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Propagation of Electron-Acoustic Solitary Waves in Weakly Relativistically Degenerate Fermi Plasma

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Abstract**—**Using one dimensional Quantum hydrodynamic (QHD) model Korteweg de Vries (KdV) solitary excitations of electron-acoustic waves (EAWs) have been examined in two-electron-populated relativistically degenerate super dense plasma. It is found that relativistic degeneracy parameter influences the conditions of formation and properties of solitary structures.

Keywords**—**Relativistic Degeneracy, Electron-Acoustic Waves, Quantum Plasma, KdV Equation.

I. INTRODUCTION

THE Electron acoustic waves (EAWs) are high frequency (in comparison with ion plasma frequency) electrostatic modes which occur in plasmas containing two distinct groups of electrons in which cold electrons provide the inertia and the restoring force comes from the hot electron pressure. The phase speed of EAWs is much larger than the thermal speed of cold electrons but much smaller than the thermal speed of hot electrons. Here ions may be regarded as forming a uniform neutralizing background. Since plasmas with two groups of electrons are known to occur in both space plasmas and laboratory experiments the EAWs play an important role in these environments. In recent years the study on the nonlinear evolution of EAWs has gained momentum with a view to explain the observation of moving EAW related structures reported by various space-craft missions [1-20]. Most of the works on EAWs are for classical nonrelativistic plasmas. The matter in some compact astrophysical objects (e.g. white dwarfs, neutron stars, magnetars etc.) exists in extreme conditions of density. In such situation the average inter-Fermion distance is comparable to or less than the thermal de Broglie wavelength and hence quantum degeneracy effects become important.

In such extreme conditions of density the electron Fermi energy E_{Fe} [$=\hbar^2\left(3\pi^2 n_e\right)^{3/2}\big/2m_e$] may become comparable to the electron rest mass energy [m_ec^2] and the electron speed can approach the speed of light (c) in vacuum. At extreme high densities the thermal pressure of electrons may be negligible as compared to the Fermi degeneracy pressure which arises due to implications of Pauli's exclusion principle. So the plasma in the interior of such compact astrophysical objects is both degenerate and relativistic. Under such conditions quantum and relativistic effects are unavoidable. Recently a large number of theoretical investigations have been made of the linear and nonlinear propagation of various electrostatic modes in degenerate quantum plasmas by using the quantum hydrodynamic model [21-41]. But to the best of our knowledge no investigation has been made of the nonlinear properties of electron-acoustic waves in weakly relativistic degenerate quantum plasmas. The purpose of the present paper is to investigate the linear and nonlinear properties of EAWs in relativistically degenerate dense quantum plasma consisting of two distinct groups of electrons and stationary ions. The paper is organized in the following way: in section II the basic set of quantum hydrodynamic equations are presented; in section III the linear dispersion characteristics is investigated; in section IV the Korteweg deVries equation is derived by using the standard perturbation techniques; in section V we discuss the dependence of soliton properties on different plasma parameters. The paper ends up with some concluding remarks.

II. BASIC FORMULATION

Electron-acoustic waves are considered to propagate in an unmagnetized three component completely degenerate dense plasma consisting of two groups of relativistic electrons at different temperatures and stationary cold ions forming a uniform neutralizing background. For electrons the thermal pressure is assumed to be negligible as compared to the degeneracy pressure which arises due to the implications of Pauli Exclusion Principle. In degenerate plasmas the rate of electron-ion collisions is limited due to the Pauli blocking mechanism which allows only degenerate particles with energies limited to a narrow range around the Fermi energy to interact, hence the plasma may be considered to be almost collision-less. Following Chandrasekhar (1939) the electron degeneracy pressure in fully degenerate and relativistic configuration can be expressed in the following form:

$$P_{eh}=\left(\pi m_e^4 c^5\big/3h^3\right)\left[R_{eh}(2R_{eh}^2-3)\sqrt{1+R_{eh}^2}+3\sinh^{-1}R_{eh}\right] \qquad (1)$$

