Design and Analysis of an Automobile Bumper with the Capacity of Energy Release Using GMT Materials

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Abstract—Bumpers play an important role in preventing the impact energy from being transferred to the automobile and passengers. Saving the impact energy in the bumper to be released in the environment reduces the damages of the automobile and passengers.

The goal of this paper is to design a bumper with minimum weight by employing the Glass Material Thermoplastic (GMT) materials. This bumper either absorbs the impact energy with its deformation or transfers it perpendicular to the impact direction.

To reach this aim, a mechanism is designed to convert about 80% of the kinetic impact energy to the spring potential energy and release it to the environment in the low impact velocity according to American standard¹. In addition, since the residual kinetic energy will be damped with the infinitesimal elastic deformation of the bumper elements, the passengers will not sense any impact. It should be noted that in this paper, modeling, solving and result's analysis are done in CATIA, LS-DYNA and ANSYS V8.0 software respectively.

Keywords—Bumper, Composite material, Energy Release, GMT, Impact

I. INTRODUCTION

THE automobile industry has been improved significantly since 1953 by emerging the composite materials [1]. Since it is proved that the composite materials can achieve the desirable properties such as low weight, high fatigue strength, easy forming and high strength, they are suitable for material replacing [1]. Although the composites have some undesirable properties such as relatively long time processing, expensive raw materials and low surface finish quality, its light weight is the major reason for the increasing application of the composite materials in the automobile industry [1]. In the mass production of vehicles, the light weight of components results in a significant reduction of the fuel consumption and consequently the reduction of the CO2 and other emissions.

Since the experimental tests, particularly at full-scale, are very costly and require highly specialized test facilities and also the model being evaluated inevitability will suffer

¹ -United State National Highway Traffic Safety Administration (49 CFR. Part 581 Bumper Standard) extensive damages, utilizing the crash simulation seems to be crucial.

Today, numerical models reproduce all details of vehicles, and also include passengers. With such analysis and employing upwards of 1,000,000 elements, it may still take a few days to solve the problem even by the modern multiprocessor computers [1]. Nowadays, with the development of the automobile technology, more and more light weighting materials like the Glass Material Thermoplastic (GMT) are applied to the automobile body. [2]. GMT provides a high strength to- weight ratio, chemical / corrosion resistance, and excellent impact properties at both low and high temperatures [3]. Compared to metals, GMT offers greater design flexibility, lower tooling costs, and opportunities for part consolidation. Compared with thermoset composites, GMT improves productivity with shorter molding cycle time, greater impact resistance, recyclability (melt reprocess ability), and elimination of controlled-storage requirements [3]. GMT has rate dependant properties, which is due to the viscoelastic properties of Polypropylene which is used as matrix in the material. As a result of this viscoelastic behavior, the strength (and energy absorption) at practically encountered deformation rates in crash loaded automotive parts is significantly higher than the "quasi-static" strength measured at an elongation rate of 0.001 (1/s) [4].

There are three principle types of GMT, including continues glass fiber, chopped glass fiber and unidirectional glass fiber [5]. The use of GMT in high-impact, structural applications in the automotive and transportation industry is well documented [6-8].

The bumper system is a structural component, which contributes to the crashworthiness or occupant's protection during a front or rear collision. There is an interest among the researchers to move from conventional materials such as plastic, aluminum, or steel to materials such as polymeric based composites in the bumper system. For instance, a composite material bumper system has been made using sheet molding compound (SMC) with random chopped glass fiber composites [9]. Minaudo et al. (1997) developed a one piece, injection molded, thermoplastic rear bumper system with pole impact protection [10]. Clark et al. (1991) described their extensive work on bumper beams using continuous glass fiber composites to study the stress contour in the component [11]. Cheon et al. (1999) developed the composite bumper beam for a passenger car. The material used was glass fiber epoxy

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composite material, except for the elbow section [12]. And then Gilliard et al. (1999) developed an I-section beam with 40% chopped glass fiber GMT [13].

Over the last few years, some factors have made this application more interesting for GMT, which are as follows [14]:

1. Increasing demands of the vehicle weight reduction: Reduction in fuel consumption and in addition to, since, the bumper is far from the center of gravity of the vehicle so it's weight is also critical to the inertia and as a result to the vehicle handling.

