

# Code-Aided Turbo Channel Estimation for OFDM Systems with NB-LDPC Codes

Ł. Januszkiewicz, G. Bacci, H. Gierszal, M. Luise

**Abstract**—In this paper channel estimation techniques are considered as the support methods for OFDM transmission systems based on Non Binary LDPC (Low Density Parity Check) codes. Standard frequency domain pilot aided LS (Least Squares) and LMMSE (Linear Minimum Mean Square Error) estimators are investigated. Furthermore, an iterative algorithm is proposed as a solution exploiting the NB-LDPC channel decoder to improve the performance of the LMMSE estimator. Simulation results of signals transmitted through fading mobile channels are presented to compare the performance of the proposed channel estimators.

**Keywords**—LDPC codes, LMMSE, OFDM, turbo channel estimation.

## I. INTRODUCTION

THE need for future communication systems to support mobile users with complete real time access to broadband services calls for robustness against fast frequency-selective multipath fading. An effective solution to this impairment is to adopt the orthogonal frequency division multiplexing (OFDM) as the multiplexing strategy, used by advanced systems such as the IEEE 802.16 and LTE (Long Term Evolution) standards [1][2]. To improve the efficiency of OFDM systems in terms of energy expenditure and bandwidth occupation, we can resort to a recently investigated class of forward error correction (FEC) techniques, the NB-LDPC codes that operate over high-order finite Galois fields  $GF(q)$  [3].

To exploit the potentiality of NB-LDPC coding in terms of word-error rate (WER) or bit-error rate (BER) performance, we can adopt an iterative channel estimation using the information produced at the output of the channel decoder to mitigate the distortions introduced by the wireless channel selectivity. Channel estimation can be introduced both, in the time domain and in the frequency domain. The framing formats dictated by current standard technologies in fact adopt a low “density” of pilot symbols with known data in order not to decrease the spectral efficiency of the system. To artificially increase the density of known data, we can design more efficient algorithms that use the outputs of the channel decoder

and possibly perform iterative estimation that goes in parallel with the decoding iterations.

Recently, many channel estimation techniques have adopted the “turbo” principle [4]. The idea behind this approach is to use the decisions coming out of the decoding stage to feed the channel estimator. Hence, in a recursive fashion, this new estimate of the channel is fed back to the decoder, and a new decision on the estimated transmitted symbols is taken, and so on. Just to mention a few significant examples, authors in [5] derive an iterative channel estimation technique for a pilot-aided BPSK system over flat-fading channels, using the feedback from the decoder. This approach is generalized in [6] to the case of frequency and time selectivity, where joint iterative equalization and decoding is used. In the field of multicarrier modulation, relevant results can be found in [7][8], which propose iterative joint data detection and channel estimation schemes for OFDM systems under double channel selectivity.

This paper elaborates on the “turbo” approach to derive an iterative channel estimation algorithm based on the soft and hard outputs of the NB-LDPC decoder. The remainder of the paper is structured as follows. Section II describes the OFDM system operating over a time-varying frequency-selective scenario considered throughout this work. Section III illustrates some (non-iterative) standard channel estimation techniques that serve as a benchmark for the proposed estimation scheme, which is derived in Section IV. Section V shows some numerical results, whereas some conclusions are drawn in Section VI.

## II. SYSTEM DESCRIPTION

This section describes the system model considered throughout this paper, focusing mostly on the channel model and the receiver stage for the sake of brevity. At the transmitter side, the source message vector  $\mathbf{u}$ , containing  $K$   $GF(q)$  symbols, is encoded into a vector  $\mathbf{c}$ , containing  $N$   $GF(q)$  symbols, by a NB-LDPC encoder with a rate  $R = K/N$  [3]. A NB-LDPC code is defined in terms of a very sparse pseudo-random parity check matrix, whose elements belong to a finite Galois field  $GF(q)$ . The way the encoding process acts is very similar to that employed by binary LDPC codes. The fundamental difference is that all operations are to be intended in the  $GF(q)$  domain.

The transmission is based on a multicarrier OFDM signal using  $N_C$  sub-carriers. We consider an OFDM system due to its robustness against fast frequency-selective fading. OFDM

Ł. Januszkiewicz is with ITTI Ltd., Poznan, Poland (phone: +48 61 622 6944; e-mail: ljanuszkiewicz@itti.com.pl).