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in which $R_{eh} = p_{Feh}/m_e c = \left[3h^3 n_{eh}/8\pi m_e^3 c^3\right]^{1/3} = R_{eh0} n_{eh}^{1/3}$ where $R_{eh0} = \left(n_{eh0}/n_0\right)^{1/3}$ with $n_0 = 8\pi m_e^3 c^3/3h^3 \approx 5.9\times10^{29}\,cm^{-3}$, 'c' being the speed of light in vacuum. p_{Feh} is the electron Fermi relativistic momentum. It is to be noted that in the limits of very small and very large values of relativity parameter R_{eh}, we obtain:

$$P_{eh} = \frac{1}{20}\left(\frac{3}{\pi}\right)^{2/3}\frac{h^2}{m_e} n_{eh}^{5/3} \quad (\text{for } R_{eh}\to 0)\,, \tag{2a}$$

$$P_{eh} = \frac{1}{8}\left(\frac{3}{\pi}\right)^{1/3} hc n_{eh}^{4/3} \quad (\text{for } R_{eh}\to\infty) \tag{2b}$$

Note that the degenerate electron pressure depends only on the electron number density but not on the electron temperature. The dynamics of such a plasma is governed by the following normalized quantum hydrodynamic equations:

$$\frac{\partial(n_{ec})}{\partial t} + \frac{\partial(n_{ec}u_{ec})}{\partial x} = 0 \tag{3}$$

$$\left(\frac{\partial}{\partial t} + u_{ec}\frac{\partial}{\partial x}\right)u_{ec} = \left[\frac{\partial\phi}{\partial x} + \frac{H^2}{2}\frac{\partial}{\partial x}\left[\frac{1}{\sqrt{n_{ec}}}\frac{\partial^2\sqrt{n_{ec}}}{\partial x^2}\right]\right] \tag{4}$$

$$\frac{\partial(n_{eh})}{\partial t} + \frac{\partial(n_{eh}u_{eh})}{\partial x} = 0 \tag{5}$$

$$0 = \left[\frac{\partial\phi}{\partial x} - F_{eh}\frac{\partial n_{eh}}{\partial x} + \frac{H^2}{2}\frac{\partial}{\partial x}\left[\frac{1}{\sqrt{n_{eh}}}\frac{\partial^2\sqrt{n_{eh}}}{\partial x^2}\right]\right] \tag{6}$$

$$\frac{\partial^2\phi}{\partial x^2} = \left(n_{ec} + \frac{n_{eh}}{\delta} - n_i\frac{\delta_1}{\delta}\right) \tag{7}$$

The normalization has been carried out in the following manner:
$x \to x\omega_c / c_{sh}, t \to t\omega_c, \phi \to e\phi / 2k_B T_{Fh}, n_j \to n_j / n_{j0}, n_i \to n_i / n_{i0}$ and $u_j \to u_j / c_{sh}$
in which $\omega_{ec} = \sqrt{4\pi n_{ec0} e^2 / m_e}$ is the cold electron plasma frequency, $c_{sh} = \sqrt{2k_B T_{Feh} / m_e}$ is the electron-acoustic speed. The charge neutrality at equilibrium reads $\delta = \delta_1 - 1$. It is to be noted that the parameter R_{eh0} is a measure of the relativistic effects and may be called relativistic degeneracy parameter. For weakly relativistic case $R_{j0} \ll 1$. Here $F_{eh} = \left(\chi R_{eh0}^2 \big/ \sqrt{1+R_{eh0}^2}\right)\Big/3$ is the term arising from relativistic pressure in weakly relativistic case, in which $\chi = m_e c^2 / 2k_B T_{Feh}$; H is the non-dimensional quantum diffraction parameter defined as $H = \hbar\omega_{ec} / 2k_B T_{Feh}$, where T_{Feh} is the Fermi temperatures for hot electrons; $\delta = n_{ec0} / n_{eh0}$ and $\delta_1 = Z_i n_{i0} / n_{eh0}$, in which n_{ec0}, n_{eh0} and n_{i0} are the equilibrium number densities of cold electrons, hot electrons and ions respectively.