2. Higher required energy absorption: Achieving energy absorption at bumper mounting points to protect the structures behind it in the vehicle, at low speed crash.

3. Controllable fracture behavior: Part integrity and stabilization function at very high speed crashes. At these rates primarily the deformation behavior is important.

In this research, a typical new front bumper beam on a passenger cars have been designed with GMT composite materials. This bumper absorbs impact energy with its deformation or transfers it perpendicular to the impact direction with the aid of a spring mechanism that is able to convert about 80% of the kinetic energy to the spring potential energy in low speed impacts according to American standard.

The main design concepts of this bumper are based on aerodynamic forms and frontal configuration of passenger cars. The design of spring system has done with the aid of Genetic algorithm in MATLAB V6. The CATIA data of the bumper structure have imported to LS-Dyna Ansys and analyses have done with nonlinear explicit impact modeling elements. Modeling, solving and analysis were carried out with respect to the American standard (49 CFR. Part 581 Bumper Standard) and a bumper was designed with 7.6 kg weight which has half weight compared with a similar steel bumper (with equal strength).

II. BUMPER MODEL DESIGN

There are several models and systems for bumpers of passenger cars [15]. Traditional models have corrugated open section areas for installing some car elements and increasing bending strength of the bumper. Main parts of the conventional bumper systems are depicted in Fig. 1.



Fig. 1 Configuration of common bumper type In Fig. 1:

1. Fascia: bumper fascias must be aerodynamic, light weight and aesthetically pleasing to the consumer. Usually

fascias are made of polypropylene, polyurethane or polycarbonate.

2. Energy absorbers: energy absorbers are designed to absorb a portion of the kinetic energy from vehicle collision. Its types include foam, honeycomb and mechanical ones. However mechanical absorbers have several times the weight of foam and honey comb absorber, they receive limited usage.

3. Reinforcing beam: this part is a key component of the bumper and helps absorb the kinetic energy and provide protection to the rest of the vehicle.

The designed bumper in this research is a combination of these elements. In other words, in low-speed contacts, the kinetic energy of impactor is absorbed by changing the impact force direction by the spring system (as mechanical energy absorbers) and in high speed contacts it is absorbed by deformation of conic composite cells of the bumper (as reinforcing beam).

The main elements of this bumper are as follows (see Fig. 2):

1. Front rubber tape: that is composed of polypropylene (PEP) for damping of poor contacts.

2. Fascia: it indicates the aerodynamic form of the bumper and is used as a bearing for spring system retainer.

3. Spring system: it contains 26 vertical springs for converting the kinetic energy to the spring potential energy, In addition to 4 horizontal springs for connecting the fascia to base plate.

4. Conics and base plate: they are main elements of the bumper for energy absorbing in high speed contacts (i.e. reinforcing beam).

5. Connecting plastic parts: two propylene (PEP) parts that connect the bumper base plate to the car.



Fig. 2 Schematic configuration of the desired bumper

To summarize, the working process of this bumper is as follows (see Fig. 3):

In the low speed impacts, the cover moves toward the conics to reaches its top surface and make the spring system to stretch in vertical direction as a result of cover edges sliding on the conics. The initial dimensions are calculated and selected proportionally then as a result the spring system stretches a totally 6 cm perpendicular to the impact direction.

So, it absorbs kinetic energy in the form of spring potential energy. Also, two small areas between the cover edge and the middle part of the cover have designed with thinner thicknesses (i.e. there are two lateral notch at the top and bottom corners of the cover), which guarantee easier deformation rather than the other parts of the cover, so, the cover edges movements mechanism is completely predicted and in control. For high speed contacts, the cover reaches the conics and they deforms as a composed part. There is a concavity in the cover where the plastic tape seats on and increases the bending strength of the bumper.



Fig. 3 section view of bumper – cover and spring movement during frontal impact

III. MESHING AND BOUNDARY CONDITIONS

Explicit method is a fast method for short time problems, complicated contact and impact problems and multiple nonlinearities large deformations (Dynamic & quasi static) in LS-Dyna. The CATIA V5 CAD data of the bumper model was imported to LS-Dyna Anys8.0. Then, meshing has created on a 3D model.