G. Bacci, is with Wireless Systems Engineering and Research (WISER), Livorno, Italy (e-mail: giacomo.bacci@wiser.it)

H. Gierszal is with Adam Mickiewicz University, Poznan, Poland (e-mail: gierszal@amu.edu.pl).

M. Luise is with the Wireless Systems Engineering and Research (WISER), Livorno, Italy (e-mail: marco.luise@wiser.it).

transmission system is used in advanced standards such as the IEEE 802.16 and LTE [1][2]. Depending on the mapping constellation selected and the number of sub-carriers allocated to the data transmission, every OFDM symbol is able to accommodate a number of vectors  $\mathbf{c}$ . To increase the frequency diversity of the signal, this group of encoded vectors  $\mathbf{c}$  is then interleaved on a GF( $q$ ) symbol basis, and mapped to quadrature amplitude modulation (QAM) symbols. The mapped symbols are modulated onto each OFDM block and sent to the OFDM receiver.

The OFDM signal is transmitted through a joint time- and frequency-selective fading channel with additive white Gaussian noise (AWGN). The channel is simulated according to models defined by the International Telecommunication Union (ITU) [14]. The first stage at the receiver side is an analog-to-digital converter (ADC), followed by a fast Fourier transform (FFT) block that allows the received signal to be handled in the frequency domain. This approach is particularly convenient in the case of an OFDM signal, since it makes all the required decoding operations much simpler.

The FFT operation gives the vector  $\mathbf{Y}(l) = [Y(l,1), \dots, Y(l, N_C)]$ , where  $Y(l,k) = X(l,k) \cdot H(l,k) + W(l,k)$  is the  $k$ -th received sub-carrier of the  $l$ -th OFDM symbol represented in the frequency domain, with  $X(l,k)$  being the QAM symbol mapped on the  $k$ -th sub-carrier;  $W(l,k)$  is the complex AWGN sample in the frequency domain with power  $\sigma_w^2/2$  over each component and  $H(l,k)$  is the channel response over the  $k$ -th sub-carrier of the  $l$ -th OFDM symbol.

Channel equalization is mandatory to mitigate the distortions introduced by channel selectivity. We make use of non-blind methods, which are often preferable to blind techniques, since the latter suffer from severe performance degradation in the presence of fast fading channels [9]. By extracting the pilot carriers embedded in the OFDM format from the vector  $\mathbf{Y}(l)$ , we can obtain a rough estimate of the channel in conjunction with an estimate of the noise power (the latter is used by the soft demapper, and possibly by the channel equalizer according to the equalization strategy).

The equalizer can thus process  $\mathbf{Y}(l)$ , and its output is sent to a deinterleaver after removing guards and pilot carriers. The stream is then subdivided in chunks of  $N$  symbols, corresponding to one codeword, and each chunk is sent to a soft demapper and finally to an NB-LDPC decoder. Decoding techniques for NB-LDPC codes can be borrowed by their binary counterparts by extending all operations to the field GF( $q$ ). The considered system adopts a simplified version of the extended min-sum (EMS) algorithm [10] that produces a matrix of *a-posteriori* probabilities (APP) for all coded GF( $q$ ) symbols.

To improve the system performance, we can exploit the information from the NB-LDPC decoder to refine the channel estimation. We can use either the soft information or the hard decisions (or even both at once) as an additional set of known sub-carriers (in addition to OFDM original pilots) to produce a further estimate of the channel response. In a recursive fashion, this new estimate is fed back to the decoder, and a

new decision on the transmitted symbols is taken, as detailed in Section IV.

### III. STANDARD CHANNEL ESTIMATION ALGORITHMS

Two common channel estimation algorithms are considered in this paper — LS and LMMSE (Wiener filter) [11][12]. The LS estimator exploits the pilot sub-carriers to estimate the channel impulse response. Assuming that every sub-carrier is a known pilot symbol, it is easy to calculate the estimate of the channel transfer function (CTF)  $\hat{H}_{LS}$ :

$$\hat{H}_{LS} = (X^H X)^{-1} \cdot X^H Y = X^{-1} Y = \begin{bmatrix} \frac{Y_0}{X_0} & \frac{Y_1}{X_1} & \dots & \frac{Y_{N_C-1}}{X_{N_C-1}} \end{bmatrix}^T, \quad (1)$$

where:

$$X = \begin{bmatrix} X_0 & 0 & \dots & 0 \\ 0 & X_1 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & X_{N_C-1} \end{bmatrix}, \quad Y = \begin{bmatrix} Y_0 \\ Y_1 \\ \vdots \\ Y_{N_C-1} \end{bmatrix}.$$

