generalization by using quasi-arithmetic means (Quasi-UIHA operator).

We develop the decision making approach about the selection of students in interdisciplinary studies by using the UIGHA operator. Thus, we get a very general formulation that includes a wide range of particular cases. Therefore, we get a complete view of the selection process where the results may be different depending on the interests of the decision maker. Moreover, by using interval numbers we are able to assess the problem considering that the results are not clearly known. In this case this is useful because the information given usually is a general result coming from a group of previous results that are not equal. Therefore, by using the interval numbers, we may include the most possible result but also the minimum and the maximum.

The rest of the paper is organized as follows. In Section 2, we briefly describe the interval numbers and some basic aggregation operators. Section 3 and 4 presents the new aggregation operators. In Section 5 we develop the new decision making approach and in Section 6 we summarize the main conclusions of the paper.

II. Preliminaries

A. Interval numbers

The interval numbers [31] are a very useful and simple technique for representing the uncertainty. It has been used in an astonishingly wide range of applications.

The interval numbers can be expressed in different forms. For example, if we assume a 4-tuple (a_1, a_2, a_3, a_4), that is to say, a quadruplet, we could consider that a_1 and a_4 represents the minimum and the maximum of the interval number, and a_2 and a_3, the interval with the highest probability or possibility, depending on the use we want to give to the interval numbers. Note that $a_1 \leq a_2 \leq a_3 \leq a_4$. If $a_1 = a_2 = a_3 = a_4$, then, the interval number is an exact number; if $a_2 = a_3$, it is a 3-tuple known as triplet; and if $a_1 = a_2$ and $a_3 = a_4$, it is a simple 2-tuple interval number.

In the following, we are going to review some basic interval number operations as follows. Let A and B be two triplets, where $A = (a_1, a_2, a_3)$ and $B = (b_1, b_2, b_3)$. Then:

1. $A + B = (a_1 + b_1, a_2 + b_2, a_3 + b_3)$
2. $A - B = (a_1 - b_3, a_2 - b_2, a_3 - b_1)$
3. $A \times k = (k \times a_1, k \times a_2, k \times a_3)$; for $k > 0$.
4. $A \times B = (a_1 \times b_1, a_2 \times b_2, a_3 \times b_3)$; for R^+.
5. $A \div B = (a_1 \div b_3, a_2 \div b_2, a_3 \div b_1)$; for R^+.

Note that R^+ refers to all the positive real numbers. Note also that other operations could be studied [31] but in this paper we will focus on these ones.

B. The OWA operator

The OWA operator was introduced by Yager in [10] and it provides a parameterized family of aggregation operators that include the arithmetic mean, the maximum and the minimum. It can be defined as follows.

Definition 1. An OWA operator of dimension n is a mapping $OWA: R^n \rightarrow R$ that has an associated weighting vector W of dimension n such that the sum of the weights is 1 and $w_j \in [0, 1]$, then:

$$OWA(a_1, a_2, \ldots, a_n) = \sum_{j=1}^{n} w_j b_j \quad (1)$$

where b_j is the jth largest of the a_i.

From a generalized perspective of the reordering step, we can distinguish between the descending OWA (DOWA) operator and the ascending OWA (AOWA) operator. The OWA operator is commutative, monotonic, bounded and idempotent. For further information on the OWA and its applications, see for example [1-30].

C. The IOWA operator

The IOWA operator was introduced by Yager and Filev [12] and it represents an extension of the OWA operator. Its main difference is that the reordering step is not developed with the values of the arguments a_i. In this case, the reordering step is developed with order inducing variables. The IOWA operator also includes as particular cases the maximum, the minimum and the average criteria. It can be defined as follows.

Definition 2. An IOWA operator of dimension n is a mapping $IOWA: R^n \rightarrow R$ that has an associated weighting vector W of dimension n such that the sum of the weights is 1 and $w_j \in [0, 1]$, then:

$$IOWA(\langle u_1, a_1\rangle, \langle u_2, a_2\rangle \ldots, \langle u_n, a_n\rangle) = \sum_{j=1}^{n} w_j b_j \quad (2)$$

where b_j is the a_i value of the IOWA pair $\langle u_i,a_i\rangle$ having the jth largest u_i, u_i is the order inducing variable and a_i is the argument variable.

