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Nonlinear Simulation of Harmonically Coupled Two-Beam Free-Electron Laser

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***Abstract*—**A nonlinear model of two-beam free-electron laser (FEL) in the absence of slippage is presented. The two beams are assumed to be cold with different energies and the fundamental resonance of the higher energy beam is at the third harmonic of lower energy beam. By using Maxwell's equations and full Lorentz force equations of motion for the electron beams, coupled differential equations are derived and solved numerically by the fourth order Runge–Kutta method. In this method a considerable growth of third harmonic electromagnetic field in the XUV and X-ray regions is predicted.

***Keywords*—**Free-electron laser, Higher energy beam, Lower energy beam, Two-beam

I. Introduction

NOWADAYS, there are considerable interests in the production of coherent high power short wavelength radiation in X-ray regions. The interest in X-ray free-electron laser has fueled intense interest in infrared-visible- and ultraviolet-wavelength experiments to test the basic physics and technology [1]. The advantage of X-ray FELs over high-power optical lasers is that, because of the much shorter wavelength, the diffraction limit is three orders of magnitude lower, allowing the laser to be focused to a much smaller spot-size (of the order of 1 nm) [2]. So FEL is required to generate coherent short-wavelength radiation in X-ray region. To reach this aim, injecting a resonant coherent seed field at the beginning of the FEL interaction which is significantly amplified over self-amplified spontaneous emission (SASE) is proposed. However the sources for seeding the FEL to produce high power short wavelength in the XUV and X-ray regions are not currently available so other methods are proposed to solve this problem. One way to achieve this is to use multiple injectors and combine the beams by either energy or phase stacking techniques. Initial simulations of this concept have been applied to the proposed design of the $1.5\,\mathring{A}$ Linac Coherent Light Source (LCLS) and indicate that it is a feasible procedure for such a fourth generation X-ray light source [3]. An alternative method, proposed in two-beam FEL of [6], which uses two electron beams in one-dimensional limit, has improved the output coherence of the injected seed field and has opened a new way to FEL researchers for generating coherent radiations in XUV and X-ray regions of spectrum. The mentioned paper is based on the averaged form of equations while the evolution of radiation wave number is not considered. The purpose of this study is to present a non-averaged simulation of a two-beam FEL in 1D Compton limit by using Maxwell's equations and full Lorentz force equations of motion for the electron beams. In the computational part, we have used the simulation technique of [4].

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II. The Model

Here, the same model like the one in [6] is used. First, the model is reviewed and then non-averaged equations of radiation fields and electron beams of motion will be defined. In this model two beams of electron with different energies copropagating through a magnetic planar wiggler, are considered. The electron beam energies are chosen so that the third harmonic of the lower energy beam is at the fundamental resonance of the higher energy beam. As a result of the interaction of the fundamental resonance field of the lower energy electron beam with the electrons, the electron beam is bunched [7]. When the bunching is strong enough, higher harmonics are generated. The result of this non-linear harmonic generation can be the seed for copropagating higher energy electron beam. So the seeding of the higher energy electron beam is not necessary. The coupling of the lower and higher energy beam FEL interactions will lead to the transferral properties of the longer wavelength injected seed field to the unseeded shorter harmonic wavelength. The lower energy electron beam has a Lorentz factor of γ_1 and the higher energy γ_n. It can be shown from the resonance relation that $\gamma_3 = \sqrt{3}\gamma_1$ [6]. It should be noted that the two-stream instability as another result of coupling of lower and higher energy beams has the potential of degrading beam quality. However it can be shown that the instability is either below threshold or has an insignificant effect for electron beam currents ($\geq 1kA$) and energies ($\geq 500Mev$) typical to those used in the FEL interactions presented here [6].