$(\cdot)^T$  and  $(\cdot)^H$  operators denote transposition and conjugate transposition, respectively, and  $N_C$  indicates the total number of sub-carriers in one OFDM block. The estimation (1) could be applied only when all sub-carriers are pilots. Practical OFDM blocks consist of both data sub-carriers and pilot sub-carriers. The LS estimator is used to calculate the estimate of CTF at pilot sub-carriers, and the values of CTF corresponding to the data sub-carriers can be calculated by means of linear or polynomial interpolation. Channel estimation in time can be obtained using interpolation between every other two OFDM blocks as well. This interpolation is needed for WiMAX system because pilot pattern is different in OFDM block by OFDM block.

A more sophisticated algorithm of channel estimation is performed by the LMMSE estimator (or Wiener filter). This kind of filtering is based on minimizing the expected mean-squared error between the actual and the estimated channel assuming some *a priori* knowledge of channel statistics. To estimate the CTF at data sub-carriers, spaced-time  $R(\Delta t)$  and spaced-frequency  $R(\Delta f)$  correlation functions need to be calculated. Assuming that  $\Delta t$  is the time distance between two subsequent OFDM blocks and  $\Delta f$  is the frequency distance between two considered sub-carriers,  $R(\Delta t)$  and  $R(\Delta f)$  can be calculated from the following formulas:

$$R(\Delta t) = \frac{P}{L} \sum_{i=1}^L J_0 \left( 2\pi \frac{v}{\lambda} \Delta t \right) = P \cdot J_0 \left( 2\pi \frac{v}{\lambda} \Delta t \right), \quad (2)$$

$$R(\Delta f) = \sum_{i=1}^L P_i e^{j2\pi \Delta f \tau_i}, \quad (3)$$

where  $P$  and  $L$  denote the total power and the total number of channel paths, respectively,  $P_i$  and  $\tau_i$  denote the power and the delay of the  $i$ -th path of channel, respectively,  $v$  is the vehicle speed and  $\lambda$  is the wavelength. Assuming  $R(\Delta t)$  and  $R(\Delta f)$  are known, it is easy to calculate the channel autocorrelation matrix  $R_h$  and cross-correlation vector  $r$ :

$$R_h = \begin{bmatrix} E\{\hat{H}_{P(0)}\hat{H}_{P(0)}^*} & E\{\hat{H}_{P(0)}\hat{H}_{P(1)}^*} & \dots & E\{\hat{H}_{P(0)}\hat{H}_{P(N_p-1)}^*} \\ E\{\hat{H}_{P(1)}\hat{H}_{P(0)}^*} & E\{\hat{H}_{P(1)}\hat{H}_{P(1)}^*} & \dots & E\{\hat{H}_{P(1)}\hat{H}_{P(N_p-1)}^*} \\ \vdots & \vdots & \ddots & \vdots \\ E\{\hat{H}_{P(N_p-1)}\hat{H}_{P(0)}^*} & E\{\hat{H}_{P(N_p-1)}\hat{H}_{P(1)}^*} & \dots & E\{\hat{H}_{P(N_p-1)}\hat{H}_{P(N_p-1)}^*} \end{bmatrix} \quad (4)$$

$$r = \begin{bmatrix} E\{\hat{H}_{P(0)}\hat{H}_k^*} \\ E\{\hat{H}_{P(1)}\hat{H}_k^*} \\ \vdots \\ E\{\hat{H}_{P(N_p-1)}\hat{H}_k^*} \end{bmatrix} \quad (5)$$

where  $E\{\cdot\}$  denotes the expectation operator and  $(\cdot)^*$  denotes the conjugate operator. All components of the above matrices can be given by:

$$E\{\hat{H}(t, f)\hat{H}(t + \Delta t, f + \Delta f)^*\} = R(\Delta t, \Delta f) = R(\Delta t) \cdot R(\Delta f) \quad (6)$$

where  $\hat{H}_{P(k)}$  ( $k = 0, 1, \dots, N_p - 1$ ) is the CTF value at the  $k$ -th pilot sub-carrier position. The LMMSE estimate of CTF corresponding to  $l$ -th OFDM symbol at  $k$ -th sub-carrier  $\hat{H}(l, k)_{LMMSE}$  is expressed as:

$$\hat{H}(l, k)_{LMMSE} = \left( \left( R_h + \frac{\beta}{SNR} \mathbf{I} \right)^{-1} \cdot r \right)^H \cdot \hat{H}_{PLS} \quad (9)$$