Note that it is possible to distinguish between the Descending IOWA (DIOWA) operator and the Ascending IOWA (AIOWA) operator. The IOWA operator is also monotonic, bounded, idempotent and commutative. For further reading on the IOWA, refer, e.g., to [2-4,6,11,28].

D. The UOWA operator

The UOWA operator was introduced by Xu and Da in [13] and it represents an extension of the OWA operator. Essentially, its main difference is that it uses interval numbers in the arguments to be aggregated. The reason for using this aggregation operator is that sometimes the environment is very uncertain and the information is not clear, then, it can only be assessed by using interval numbers. The UOWA operator provides a parameterized family of aggregation operators that include the uncertain maximum, the uncertain minimum and the UA, among others. It can be defined as follows.

Definition 3. Let Ω be the set of interval numbers. An UOWA operator of dimension n is a mapping $UOWA$: $\Omega^n \to \Omega$ that has an associated weighting vector W of dimension n with the following properties:

1) $\sum_{j=1}^{n} w_j = 1$
2) $w_j \in [0, 1]$

and such that:

$$UOWA(\tilde{a}_1, \tilde{a}_2 \ldots, \tilde{a}_n) = \sum_{j=1}^{n} w_j b_j \quad (3)$$

where b_j is the jth largest of the $\tilde{a}_i$, and the $\tilde{a}_i$ are interval numbers.

From a generalized perspective of the reordering step, we can distinguish between the descending UOWA (DUOWA) operator and the ascending UOWA (AUOWA) operator. The weights of these operators are related by $w_j = w^*_{n-j+1}$, where w_j is the jth weight of the DUOWA and w^*_{n-j+1} the jth weight of the AUOWA operator.

The UOWA operator is commutative, monotonic, bounded and idempotent. Different families of UOWA operators can be found by choosing a different manifestation in the weighting vector such as the median-UOWA, the olympic-UOWA or the centered-UOWA operator.

E. The IGOWA operator

The IGOWA operator was introduced in [6] and it represents a generalization of the IOWA operator by using generalized means. Then, it is possible to include in the same formulation, different types of induced operators such as the IOWA operator or the induced OWG (IOWG) operator. It can be defined as follows.

Definition 4. An IGOWA operator of dimension n is a mapping $IGOWA$: $R^n \to R$ that has an associated weighting vector W of dimension n such that $w_j \in [0, 1]$ and $\sum_{j=1}^{n} w_j = 1$, then:

$$IGOWA(\langle u_1,a_1\rangle,\ldots,\langle u_n,a_n\rangle) = \left(\sum_{j=1}^{n} w_j b_j^{\lambda} \right)^{1/\lambda} \quad (4)$$

where b_j is the a_i value of the IGOWA pair $\langle u_i,a_i\rangle$ having the jth largest u_i, u_i is the order inducing variable, a_i is the argument variable and λ is a parameter such that $\lambda \in (-\infty, \infty)$.

As we can see, if $\lambda = 1$, we get the IOWA operator. If $\lambda = 0$, the IOWG operator and if $\lambda = 2$, the IOWQA operator. Note that it is possible to further generalize the IGOWA operator by using quasi-arithmetic means. The result is the Quasi-IOWA operator.

F. The IHA operator

The induced HA (IHA) operator [4] is an extension of the HA operator that uses order inducing variables. The HA operator is an aggregation operator that uses the WA and the OWA in the same formulation. Then, in the IHA operator it is possible to consider in the same problem, a complex attitudinal character of the decision maker and its subjective probability.

Definition 5. An IHA operator of dimension n is a mapping IHA: $R^n \to R$ that has an associated weighting vector W of dimension n such that the sum of the weights is 1 and $w_j \in [0, 1]$, then:

$$IHA(\langle u_1,a_1\rangle,\ldots,\langle u_n,a_n\rangle) = \sum_{j=1}^{n} w_j b_j \quad (5)$$

where b_j is the $\hat{a}_i$ value ($\hat{a}_i = n\omega_i a_i$, $i = 1,2,\ldots,n$), of the IHA pair $\langle u_i,a_i\rangle$ having the jth largest u_i, u_i is the order inducing variable, $\omega = (\omega_1, \omega_2, \ldots, \omega_n)^T$ is the weighting vector of the a_i, *with* $\omega_i \in [0, 1]$ and the sum of the weights is 1.