Since the average thickness of the cover, conics and base plate is much smaller than the other dimensions of the part, the best element for meshing was the shell element (Shell 163). Membrane Blystchst chko-Tsy method was used for solution. (it is a fast method for membrane problems and composite material with corrugated surface). Solid 164 elements were used for impactor, plastic parts and plastic front rubber meshing.

The impactor as a steel structure was modeled by isotropic rigid pyramid solid impact elements. Spring-damper element (Combi 165) was used for spring system modeling. Each plastic part is attached to the car body by screws at four points. Hence, on the model, one-eighth of the car weight was attached to each screwed node as a point mass element (Mass 166).

There is no external force on the elements and no friction was assumed between the impactor and the bumper surfaces and the car was taken to be laying on a flat and frictionless surface and all other conditions were drawing from American Traffic Safety Administration (49 CFR. Part 581, Bumper Standard). All degrees of freedoms of the corresponding nodes on the bumper, plastic parts and plastic tape (screw points) were coupled and merged.

Finally meshing of different parts of the bumper with adequate numbers of elements have done by Mapped method. FEM specifications and element types used for the main parts of the bumper are tabulated in Table 1 and Fig. 4:

TABLE I				
FEM CHARACTERISTICS OF THE BUMPER MODELS				
Components	Element	Material	Number	Weight,
	Туре		of Nodes	Thickness
Impactor	Solid 164	Rigid	246	850 Kg
		Hardened		
		Steel		
Bumper			10250	8.5 Kg
Car	Mass 166		8	1150 Kg
Cover	Shell 163	GMT	3015	t: 2-5 mm
Conics & base plate	Shell 163	GMT	6081	t: 3.5-5 mm
Plastic parts of the bumper	Solid 164	PEP	1154	



Fig. 4 meshing of impact layout

IV. MODEL ANALYSIS

The results of some investigations by Tao & Yu [17-19] identified that the grid-domed cellular structure possesses the highest specific energy absorbing capacity among so many cell configurations (including circular and square tubular knitted, multi-layer 3D woven, non-woven spun sponded and grid-domed cells) under both quasi-static compression and impact conditions. In addition, other geometrical factors on these flat-tapped cellular composites that govern the energy-absorbing capacity, including cell height, diameter ratio of cell-top to cell-bottom, projected wall area, cell density and component content have also been optimized [17] Fig. 5 shows a cross section of this cellular structure.



Fig. 5 Schematic geometry of a basic grid-domed cellular structure

In this research an impact test has simulated (with the same test conditions that S.W. Lam has implemented [18]) for the grid-domed cell with GMT in Ansys Ls-Dyna, and the extra properties and the impact energy absorption capacity of GMT have been identified. The GMT with 30% volume fraction (V.F.) has less peak loads and more toughness in comparison with the GMT with 40% V.F. (see Fig. 6)



Fig. 6 Absorbed energy – Normal displacement of GMT grid-domed cellular sample under impact

So, the GMT with 30% V.F. with similar dimensions as usual car dampers has used for the first try of design and analysis. In accordance with the standard of straight frontal impact test, the mass of impactor must be equal to net car mass. So, initial kinetic energy of the impactor (as a result of its initial velocity) could be derived as in (1):

$$KE = \frac{1}{2}MV^{2} = \frac{1}{2} \times 850 \times (\frac{4}{3.6})^{2} = 524.69J$$
(1)

The objective of the spring design is to completely absorb the above (1) kinetic energy as the cover reaches the conics and its edges are opened and the springs are in maximum extension length. In other words, the total impact energy should be used for cover change from stage 1 to stage 2 which also cause the springs to extend simultaneously (See Fig. 7).



Fig. 7 cover movements and spring extension: stage 1-2

In the case which there are no springs, thus there isn't any resistance forces on the cover edges, the simulation shows that in changing from stage 1 to stage 2, the vertical displacement of the cover edges are more than 6 cm. in this case, stresses are negligible, except in a small area between the cover edge and the middle part of the cover (i.e. elastic hinges) and all deformations are completely elastic, so absorbed energy is negligible too.