where  $\hat{H}_{PLS}$  is a vector containing LS estimates of channel at pilot sub-carriers and  $\beta = E|X_k|^2 \cdot E|1/X_k|^2$  is a constant depending on the signal constellation. For BPSK and QPSK  $\beta = 1$  and for 16-QAM  $\beta = 17/9$ . The signal-to-noise ratio SNR is given by  $SNR = E|X_k|^2 / \sigma_w^2$ , where  $\sigma_w^2$  is the AWGN variance. If only one-dimensional (1D) estimation (in frequency domain) is considered, the spaced-time correlation is equal to 1 ( $\Delta t = 0$ ), so the calculation of  $R_h$  and  $r$  becomes easier. The number ( $N_p$ ) of pilots in an OFDM block used to determine the autocorrelation matrix has a huge impact on the performance of the estimator and its computational complexity. The reduced number of elements in  $R_h$  and  $r$  can be obtained by taking only a fixed number of pilots closest to the estimated sub-carrier. A lower number of neighboring pilots used in the calculation of the estimation leads to faster processing.

IV. TURBO CHANNEL ESTIMATION APPROACH

In order to maximize the performance of channel estimation, a turbo iterative estimation structure and algorithm are proposed. This approach assumes feeding an additional information from the decoder back to the channel estimation block in every iteration of the turbo estimation process. The block diagram of this method is presented in Fig. 1.

When a single OFDM block is received and transformed to frequency domain (FFT), the set of  $N_p$  pilots is extracted from it according to known structure of the OFDM block. Those pilots are input data for the channel estimation block, which performs Wiener filtering and gives the current channel estimate on its output. In the next stage, the OFDM block is equalized and the QAM symbols are soft-demapped to the code symbols represented in the form of binary image of GF elements. In order to proceed with demapping, the LLR values are calculated according to the following equation:

$$a_i = -\frac{1}{\sigma_w^2} |y - h \cdot \mu(\alpha_i)|^2 \quad (8)$$

where  $a_i$  denotes the LLR corresponding to demapping of QAM symbol  $y$  to a binary image of the GF element  $\alpha_i$  ( $i = 0, 1, \dots, q-1$ ), and  $h$  determines the CTF value for the currently processed data sub-carrier within the OFDM block. Equation (8) applies only to 64-QAM modulation, where the mapping function  $\mu(\alpha_i)$  transforms exactly one code symbol to one QAM symbol. Transformations for other types of modulation are discussed in [13]. The computed vector of  $q$  LLR values for each single code symbol determine the probabilities of correct decoding of this code symbol. Relying on this soft-decision criterion, a fixed number ( $N_v$ ) of virtual pilots' positions (i.e. sub-carriers indices) is deduced and exploited in the turbo channel estimation process. The indices of virtual pilots can be determined in every single iteration of the estimation. After each decoding, the binary data obtained are re-mapped (using the  $\mu(\alpha_i)$  mapping function) to adequate constellation (i.e. 64-QAM) and then fed to the channel estimation block. With a wider set of pilots (including original and virtual ones) the LMMSE estimator can provide a more accurate estimation of the channel at particular sub-carriers, thus reducing the decoding error rate.

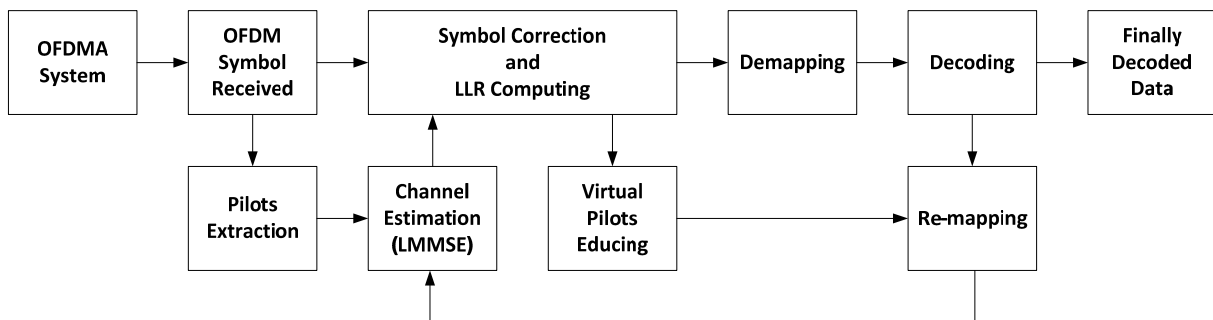


Fig. 1 Block diagram of turbo iterative channel estimation.