From a generalized perspective of the reordering step, we can distinguish between the descending IHA (DIHA) operator and the ascending IHA (AIHA) operator. The weights of these operators are related by $w_j = w^*_{n-j+1}$, where w_j is the jth weight of the DIHA and w^*_{n-j+1} the jth weight of the AIHA operator. Different families of IHA operators are found by using a different manifestation in the weighting vector such as the step-IHA operator, the window-IHA operator, the median-IHA operator, the centered-IHA operator, etc.

G. The IGHA operator

The IGHA operator is a generalization of the IHA operator by using generalized means. It includes in the same formulation the weighted generalized mean and the IGOWA operator. It also uses order inducing variables in the

reordering process. Then, this operator includes the WA, the OWA, the IOWA and the IOWG operator as special cases. It is defined as follows.

Definition 6. An IGHA operator of dimension n is a mapping $IGHA$: $R^n \rightarrow R$ that has an associated weighting vector W of dimension n such that the sum of the weights is 1 and $w_j \in [0, 1]$, then:

$$IGHA(\langle u_1,a_1\rangle,\ldots,\langle u_n,a_n\rangle) = \left(\sum_{j=1}^{n} w_j b_j^{\lambda}\right)^{1/\lambda} \quad (6)$$

where b_j is the $\hat{a}_i$ value ($\hat{a}_i = n\omega_i a_i$, $i = 1,2,\ldots,n$), of the IHA pair $\langle u_i,a_i\rangle$ having the jth largest u_i, u_i is the order inducing variable, $\omega = (\omega_1, \omega_2, \ldots, \omega_n)^T$ is the weighting vector of the a_i, with $\omega_i \in [0, 1]$ and the sum of the weights is 1, and λ is a parameter such that $\lambda \in (-\infty, \infty)$.

From a generalized perspective of the reordering step, we can distinguish between the descending IGHA (DIGHA) operator and the ascending IGHA (AIGHA) operator. The IGHA operator is commutative, monotonic and idempotent.

III. The Uncertain Induced Generalized Hybrid Averaging Operator

The UIGHA operator is a generalization of the IGHA operator by using uncertain information given in the form of interval numbers. It includes in the same formulation the weighted generalized mean and the IGOWA operator. It also uses order inducing variables in the reordering process. Then, this operator includes the uncertain WA (UWA), the UOWA, the uncertain IOWA (UIOWA) and the uncertain IOWG (UIOWG) operator as special cases. It is defined as follows.

Definition 7. Let Ω be the set of interval numbers. An UIGHA operator of dimension n is a mapping $UIGHA$: $\Omega^n \rightarrow \Omega$ that has an associated weighting vector W of dimension n with the following properties:

1) $\sum_{j=1}^{n} w_j = 1$
2) $w_j \in [0, 1]$

and such that:

$$UIGHA(\tilde{a}_1, \tilde{a}_2\ldots, \tilde{a}_n) = \left(\sum_{j=1}^{n} w_j b_j^{\lambda}\right)^{1/\lambda} \quad (7)$$

where b_j is the $\hat{a}_i$ value ($\hat{a}_i = n\omega_i\tilde{a}_i$, $i = 1,2,\ldots,n$), of the UIGHA pair $\langle u_i,\tilde{a}_i\rangle$ having the jth largest u_i, u_i is the order inducing variable, the $\tilde{a}_i$ are interval numbers, $\omega = (\omega_1, \omega_2, \ldots, \omega_n)^T$ is the weighting vector of the $\tilde{a}_i$, with $\omega_i \in [0, 1]$ and the sum of the weights is 1, and λ is a parameter such that $\lambda \in (-\infty, \infty)$.

Note that the reordering of the arguments has an additional difficulty because now we are using interval numbers. Then, in some cases, it is not clear which interval number is higher, so we need to establish an additional criteria for reordering the interval numbers. For simplicity, we recommend the following criteria. For 2-tuples, calculate the arithmetic mean of the interval: $(a_1 + a_2) / 2$. For 3-tuples and more, calculate a weighted average that gives more importance to the central values. That is, $(a_1 + 2a_2 + a_3) / 4$. Then, for 4-tuples we could calculate: $(a_1 + 2a_2 + 2a_3 + a_4) / 6$. And so on. In the case of tie, we will select the interval with the lowest increment ($a_2 - a_1$). For 3-tuples and more we will select the interval with the highest central value. Note that for 4-tuples and more we need to calculate the average of the central values following the initial criteria.