III. Dispersion Characteristics

In order to investigate the nonlinear behaviour of electron-acoustic waves we make the following perturbation expansion for the field quantities n_{eh}, u_{eh}, n_{ec}, u_{ec} and ϕ about their equilibrium values:

$$\begin{bmatrix} n_j \\ u_j \\ \phi \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + \varepsilon\begin{bmatrix} n_j^{(1)} \\ u_j^{(1)} \\ \phi^{(1)} \end{bmatrix} + \varepsilon^2\begin{bmatrix} n_j^{(2)} \\ u_j^{(2)} \\ \phi^{(2)} \end{bmatrix} + \ldots\ldots \tag{8}$$

Substituting the expansion (8) in Eqs. (3)-(7) and then linearizing and assuming that all the field quantities vary as $e^{i(kx-\omega t)}$, we get for normalized wave frequency ω and wave number k, the following linear dispersion relation :

$$\omega^2 = \frac{\delta k^2(F_{eh} + H^2k^2/4)}{1+\delta k^2(F_{eh} + H^2k^2/4)} + \frac{H^2k^4}{4} \tag{9}$$

In the long wavelength limit (i.e. k→0):

$$\omega = k\sqrt{\delta F_{eh}} = kR_{eh0}\sqrt{\delta\,\chi/3}\left(1+R_{eh0}^2\right)^{-1/4} \tag{10}$$

The long wave phase speed is:

$$V_0 = \omega/k = R_{eh0}\sqrt{\delta\,\chi/3}\left(1+R_{eh0}^2\right)^{-1/4} \tag{11}$$

It represents the long wave dispersion character of EAWs in a quantum-relativistic plasma composed of inertia less hot electrons, inertial cold electrons and stationary ions. We numerically examine the behaviour of the dispersion relation (9) with respect to the variations of R_{eh0}, δ and H. Fig. 1(a-c) shows the variation of ω with k for different values of the relativity parameter R_{eh0}, δ and H respectively. It is shown that the wave frequency ω increases with increase in all three of them.

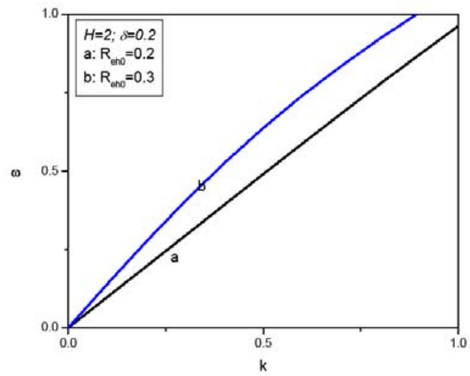

Fig. 1(a) Dispersion curves for different values of the relativity parameter R_{eh0} keeping δ and H constant

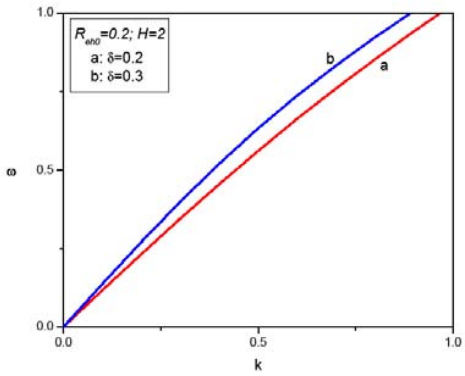

Fig. 1(b) Dispersion curves for different values of the equilibrium cold to hot electron density δ keeping R_{eh0} and H constant