Additionally, to increase the accuracy of estimation 2D Wiener filtering was applied in the turbo estimation algorithm, where the reference data for filtering (pilots and virtual pilots) were taken from the current and previous OFDM blocks. Due to it an additional channel state information (CSI) could be explored.

V. SIMULATIONS AND RESULTS

Simulations are based on the WiMAX platform with the following system parameters:

- number of sub-carriers in the OFDM symbol  $N_C = 2048$ , in which there are 1680 data sub-carriers and 240 pilot sub-carriers;
- system bandwidth  $B = 20$  MHz;
- sampling frequency  $f_s = 22.4$  MHz;
- sub-carrier spacing  $\Delta f = f_s / N_C = 10.9375$  kHz;
- carrier frequency  $f_c = 2.5$  GHz.

Pilot spacing pattern is adopted from IEEE 802.16e standard (OFDMA DL PUSC mode), where the pilot allocation scheme is different for odd and even OFDM symbols. The FEC technique exploiting NB-LDPC codes is applied, with Galois field size equal to  $q = 64$ . The length of the codeword is 96 Galois symbols and the code rate is set to  $R = 0.5$ . The results are achieved by simulations of data transmission through the ITU Vehicular A (modified) mobile channel reflecting the vehicle speed of 30 km/h (8.33 m/s). A LMMSE estimation of channel is obtained using 30 pilots closest to the currently estimated sub-carrier (30-pilot window), offering a trade-off between the performance and computational complexity.

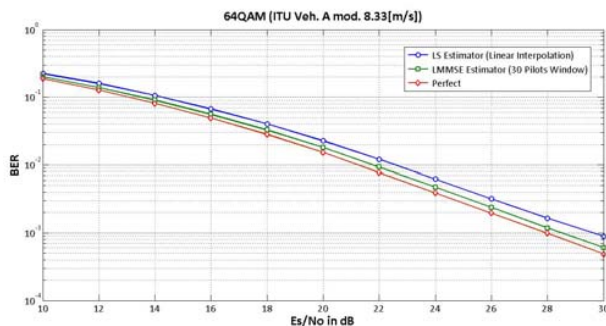


Fig. 2 Bit Error Rate for 64QAM and mobile channel (vehicle speed 30 km/h).

Fig. 2 presents the value of BER as a function of  $E_s/N_0$  (SNR) for both types of estimators and the 64-QAM modulation scheme. Results show that the LMMSE estimator can provide better performance in the whole range of SNR values. The performance difference between LMMSE and LS estimators is slightly increasing for higher SNR values. The channel estimation accuracy of both tested techniques is compared each other in Fig. 3. In order to emphasize the difference between the LS and LMMSE, the pilots affected by the noise are presented as well.

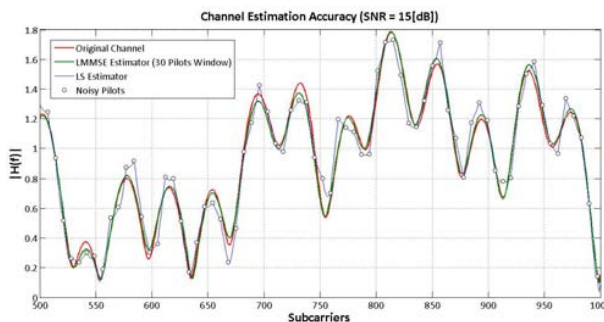


Fig. 3 Comparison of channel estimation accuracy for SNR = 15 dB.

During the simulations of the turbo iterative estimation algorithm, the number of additional virtual pilots was equal to  $N_V = 3N_p$ . The word error rate was measured for different numbers of additional iterations (up to 6). Fig. 4 shows that after the first iteration there is already a slight gain of 0.1-0.2 dB in the transmission performance (Fig. 4). Further iterations lead to a slightly larger gain, which is increasing in the higher range of SNR values.

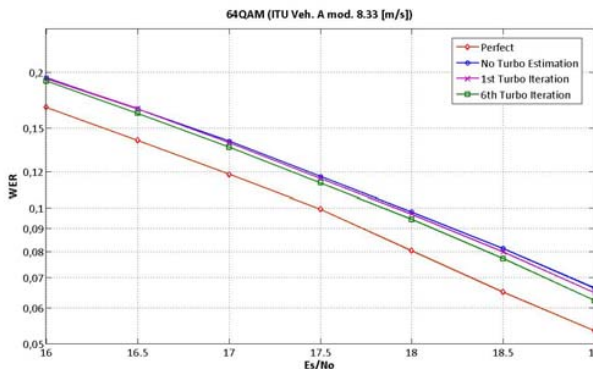


Fig. 4 Word Error Rate for the turbo iterative channel estimation.