Note also that in more complex analysis it would be possible to consider that the weights w_j, the weights ω_i and the parameter λ are interval numbers.

If B is a vector corresponding to the ordered arguments b_j^{λ}, we shall call this the ordered argument vector and W^T is the transpose of the weighting vector, then, the UIGHA operator can be expressed as:

$$UIGHA(\tilde{a}_1, \tilde{a}_2\ldots, \tilde{a}_n) = \left(W^T B\right)^{1/\lambda} \quad (8)$$

Note that if the weighting vector is not normalized, i.e., $W = \sum_{j=1}^{n} w_j \neq 1$, then, the UIGHA operator can be expressed as:

$$UIGHA(\tilde{a}_1, \tilde{a}_2\ldots, \tilde{a}_n) = \left(\frac{1}{W}\sum_{j=1}^{n} w_j b_j^{\lambda}\right)^{1/\lambda} \quad (9)$$

From a generalized perspective of the reordering step, it is possible to distinguish between the descending UIGHA (DUIGHA) operator and the ascending UIGHA (AUIGHA) operator. The weights of these operators are related by $w_j = w^*_{n-j+1}$, where w_j is the jth weight of the DUIGHA and w^*_{n-j+1} the jth weight of the AUIGHA operator. As we can see, the main difference is that in the AUIGHA operator, the elements b_j ($j = 1, 2, \ldots, n$) are ordered in an increasing way: $b_1 \leq b_2 \leq \ldots \leq b_n$ while in the DUIGHA (or UIGHA) they are ordered in a decreasing way.

The UIGHA operator is commutative, monotonic and idempotent. It is commutative because any permutation of the arguments has the same evaluation. That is, $UIGHA(\langle u_1,\tilde{a}_1\rangle, \langle u_2,\tilde{a}_2\rangle\ldots, \langle u_n,\tilde{a}_n\rangle) = UIGHA(\langle u_1,d_1\rangle, \langle u_2,d_2\rangle\ldots, \langle u_n,d_n\rangle)$, where $(d_1,\ldots, d_n)$ is any permutation of the arguments $(\tilde{a}_1,\ldots, \tilde{a}_n)$. It is monotonic because if $\tilde{a}_i \geq d_i$, for all $\tilde{a}_i$, then, $UIGHA(\langle u_1,\tilde{a}_1\rangle, \langle u_2,\tilde{a}_2\rangle\ldots, \langle u_n,\tilde{a}_n\rangle) \geq UIGHA(\langle u_1,d_1\rangle, \langle u_2,d_2\rangle\ldots, \langle u_n,d_n\rangle)$. It is idempotent because if $\tilde{a}_i = a$, for all $\tilde{a}_i$, then, $UIGHA(\langle u_1,\tilde{a}_1\rangle, \langle u_2,\tilde{a}_2\rangle\ldots, \langle u_n,\tilde{a}_n\rangle) = a$.

Another interesting issue when analysing the UIGHA operator is the problem of ties in the reordering step. In order to solve this problem, we recommend the policy developed by Yager and Filev [12] where they replace each argument of the tied IOWA pairs by their average. For the UIGHA operator, instead of using the arithmetic mean, we will replace each argument of the tied UIGHA pairs by its uncertain generalized

mean. Then, depending on the parameter λ, we will use a different type of mean to replace the tied arguments.

As it is explained in [12] for the IOWA operator, when studying the order inducing variable of the UIGHA operator, we should note that the values used can be drawn from a space such that the only requirement is to have a linear ordering. Then, it is possible to use different kinds of attributes for the order inducing variables that permit us, for example, to mix numbers with words in the aggregations. Note that in some situations it is possible to use the implicit lexicographic ordering associated with words such as the ordering of words in dictionaries.

Another interesting issue to analyze are the measures for characterizing the weighting vector W. Following a similar methodology as it has been developed for the OWA [2,10] and the GOWA operator [2,15], we can formulate the attitudinal character, the entropy of dispersion, the divergence of W and the balance operator.