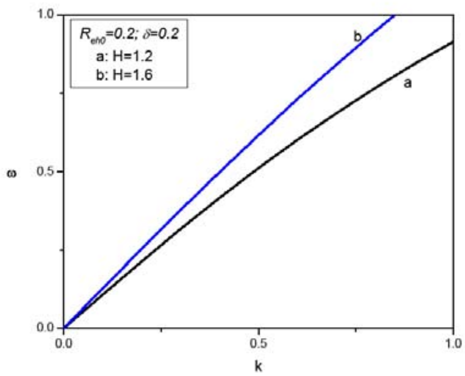

Fig. 1(c) Dispersion curves for different values of the quantum diffraction parameter H keeping δ and R_{eh0} constant

IV. KDV Solitary Wave Structures

In order to study the nonlinear behaviour of electron acoustic waves we follow the standard reductive perturbation technique we obtain the desired Korteweg de Vries (KdV) equation:

$$\frac{\partial\phi}{\partial\tau}+A\phi\frac{\partial\phi}{\partial\xi}+B\frac{\partial^3\phi}{\partial\xi^3}=0 \tag{12}$$

where

$$A=-\frac{3}{2V_0}=-\frac{3\cdot R_{eh0}\sqrt{\delta\chi/3}}{2\cdot\left(1+R_{eh0}^2\right)^{1/4}} \tag{13a}$$

$$B=\frac{V_0^4-(1+\delta)H^2/4}{2V_0}=\frac{\left[\left(\frac{R_{eh0}^4\delta^2\chi^2}{9\left(1+R_{eh0}^2\right)}\right)-(1+\delta)\frac{H^2}{4}\right]}{2\frac{R_{eh0}\sqrt{\delta\chi/3}}{\left(1+R_{eh0}^2\right)^{1/4}}} \tag{13b}$$

To find the solution of Eq. (12) we transform the independent variables ξ and τ into one variable η = ξ - M τ where M is the normalized constant speed of the wave frame. Applying the boundary conditions that as η → ± ∞; $\phi,\frac{\partial\phi}{\partial\eta},\frac{\partial^2\phi}{\partial^2\eta}\to 0$ the possible stationary solution of Eq. (12) is obtained as:

$$\phi=\phi_m\sec h^2\left(\frac{\eta}{\Delta}\right) \tag{14}$$

where the amplitude ϕ_m and width Δ of the solution are given by:

$$\phi_m=3M/A \tag{15a}$$

and $$\Delta=\sqrt{4B/M} \tag{15b}$$

The solitary wave structure is formed due to a delicate balance between dispersive and nonlinear effects. Relative strength of these two effects determines the characteristic of such solitary wave structure. The coefficients A and B, corresponding to the nonlinear effect and dispersive effect play a crucial role in determining the solitary wave structure. So it is important to study the dependence of these coefficients on different physical parameters. From Eqs. (13a) and (13b) it is clear that both the nonlinear and dispersion coefficients get modified due to the inclusion of relativistic effect whereas the quantum effect enters only into the dispersion coefficient. Both these coefficients depend on δ, the equilibrium cold-to-hot electron concentration ratio. For a given H and δ there exists a critical value of the relativity parameter R_{eh0} at which the dispersion coefficient vanishes. This critical value of R_{eh0} is given by:

$$\left(R_{eh0}\right)_c=\frac{9(1+\delta)\frac{H^2}{4}+\sqrt{81(1+\delta)^2\frac{H^4}{16}+36\delta^2\chi^2(1+\delta)\frac{H^2}{4}}}{2\delta^2\chi^2} \tag{16}$$

No solitary structure is possible for $R_{eh0}<\left(R_{eh0}\right)_c$. Note that the critical value of the relativity parameter depends on both δ and H. From Eqs. (13)- (15) it is obvious that the degenerate plasma under consideration supports only rarefactive solitary wave structures which are associated with negative potentials. Fig. 2(a) shows electron-acoustic solitary profiles for different values of the relativistic degeneracy parameter R_{eh0} (which is directly proportional to the plasma number density) for fixed values of M, δ and H. It shows that both the amplitude and width of the soliton increase with increase of R_{eh0}.