VI. CONCLUSIONS

The LMMSE estimator outperforms the simpler LS; for  $BER = 10^{-3}$  the gain is over 1 dB. Moreover the LMMSE is a better solution for the presented turbo algorithm, because this kind of filtering is more proof against hard-decision errors of the decoder.

The simulation results show that the turbo channel estimation algorithm improves the performance of channel estimation. Introducing additional reference data in the shape of virtual pilots may be used to decrease the number of original pilot sub-carriers and thus expand the spectral efficiency of the whole system. Performing the first few iterations of the turbo algorithm can lead to an additional gain of about 0.3 dB and in consequence to a better word error rate. Additional simulations of channel estimation performed in two dimensions (2D Wiener filtering) showed the potential of the presented turbo approach. This modification allowed increasing the gain up to 0.5 dB.

It is important to notice that in practical implementation of

turbo channel estimation, the number of iterations has to be limited. Too many iterations can lead to overall delay in the transmission chain.

## REFERENCES

- [1] LTE Document, "3GPP; Technical Specification Group Radio Access Network; Evolved Universal Terrestrial Radio Access (E-UTRA); Physical Channels and Modulation (Release 8)", 3GPP TS 36.211 V8.5.0 (2008-12).
- [2] M. Davey and D. MacKay, "Low density parity check codes over GF(q)," *IEEE Commun. Lett.*, vol. 2, no. 6, pp. 165–167, June 1998.
- [3] C. Douillard, M. Jézéquel, C. Berrou, A. Picart, P. Didier, A. Glavieux, "Iterative correction of intersymbol interference: Turbo-equalization," *Eur. Trans. Commun.*, vol. 6, no. 5, pp. 507-511, Sept.-Oct. 1995.
- [4] M. Valenti and B. Woerner, "Iterative channel estimation and decoding of pilot symbol assisted turbo codes over flat-fading channels," *IEEE Select. Areas Commun.*, vol. 19, no. 9, pp. 1697-1705, Sept. 2001.
- [5] M. Flanagan and A. Fagan, "Iterative channel estimation, equalization, and decoding for pilot-symbol assisted modulation over frequency selective fast fading channels," *IEEE Trans. Veh. Technol.*, vol. 56, no. 4, pp. 1661-1670, July 2007.
- [6] X. Nguyen and J. Choi, "Iterative channel estimation and turbo decoding in OFDM systems," in *Proc. IEEE Veh. Technol. Conf.-Fall, Dallas, TX*, pp. 166-170, Sept. 2005.
- [7] Y.-N. Lee, A. Ashikhmin, and J.-T. Chen, "Impact of soft channel construction on iterative channel estimation and data decoding for multicarrier systems," *IEEE Trans. Wireless Commun.*, vol. 7, no. 7, pp. 2762-2760, July 2008.
- [8] R. Otnes and M. Tüchler, "Iterative channel estimation for turbo equalization of time-varying frequency-selective channels," *IEEE Trans. Wireless Commun.*, vol. 3, no. 6, pp. 1918-1923, Nov. 2004.
- [9] A. Voicila, D. Declercq, F. Verdier, M. Fossorier, P. Urard, "Low-complexity, low-memory EMS algorithm for non-binary LDPC codes," in *Proc. IEEE Int. Conf. Commun. (ICC), Glasgow, UK, June 2007*, pp. 671–676.
- [10] M. S. Akram, "Pilot-based Channel Estimation in OFDM Systems," Master Thesis, Nokia Mobile Phones, 2007.
- [11] W. Zhou, W. H. Lam, "A Fast LMMSE Channel Estimation Method for OFDM Systems," Department of Electrical and Electronics Engineering, The University of Hong Kong, Hong Kong, 2009.
- [12] Henryk Gierszal, Witold Hołubowicz, Łukasz Kiedrowski, Adam Flizikowski, "Non-Binary LDPC Coding for IMT-Advanced System," 4th International Conference on Broadband Communication, Information Technology & Biomedical Applications, BroadBandCom'09, 15-18 July 2009, Wrocław, ID 197
- [13] "IEEE 802.16m Evaluation Methodology Document (EMD)," 2008-07-03.