The entropy of dispersion measures the amount of information being used in the aggregation.

$$H(W) = -\sum_{j=1}^{n} w_j \ln(w_j) \tag{10}$$

For example, if $w_j = 1$ for some j, known as step-UIGHA, then $H(W) = 0$, and the least amount of information is used.

The divergence of W measures the divergence of the weights against the attitudinal character measure. It is useful in some exceptional situations when the attitudinal character and the entropy of dispersion are not enough to correctly analyze the weighting vector of an aggregation.

$$Div(W) = \sum_{j=1}^{n} w_j \left(\frac{n-j}{n-1} - \alpha(W) \right)^2 \tag{11}$$

The balance operator measures the balance of the weights against the orness or the andness.

$$Bal(W) = \sum_{j=1}^{n} \left(\frac{n+1-2j}{n-1} \right) w_j \tag{12}$$

It can be shown that $Bal(W) \in [-1, 1]$. Note that for the optimistic criteria, $Bal(W) = 1$, and for the pessimistic criteria, $Bal(W) = -1$.

A further interesting issue to consider is the different families of UIGHA operator that are found by analysing the weighting vector W and the parameter λ. If we look to the parameter λ, we get the following particular cases.

- The UIHA operator if λ = 1 (arithmetic).
- The UIHGA operator if λ approaches to 0 (geometric).
- The UIHQA operator if λ = 2 (quadratic).
- The UIHHA operator if λ = –1 (harmonic).
- Etc.

And if we look to the weighting vector W, we get the following ones.

- The uncertain hybrid maximum ($w_1 = 1$ and $w_j = 0$, for all $j \neq 1$).
- The uncertain hybrid minimum ($w_n = 1$ and $w_j = 0$, for all $j \neq n$).
- The uncertain generalized mean ($w_j = 1/n$, and $\omega_j = 1/n$, for all $\tilde{a}_i$).
- The uncertain weighted generalized mean ($w_j = 1/n$, for all $\tilde{a}_i$).
- The UIGOWA operator ($\omega_j = 1/n$, for all $\tilde{a}_i$)
- The uncertain induced generalized hybrid Hurwicz criteria ($w_1 = \alpha$, $w_n = 1 - \alpha$ and $w_j = 0$, for all $j \neq 1, n$).
- The step-UIGHA ($w_k = 1$ and $w_j = 0$, for all $j \neq k$).
- The olympic-UIGHA operator ($w_1 = w_n = 0$, and $w_j = 1/(n-2)$ for all others).
- The general olympic-UIGHA operator ($w_j = 0$ for $j = 1, 2, \ldots, k, n, n-1, \ldots, n-k+1$; and for all others $w_{j^*} = 1/(n-2k)$, where $k < n/2$).
- The S-UIGHA ($w_1 = (1/n)(1 - (\alpha + \beta) + \alpha$, $w_n = (1/n)(1 - (\alpha + \beta) + \beta$, and $w_j = (1/n)(1 - (\alpha + \beta)$ for $j = 2$ to $n - 1$ where $\alpha, \beta \in [0, 1]$ and $\alpha + \beta \leq 1$).
- The centered-UIGHA (if it is symmetric, strongly decaying from the center to the maximum and the minimum, and inclusive).
- Etc.

Note that other families of UIGHA operator could be found following a similar methodology as it has been develop in a wide range of papers for the OWA operator and its extensions [2-6,9-10,15,24-30].

IV. The Quasi-UIHA Operator

A further generalization of the UIGHA operator is possible by using quasi-arithmetic means in a similar way as it was done for the IGOWA [6]. The result is the Quasi-UIHA operator which is a hybrid version of the Quasi-IOWA operator. It can be defined as follows.