Related to the stages 1 & 2 and numbers of conics, prescribed parameters are: δ (springs displacement) and ns (number of springs in the system) which are held between conics. Other parameters are identified by utilizing a genetic algorithm with the consideration on the minimum weight of spring system (and so minimum weight of bumper).

Mathematical model of the problem will be as in equation (2) to (5):

1. Variable vector

$$\{X\} = \{x_1, x_2, x_3\} = \{N, d, D\}$$
(2)

In which "N" is the number of active coils, "D" is mean spring diameter and "d" is spring wire diameter.

1. Objective function

$$f(X) = W = 19.368 \times 10^{-6} \times n_s \times \{x_1 \times x_2^2 \times x_3\}$$
(3)

2. Equal boundary condition equation

$$g_1(X) = n_s \times x_2^4 \times \delta^2 / (x_1 \times x_3^2) = 106.94$$
 (4)

3. Non equal boundary condition equations

$$g_{2}(X) = k_{s} \times \delta \times x_{2} / (x_{1} \times x_{3}^{2}) \le (S_{sy} / 24.98 \times 10^{3})$$
(5)

$$g_{3}(X) = \frac{x_{2}^{2}}{x_{1} \times x_{3}^{3}} \times \delta \times [(\frac{x_{3}}{x_{3} - x_{2}})\frac{4x_{3}}{x_{2}} + 1] \le (S_{y}/12.49103)$$
(6)

" k_s " is the spring constant, " S_{sy} " is maximum permissible shear stress and " S_y " is the yield strength.

So, solving a weight optimization problem with 3 boundary condition equations will clarify the spring system.

For a bumper with minimum weight, it is necessary to have the least thicknesses for its different parts. With respect to the mounting procedure of the bumper on the car, one can consider it as a beam with a hinge at each ends, and the maximum pressure and impact load is at the middle of the beam and the major loading condition is bending.

Since the base bumper surface behind the conics were assumed to take the most bending and deflections, the maximum thickness in comparison with the other composite parts were implemented in this area.

In fact, the crash test was done for 3 types of thicknesses and the energy absorb capacity, deflection, deformation and the maximum stress of the bumper were taken into account and compared case by case. In all of these tests, the middle part of the cover had the same thickness of the conics. Also, the thickness of the notch on the cover was just smaller than it is considered as elastic-plastic hinge.

In case 1, it has considered 2.5 mm thickness for the cover and conics, 2 mm for the hinges and 4 mm for the base plate of the bumper. And it was shown that the cover ends converge a little to each other and spring tension seems to overcome the bending strength of the cover edges. Thus the spring system cannot play its expected role. Since the spring system has converted the minimum of kinetic energy of the impactor to the potential energy, bumper deflection reached to 19 cm after 0.2 s (T1).

To avoid this, the base plate thickness was increased from 4 to 4.5 mm, cover thickness to 3.5 mm and cover edges thickness to 4 mm. In this case, since the kinetic energy of impactor has not completely damped, so the impactor continue to its forward movement even after that cover opens completely and reaches the conics. Also yield was reported for some parts of base plate and this causes a long deflection in the bumper and as a result speeding up of the conics and then spring return (T2).

Finally to overcome this problem and provide the minimum weight for the bumper, the base plate thickness and the cover edges was increased to 5 mm and 4.5 mm respectively. The structure was tested again and no yield was reported. Also, after 0.17 s, the elastic deflection in the bumper was less than 3 cm and the impactor was stopped, so the aim was reached. Since the springs are at the highest extension, the impactor moves backward a little after a bit passed time (T3).

Modification process is shown by diagrams (Fig. 8-11) forward:



Fig. 8 Max Von Mises in the cover (T1-T3)



Fig. 9 Max displacement in impact direction in the cover (T1-T3)



Fig. 10 Max displacement in vertical direction at the cover edges (T1-T3)



Fig. 11 Impactor speed in X direction (T1-T3)

By analyzing these diagrams, it was shown that at (t=0.054 s) when the impactor and cover reach the conics (i.e. during cover opening and spring extension), more than 80% of impactor kinetic energy was absorbed and the remaining energy (Vx = 467 mm/s) was damped by a general deflection of the bumper and by elastic and plastic deflections of the cover. Displacement in X direction which is a combination of cover translation and deformation for a node in the middle of the bumper on the plastic tape vs. time has shown by (Fig. 12).