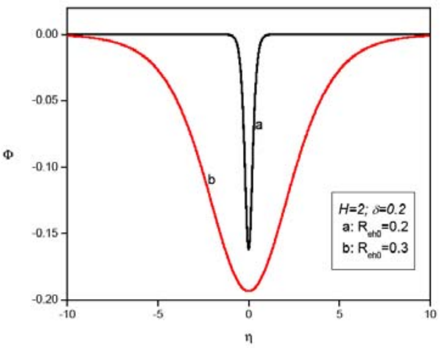

Fig. 2(a) Electron-acoustic solitary profiles for different values of the relativistic degeneracy parameter R_{eh0} for fixed values of M, δ and H

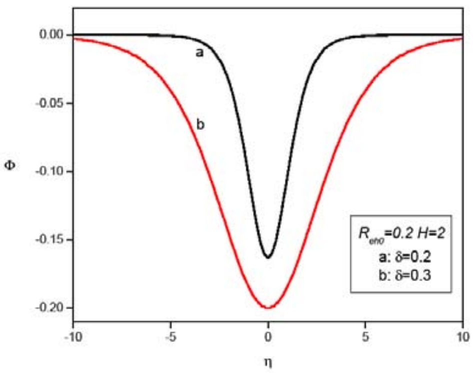

Fig. 2(b) Electron-acoustic solitary profiles for different values of the equilibrium cold-to-hot electron concentration ratio δ for fixed values of R_{eh0} , M and H

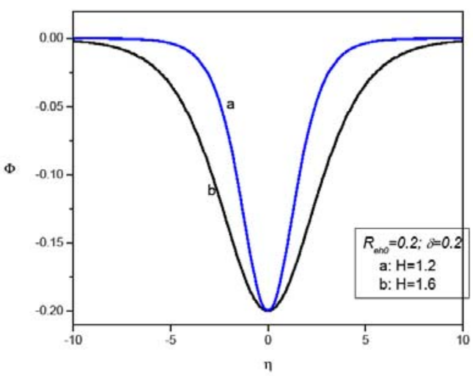

Fig. 2(c) Electron-acoustic solitary profiles for different values of the quantum diffraction parameter H for fixed values of M, δ and R_{eh0}

Fig. 2(b) shows solitary structures for different values of δ keeping R_{eh0}, M and H constant. It is observed that with increase in δ both the amplitude and width of the soliton increase. Fig. 2(c) shows solitary structures for different values of H keeping other parameters fixed. It shows that the soliton width increases with increase in the value of H but its amplitude is independent of H. The amplitude of electron-acoustic solitary structure increases with increase in R_{eh0} and δ, but it is independent of H. On the other hand the width of the soliton increases with increase in R_{eh0} , δ or H.

V. Discussion and Conclusion

Using QHD model the linear and nonlinear propagation characteristics of EAWs are investigated in a relativistic degenerate dense plasma consisting of two distinct groups of electrons and stationary ions. It is shown that the plasma under consideration can support only rarefactive solitary waves under certain restricted regions of plasma parameters. The soliton properties are shown to depend significantly on the relativistic degeneracy parameter R_{eh0}, the equilibrium cold-to-hot electron density ratio δ and also the quantum diffraction parameter H. The present investigation may be helpful in understanding the basic features of electron-acoustic waves in super dense astrophysical objects like white dwarfs, neutron stars as well as in the future intense laser-solid plasma experiments where the relativistic electron degeneracy effects become important. Finally we would like to point out that the investigation presented here may be helpful in the understanding of the basic features of long wavelength electron plasma waves in dense plasmas such as can be found in white dwarfs, neutron stars and intense laser-solid plasma experiments.

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