Definition 8. Let Ω be the set of interval numbers. A Quasi-IHA operator of dimension n *is* a mapping *QIHA:* $\Omega^n \rightarrow \Omega$ that has an associated weighting vector W of dimension n such that the sum of the weights is 1 and $w_j \in [0, 1]$, then:

$$Quasi\text{-}UIHA(\langle u_1, a_1 \rangle, \ldots, \langle u_n, a_n \rangle) = g^{-1}\left(\sum_{j=1}^{n} w_j g\left(b_{(j)} \right) \right) \tag{13}$$

where b_j is the $\hat{a}_i$ value ($\hat{a}_i = n\omega_i \tilde{a}_i$, $i = 1,2,\ldots,n$), of the Quasi-UIHA pair $\langle u_i, \tilde{a}_i \rangle$ having the jth largest u_i, u_i is the order inducing variable, the $\tilde{a}_i$ are interval numbers, $\omega = (\omega_1, \omega_2, \ldots, \omega_n)^T$ is the weighting vector of the $\tilde{a}_i$, with $\omega_i \in [0, 1]$ and the sum of the weights is 1, and g is a strictly continuous monotonic function.

As we can see, we replace b^{λ} with a general continuous strictly monotone function $g(b)$. In this case, the weights of

the ascending and descending versions are also related by $w_j = w^*_{n-j+1}$, where w_j is the jth weight of the Quasi-DUIHA and w^*_{n-j+1} the jth weight of the Quasi-AUIHA operator.

Note that all the properties and particular cases commented in the UIGHA operator, are also included in this generalization. For example, we could study different families of Quasi-UIHA operators such as the Quasi-UIOWA, the Quasi-UWA, the Quasi-step-UIHA, the Quasi-median-UIHA, the Quasi-olympic-UIHA, the Quasi-centered-UIHA, etc.

V. Illustrative Example

In the following, we are going to develop an illustrative example of the new approach in a decision making problem. We will study a problem of selection of students in interdisciplinary studies. We are going to consider a master that combines business, economics and law in its program. Therefore, the requirements for entering the program include knowledge in business, economics and law. Note that other decision making applications could be developed in educational management or in other business problems such as the selection of financial products, the selection of human resources, in strategic management, etc.

Assume that a master program that has selected almost all his students wants to select the last two students for the program. They have five available students that can be selected.

- S_1 = Student A.
- S_2 = Student B.
- S_3 = Student C.
- S_4 = Student D.
- S_5 = Student E.

In order to evaluate these students the recruiters consider five main characteristics that are relevant for the selection process.

- C_1 = Knowledge of the student in business. This characteristic is a general evaluation of the business knowledge of the student according to the number of courses taken and their results.
- C_2 = Knowledge of the student in economics. This characteristic is a general evaluation of the economic knowledge of the student according to the number of courses taken and their results.
- C_3 = Knowledge of the student in law. This characteristic is a general evaluation of the law knowledge of the student according to the number of courses taken and their results.
- C_4 = Other knowledge of the student relevant to the program. It includes other courses or similar taken by the student that are relevant to the program.
- C_5 = Other variables. It includes the motivation of the student, age, citizenship, dedication to the program, scholarships, etc.

The recruiters evaluate the students given marks to each characteristic from 0 to 100, being 100 the best result. The results obtained depending on the characteristic C_i and the student S_k are shown in Table 1.

TABLE I
EVALUATION OF THE RESULTS

	C_1	C_2	C_3	C_4	C_5
S_1	(50,60,70)	(60,70,80)	(80,90,100)	(40,50,60)	(70,80,90)
S_2	(90,100,110)	(90,100,110)	(40,50,60)	(40,50,60)	(40,50,60)
S_3	(60,70,80)	(50,60,70)	(80,90,100)	(50,60,70)	(50,60,70)
S_4	(70,80,90)	(80,90,100)	(60,70,80)	(60,70,80)	(30,40,50)
S_5	(50,60,70)	(70,80,90)	(60,70,80)	(50,60,70)	(70,80,90)

In this problem, the recruiters assume the following weighting vectors: W = (0.1, 0.2, 0.2, 0.2, 0.3) and ω = (0.4, 0.2, 0.2, 0.1, 0.1). Due to the fact that the attitudinal character is very complex because it involves the opinion of different members of the board of directors, the recruiters use order inducing variables to express it. The results are represented in Table 2.

TABLE II
ORDER INDUCING VARIABLES

	C_1	C_2	C_3	C_4	C_5
S_1	20	25	16	22	14
S_2	16	14	12	18	19
S_3	23	21	19	17	25
S_4	20	16	12	22	24
S_5	12	13	15	16	19

With this information, we can aggregate the expected results for each student in order to take a decision. In Table 3, we present different results obtained by using different types of UIGHA operators. We consider the uncertain average, the uncertain weighted average (UWA), the UOWA, the UIOWA and the UIHA operator.