III. Field Equations

In this section we define the field equations which are derived from Maxwell's equations in 1D Compton limit. The tapered and planar wiggler magnetic fields are described as below:

$$\mathbf{B}_w(z) = B_w(z)\sin(k_w z)\hat{\mathbf{e}}_y, \quad (1)$$

$$B_w(z)=\begin{cases} B_w \sin^2\left(\dfrac{k_w z}{4N_w}\right) & 0 \le z \le N_w\lambda_w \\ B_w & N_w\lambda_w < z \end{cases},$$

where $B_w(z)$ refers to the wiggler amplitude and $k_w = 2\pi/\lambda_w$ is the wiggler wave number. As discussed in previous papers, only the odd harmonic components with $h = 1,3,5,...$ are considered [7]. If the third harmonic of the lower energy beam is at the fundamental resonance of the higher energy beam, the strongest coupling occurs in $h = n = 3$.The coupling of the higher harmonics are neglected, as their frequencies are not equal to any fundamental resonance. The vector potential and phase of radiation fields are written as

$$\delta\mathbf{A}_1(z,t) = \delta A_1(z)\cos\alpha_{+1}\hat{\mathbf{e}}_x, \tag{2}$$

$$\delta\mathbf{A}_3(z,t) = \delta A_3(z)\cos\alpha_{+3}\hat{\mathbf{e}}_x, \tag{3}$$

$$\alpha_{+1} = \int_0^z k_{+1}(z')dz' - \omega t, \tag{4}$$

$$\alpha_{+3} = \int_0^z k_{+3}(z')dz' - 3\omega t, \tag{5}$$

$$\frac{d\alpha_{+1}}{d\bar{z}} = \bar{k}_{+1} - \frac{\omega\gamma_1}{u_{z(low)}}, \tag{6}$$

$$\left.\frac{d\alpha_{+3}}{d\bar{z}}\right|_{low} = \bar{k}_{+3} - \frac{\omega\gamma_1}{u_{z(low)}}, \tag{7}$$

$$\frac{d\phi_{+1}}{d\bar{z}} = \left.\frac{d\alpha_{+3}}{d\bar{z}}\right|_{high} = \bar{k}_{+3} - \frac{\omega\gamma_3}{u_{z(high)}}, \tag{8}$$

where $u_{z(low)}$ and $u_{z(high)}$ are the axial velocities of each electron in lower and higher energy beams. Besides, the amplitudes and wave numbers of the radiation are assumed to vary slowly with z.

In the Coulomb gauge, the Maxwell-Poisson equations are of the form [9]

$$\left(\frac{\partial^2}{\partial\bar{z}^2} - \frac{\partial^2}{\partial\bar{t}^2}\right)\left(\sum_{h,odd}^{n=3}\delta a_h\right) = -4\pi\delta J_x(\bar{z},\bar{t}), \tag{9}$$

where $\delta a_h = e\delta A_h/mc^2$, $\bar{z} = k_w z$, $\bar{t} = tk_w c$ and $\delta J_x(\bar{z},\bar{t})$ is the current density. The current density can be written as an average over the entry time $\bar{t}_0$ (the time at which an electron crosses the $\bar{z} = 0$ plane) [9].

$$\delta\mathbf{J}(\bar{z},\bar{t}) = \frac{-\bar{\omega}_b^{\,2}\beta_{z0}}{4\pi}\int_{-\infty}^{+\infty}\frac{d\bar{t}_0\,\sigma(\bar{t}_0)\boldsymbol{\beta}(\bar{t},\bar{t}_0)\delta\left[\bar{t}-\tau(\bar{z},\bar{t}_0)\right]}{\left|\beta_z(\bar{t},\bar{t}_0)\right|}, \tag{10}$$

where $\bar{\omega}_b^{\,2} = 4\pi n_b e^2/mk_w^{\,2}c^2$ and $\boldsymbol{\beta}(\bar{t},\bar{t}_0)$ is the velocity of each electron at time $\bar{t}$ which has entered the plane at time $\bar{t}_0$ and $\sigma(\bar{t}_0)$ is the distribution in entry times and

$$\tau(\bar{z},\bar{t}_0) = \bar{t}_0 + \int_0^{\bar{z}}\beta_z^{-1}(\bar{z}',\bar{t}_0)\,d\bar{z}'. \tag{11}$$