Fig. 12 Max displacement in impact direction in the cover versus time

Fig. 12 clearly depicts that when impactor reaches the conics, displacement in X direction increases with a low rate i.e. the bumper reaches its maximum deformation. Since all deformations are in elastic range, the bumper parts back to the initial states, gradually.

In order to observe the stress wave distribution in the bumper, two paths were defined in the vertical and horizontal directions on the cover as shown in Fig. 13, Fig. 14 (a) & Fig. 14 (b).



Fig. 13 Paths on the cover, vertical and horizontal



Fig. 14 (a) Paths on the cover, horizontal nods



Fig. 14 (b) Paths on the cover, vertical nods

Then stress distributions on vertical and horizontal path taken at time intervals of 0.25 s shown in Fig. 15 & Fig. 16 respectively.



Fig. 15 Von Mises stress distribution along horizontal path



Fig. 16 Von Mises stress distribution along vertical path

The maximum stress distribution occurs at t=0.052 s which correspond with the time that cover reaches to the conics.

On vertical path, the stresses are a combination of compression in longitudinal & lateral directions and tension in lateral direction. Maximum Von Mises stress occurs at location 4 since the elements are under compression on the contacting surface with the impactor and under tension on the lateral direction. The next extremes are for points 2 and 6 because of stress concentration due to relative sharp corner and implemented notch (hinge) respectively. Points 7, 8 & 9 are those on the cover edge after the hinge and hence yielding Von Mises stresses of nearly zero. In horizontal path, stresses are more tension than compression and also stress wave fluctuate and decreases along the path from the center of the bumper to each side.

Safety factor can be derived with the aid of maximum Von Mises stress and GMT yield strength. By utilizing this concept, the safety factor in vertical path has shown in Fig. 17. Minimum value occurs at point 4 and is about 1.5 that is generally satisfactory.



Fig. 17 Von Safety factor of the bumper along vertical path

If this bumper was made by steal instead of GMT with yield strength of 230 Mpa, then to achieve the same safety factor (i.e. to have the same strength for the bumper structure), generally all the thicknesses should be decreased by a reduction factor of 0.34 (The inverse of yield strength of steel divided by GMT). Fig. 18 shows that for equal safety factor steel bumper has a bit better energy absorption capacity rather than GMT, although by comparing steel & GMT density and implementing thicknesses a weight reduction of about ¹/₂ is achieved that is satisfactory for using GMT instead of steel.



Fig. 18 Kinetic energy of the impactor during the impact with steel and GMT bumper

V.SUMMARY

There are many effective factors in selection of a bumper system. The most important one is its ability to absorb impact energy especially in high speed crashes according to legal standards [R8]. Weight, manufacturability and price have secondary importance. Although the bumpers are designed for low speed impacts, in high speed crashes, bumper is the first part for energy absorption and also replacement. The GMT offers more suitable material at lower cost and easier production process in comparison with conventional metals. Also, it can form large and complex parts with appropriate dimensional stability in a short shaping cycling.

A commercial short-fiber composite bumper made of GMT material with a mechanical spring mechanism (as energy absorber mechanism) is designed under frontal impact test according to American bumper standard. It is revealed that by utilizing this mechanical energy absorber the bumper is able to convert about 80% of the kinetic impact energy to spring potential energy and release it to the environment in the low impact velocity. The residual kinetic energy will be damped with infinitesimal elastic deformation of bumper elements. So, the passengers will not sense any impact. Finally, steel bumper (as a conventional material) was compared with the GMT and the results showed inappropriate weight increase of about two times of the GMT bumper with the same safety factors.

However in high speed crashes, GMT conical part of the bumper was desired to absorb the kinetic energy of the impactor as much as possible, the authors believe more practical tests and simulations should be carried out to verify the advantages and stability of the proposed structure.

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