TABLE II
AGGREGATED RESULTS

	UA	UWA	UOWA	UIOWA	UIHA
S_1	(60,70,80)	(63,73,83)	(64,74,84)	(59,69,79)	(61,71,81)
S_2	(58,68,78)	(74,84,94)	(65,75,85)	(60,70,80)	(68,78,88)
S_3	(58,68,78)	(65,75,85)	(61,71,81)	(58,68,78)	(60,70,80)
S_4	(60,70,80)	(71,81,91)	(65,75,85)	(57,67,77)	(60,70,80)
S_5	(60,70,80)	(63,73,83)	(62,72,82)	(62,72,82)	(51,61,71)

If we establish an ordering of the alternatives, a typical situation if we want to consider more than one alternative, then, we get the following results shown in Table 4.

TABLE IV
ORDERING OF THE STUDENTS

UA	$S_1=S_4=S_5\succ S_2=S_3$	*UIOWA*	$S_5\succ S_2\succ S_1\succ S_3\succ S_4$
UWA	$S_2\succ S_4\succ S_3\succ S_1=S_5$	*UIHA*	$S_2\succ S_1\succ S_4\succ S_3\succ S_5$
UOWA	$S_2=S_4\succ S_1\succ S_5\succ S_3$		

As we can see, depending on the aggregator operator used, the ordering of the students may be different. Note that the main advantage of using the UIGHA operator is that we can consider a wide range of particular cases such as the UA, the UWA, the UOWA, the UIOWA and the UIHA operator. As each case may give different results, the decision maker will select for his decision the one that is closest to his interests but he will be able what can happen in other potential situations that may occur in the future.

VI. Conclusions

We have presented a new method for the selection of students in interdisciplinary studies when the available information is very uncertain and can be assessed with interval numbers. We have developed a new general aggregation operator that it is very useful in the selection process because it is able to consider a wide range of particular cases according to the interests of the decision maker. We have called it the UIGHA operator. It is an aggregation operator that unifies the OWA operator and the weighted average in the same formulation. Moreover, it also uses order inducing variables in order to assess complex attitudinal characters of the decision maker and generalized means that include a wide range of particular cases obtaining a more robust formulation. Furthermore, it also uses uncertain information represented with interval numbers in order to assess the uncertain environment with uncertain information that it is not clearly known. The main advantage of this approach is that we can show a wide range of scenarios to the decision maker according to its interests, especially to its degree of optimism. Moreover, we are able to consider mathematically, the degree of importance of the different characteristics considered in the selection process.

In future research, we expect to develop further extensions to this approach by considering other theoretical approaches and other potential applications such as in the selection of professors and projects, in the selection of new educational methodologies, etc.

References

[1] G. Beliakov, A. Pradera, and T. Calvo, *Aggregation Functions: A guide for practitioners*, Springer-Verlag, Berlin, 2007.

[2] J.M. Merigó, *New Extensions to the OWA Operators and its application in decision making*, PhD Thesis (in Spanish), Dept. Business Administration, Univ. Barcelona, Barcelona, Spain, 2008.

[3] J.M. Merigó, and M. Casanovas, "Induced aggregation operators in decision making with Dempster-Shafer belief structure", *Int. J. Intelligent Systems* (to be published).

[4] J.M. Merigó, M. Casanovas, "The induced generalized hybrid averaging operator and its application in financial decision making", *International Journal of Business, Economics, Finance and Management Sciences* (submitted for publication).

[5] J.M. Merigó, M. Casanovas, "Uncertain decision making with Dempster-Shafer theory", In *Proceedings of the IPMU International Conference*, Torremolinos – Málaga, Spain, 2008, pp. 425-432.

[6] J.M. Merigó, and A.M. Gil-Lafuente, "The induced generalized OWA operator", *Information Sciences*, vol. 179, pp. 729-741, 2009.

[7] Z.S. Xu, "A Note on Linguistic Hybrid Arithmetic Averaging Operator in Multiple Attribute Group Decision Making with Linguistic Information", *Group Decision and Negotiation*, vol. 15, pp. 593-604, 2006.

[8] Z.S. Xu, "An approach based on the uncertain LOWG and induced uncertain LOWG operators to group decision making with uncertain multiplicative linguistic preference relations", *Decision Support Systems*, vol. 41, pp. 488-499, 2006.