The system is assumed to be quasistatic, so that particles which enter the interaction region at time $\bar{t}_0$ separated by integral multiples of a wave period will execute identical orbits [8]. By substitution of (2) and (3) into (9) a set of coupled nonlinear differential equations are derived for δa_h , k_{+h} and Γ_{+h} , where Γ_{+h} defines the growth rate. For the fundamental radiation, the equations are

$$\frac{d\delta a_1}{d\bar{z}} = \bar{\Gamma}_{+1}\delta a_1, \tag{12}$$

$$\frac{d\bar{\Gamma}_{+1}}{d\bar{z}} = -\bar{w}^2 - \bar{\Gamma}_{+1}^{\,2} + \bar{k}_{+1}^{\,2} + 2\bar{\omega}_b^{\,2}\beta_{z0}\left\langle\frac{u_{x(low)}\cos\alpha_{+1}}{\left|u_{z(low)}\right|}\right\rangle\Big/\delta a_1, \tag{13}$$

$$\frac{d\bar{k}_{+1}}{d\bar{z}} = -2\bar{k}_{+1}\bar{\Gamma}_{+1} - 2\bar{\omega}_b^{\,2}\beta_{z0(low)}\left\langle\frac{u_{x(low)}\sin\alpha_{+1}}{\left|u_{z(low)}\right|}\right\rangle\Big/\delta a_1, \tag{14}$$

and the equations for the third harmonic are

$$\frac{d\delta a_3}{d\bar{z}} = \bar{\Gamma}_{+3}\delta a_3, \tag{15}$$

$$\frac{d\bar{\Gamma}_{+3}}{d\bar{z}} = -(3\bar{w})^2 - \bar{\Gamma}_{+3}^{\,2} + \bar{k}_{+3}^{\,2} + 2\bar{\omega}_b^{\,2}\beta_{z0(low)}\left\langle\frac{u_{x(low)}\cos\alpha_{+3}}{\left|u_{z(low)}\right|}\right\rangle\Big/\delta a_3 + 2\bar{\omega}_{b3}^{\,2}\beta_{z0(high)}\left\langle\frac{u_{x(high)}\cos\varphi_{+1}}{\left|u_{z(high)}\right|}\right\rangle\Big/\delta a_3, \tag{16}$$

$$\frac{d\bar{k}_{+3}}{d\bar{z}} = -2\bar{k}_{+3}\bar{\Gamma}_{+3} - 2\bar{\omega}_b^{\,2}\beta_{z0(low)}\left\langle\frac{u_{x(low)}\sin\alpha_{+3}}{\left|u_{z(low)}\right|}\right\rangle\Big/\delta a_3 - 2\bar{\omega}_{b3}^{\,2}\beta_{z0(high)}\left\langle\frac{u_{x(high)}\sin\varphi_{+1}}{\left|u_{z(high)}\right|}\right\rangle\Big/\delta a_3, \tag{17}$$

where $\psi_0 = -\bar{\omega}\bar{t}_0$ is an initial pondermotive phase and the averaging operator is defined as

$$\langle(...)\rangle = \frac{1}{2\pi}\int_{-\pi}^{+\pi} d\psi_0 \sigma(\psi_0)(...) . \quad (18)$$

In (10)-(17), β_{z0} is the initial axial velocity of electrons. Observe that the electron beams are mono-energetic with a vanishing pitch-angle spread and it is assumed that the electrons of each beam have the same initial axial velocities. Current densities of electron beams are different so we need to specify the beam current ratio, $R_3 = I_3/I_1$. Since the current densities are not equal, the electron beam plasma frequencies of electron beams will be related by $\bar{\omega}_{b3} = \bar{\omega}_b\sqrt{R_3\,\gamma_3(\gamma_1^2-1)/\gamma_1(\gamma_3^2-1)}$. In (12)-(14), the coupling of the lower energy electrons to fundamental and higher harmonic fields is seen. However, (15)-(17) indicate that the higher energy electrons only couple to the harmonic field A_3.

IV. Dynamic Equations

In order to complete the formulation, the electron orbit equations should be considered in the presence of fluctuating fields. The Lorentz force equation with the electric and magnetic fields associated with the vector potentials are used to advance the electrons in z. In the below equations we have used dimensionless variables $\bar{\Omega}_w = eB_w/mk_wc^2$, $\mathbf{u} = \mathbf{P}/mc = \gamma\boldsymbol{\beta}$ and $\bar{k}_+ = k_+/k_w$. For the slower electrons of the lower energy beam, the equations are of the form

$$\frac{du_{x(low)}}{d\bar{z}} = \bar{\Omega}_w(\bar{z})\sin(\bar{z}) + \delta a_1\left\{\frac{\gamma_1\bar{w}}{u_{z(low)}} - \bar{k}_{+1}\right\}\sin\alpha_{+1} + \frac{d\delta a_1}{d\bar{z}}\cos\alpha_{+1} + \delta a_3\left\{\frac{\gamma_1(3\bar{w})}{u_{z(low)}} - \bar{k}_{+3}\right\}\sin\alpha_{+3} + \frac{d\delta a_3}{d\bar{z}}\cos\alpha_{+3} , \quad (19)$$

$$\frac{du_{y(low)}}{d\bar{z}} = 0 , \quad (20)$$

$$\frac{du_{z(low)}}{d\bar{z}} = -\bar{\Omega}_w(\bar{z})\sin(\bar{z})\frac{u_{x(low)}}{u_{z(low)}} - \frac{u_{x(low)}}{u_{z(low)}}\left\{\frac{d\delta a_1}{d\bar{z}}\cos\alpha_{+1} - \bar{k}_{+1}\delta a_1\sin\alpha_{+1}\right\} - \frac{u_{x(low)}}{u_{z(low)}}\left\{\frac{d\delta a_3}{d\bar{z}}\cos\alpha_{+3} - \bar{k}_{+3}\delta a_3\sin\alpha_{+3}\right\} , \quad (21)$$

and for the faster electrons which belong to the higher energy beam, the equations are as the followings

$$\frac{du_{x(high)}}{d\bar{z}} = \bar{\Omega}_w(\bar{z})\sin(\bar{z}) + \delta a_3\left\{\frac{\gamma_3(3\bar{w})}{u_{z(high)}} - \bar{k}_{+3}\right\}\sin\varphi_{+1} + \frac{d\delta a_3}{d\bar{z}}\cos\varphi_{+1} , \quad (22)$$

$$\frac{du_{y(high)}}{d\bar{z}} = 0 , \quad (23)$$

$$\frac{du_{z(high)}}{d\bar{z}} = -\bar{\Omega}_w(\bar{z})\sin(\bar{z})\frac{u_{x(high)}}{u_{z(high)}} - \frac{u_{x(high)}}{u_{z(high)}}\left\{\frac{d\delta a_3}{d\bar{z}}\cos\varphi_{+1} - \bar{k}_{+3}\delta a_3\sin\varphi_{+1}\right\} . \quad (24)$$

In (19)-(21), the coupling of the lower energy electrons to both fundamental and harmonic fields is seen. However, (22)-(24) indicates that the higher energy electrons only couple to the harmonic field A_3.

V. Numerical Simulation

In this section, a set of self-consistent first-order differential equations are solved numerically by the fourth order Runge–Kutta method in order to demonstrate the evolution of the coupled two-beam FEL system. The initial imposed conditions on electron beams are chosen such that the electrons are uniformly distributed for $-\pi \le \psi_0 \le \pi$ [4]. Since the steady-state amplifier model is considered, the initial amplitude of the vector potential for an un-bunched beam can be selected arbitrarily to represent the amplitude of the injected signal [5]. The seed field at the longer wavelength is $\delta a_1(\bar{z}=0) = 10^{-7}$ which is two orders of magnitude greater than that of the third harmonic. As mentioned before the shorter wavelength harmonic field is seeded when the lower energy beam is bunched. In this process the harmonic seed retains the coherence properties of the initial radiation seed field. Other parameters which are used in this paper are $\bar{\Omega}_w/\gamma = 0.05$, $\bar{\omega}_b^{\,2}/\gamma = 0.1$ and $N_w = 10$.