[9] R.R. Yager, and J. Kacprzyck, *The Ordered Weighted Averaging Operators: Theory and Applications*, Kluwer Academic Publishers, Norwell, MA, 1997.

[10] R.R. Yager, "On Ordered Weighted Averaging Aggregation Operators in Multi-Criteria Decision Making", *IEEE Trans. Systems, Man and Cybernetics*, vol. 18, pp. 183-190, 1988.

[11] Z.S. Xu, and Q.L. Da, "An Overview of Operators for Aggregating Information", *Int. J. Intelligent Systems*, vol. 18, pp. 953-969, 2003.

[12] R.R. Yager, and D.P. Filev, "Induced ordered weighted averaging operators", *IEEE Trans. Syst. Man Cybern.*, vol. 29, pp. 141-150, 1999.

[13] Z.S. Xu, and Q.L. Da, "The Uncertain OWA Operator", *Int. J. Intelligent Systems*, vol. 17, pp. 569-575, 2002.

[14] N. Karayiannis, "Soft Learning Vector Quantization and Clustering Algorithms Based on Ordered Weighted Aggregation Operators", *IEEE Trans. Neural Networks*, vol. 11, 1093-1105, 2000.

[15] R.R. Yager, "Generalized OWA Aggregation Operators", *Fuzzy Opt. Decision Making*, vol. 3, pp.93-107, 2004.

[16] G. Beliakov, "Learning Weights in the Generalized OWA Operators", *Fuzzy Opt. Decision Making*, vol. 4, pp. 119-130, 2005.

[17] T. Calvo, G. Mayor, and R. Mesiar, *Aggregation Operators: New Trends and applications*, Physica-Verlag, New York, 2002.

[18] J. Fodor, J.L. Marichal, and M. Roubens, "Characterization of the ordered weighted averaging operators", *IEEE Trans. Fuzzy Systems*, vol. 3, pp. 236-240, 1995.

[19] J.M. Merigó, and M. Casanovas, "The fuzzy generalized OWA operator", In *Proceedings of the Conference SIGEF 2007*, pp. 504-517, Poiana-Brasov, Romania, 2007.

[20] J.M. Merigó, M. Casanovas, "The uncertain generalized OWA operator and its application in the selection of financial strategies", In *Proceedings of the International Conference AEDEM 2007*, pp. 547-556, Krakow, Poland, 2007.

[21] J.H. Wang, and J. Hao, "A new version of 2-tuple fuzzy linguistic representation model for computing with words", *IEEE Trans. Fuzzy Systems*, vol. 14, pp. 435-445, 2006.

[22] P.A. Schaefer, and H.B. Mitchell, "A generalized OWA operator", *Int. J. Intelligent Systems*, vol. 14, pp. 123-143, 1999.

[23] Z.S. Xu, "A method based on linguistic aggregation operators for group decision making with linguistic preference relations", *Information Sciences*, vol. 166, pp. 19-30, 2004.

[24] R.R. Yager, "On generalized measures of realization in uncertain environments", *Theory and Decision*, vol. 33, pp. 41-69, 1992.

[25] R.R. Yager, "Families of OWA operators", *Fuzzy Sets and Systems*, vol. 59, pp. 125-148, 1993.

[26] R.R. Yager, "Quantifier Guided Aggregation Using OWA operators", *Int. J. Intelligent Systems*, vol. 11, pp. 49-73, 1996.

[27] R.R. Yager, E-Z OWA weights, In *Proceedings of the 10th International Fuzzy Systems Association (IFSA) World Congress*, Istanbul, Turkey, pp. 39-42, 2003.

[28] R.R. Yager, "Induced aggregation operators", Fuzzy Sets and Systems, vol. 137, pp. 59-69, 2003.

[29] R.R. Yager, "Centered OWA operators", *Soft Computing*, vol. 11, pp. 631-639, 2007.

[30] R.R. Yager, and D.P. Filev, "Parameterized "andlike" and "orlike" OWA operators", *Int. J. General Systems*, vol. 22, pp. 297-316, 1994.

[31] R. Moore, *Interval analysis*, Prentice-Hall, Englewood Cliffs, NJ, 1966.

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