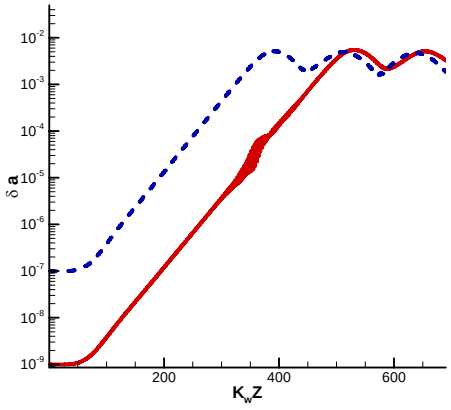

Fig. 1 Evolution of fundamental (dashed line) and third harmonic (solid line) radiation amplitudes as a function of k_wz for $n=3$ and $R_3=5$

In Fig. 1 the evolution of fundamental and third harmonic radiation amplitude with k_wz is plotted. The third harmonic is rapidly amplified by about two orders of magnitude more than

the fundamental until it reaches a saturation point. It can be expected that harmonic bunching of the lower energy beam which leads to the amplification of δa_3, retains the coherence properties of initial radiation seed at the fundamental. After the process of seeding, the harmonic radiation field is amplified exponentially until the saturation occurs.

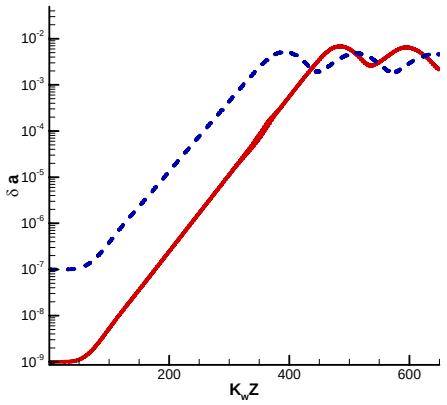

Fig. 2 Evolution of fundamental (dashed line) and third harmonic (solid line) radiation amplitudes as a function of $k_w z$ for $n = 3$ and $R_3 = 7$

In Fig. 2, the evolution of fundamental and third harmonic radiation amplitude with $k_w z$ is plotted for $R_3 = 7$. It is seen that the third harmonic radiation is increased more than the fundamental and the saturation length of shorter wavelength radiation is decreased.

VI. Conclusion

A non-averaged simulation of Compton FEL with planar wiggler is presented in the absence of slippage. By using two-beam FEL, a source for generating intensified higher harmonics with shorter wavelengths in the XUV and X-ray regions, was obtained. In this method, coherent properties of the injected seed field are transferred to the shorter wavelength radiation as a result of the interaction of the two beams. It is important to note that in the present study, Maxwell's and full Lorentz force equations are used and the variation of radiation wave number is included.

References

[1] P. G. O'Shea and H. P. Freund, "Free-Electron Lasers: Status and Applications," *SCIENCE*, vol. 292, pp. 1853-1858, June 2001.
[2] S. Krishnagopal , V. Kumar, S. Maiti , S. S. Prabhu and S. K. Sarkar, "Free-electron lasers," *Curr. Sci.*, vol. 87, pp. 1066-1078, October 2004.
[3] H. P. Freund, D. Douglas, P.G. O'Shea, "Multiple-beam free-electron lasers ," *Nucl. Instr. Meth. Phys. Res. A*, vol. 507, pp. 373–377, 2003.
[4] M. H. Rouhani and B. Maraghechi, "Efficiency enhancement in a single-pass Raman free electron laser," *Phys. Plasmas* ,vol. 16, 093110, September 2009.
[5] B. Maraghechi and M. H. Rouhani, "Simulation Of Raman Free-Electron Laser Amplifier With Planar Wiggler And Ion Channel Guiding," in *2009 Proc. FEL09 Conf.* , Liverpool, pp. 91-94.
[6] B. W. J. McNeil and G. R. M. Robb, "Two-beam free-electron laser," *Phys. Rev. E*, vol. 70, 035501, 2004.
[7] M. H. Rouhani, B. Maraghechi, and H. Saberi, "Efficiency enhancement of a two-beam free-electron laser," *Phys. Plasmas*, vol. 16, 123105, December 2009.
[8] H. P. Freund, "Nonlinear analysis of free-electron-laser amplifiers with axial guide fields," *Phys. Rev. A*, vol. 27, pp. 1977-1988. April 1983.
[9] H. P. Freund and J. M. Antonsen, *Principles of Free Electron Laser*, London: Chapman and Hall, 1996, pp. 153-244.