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Quantification of Periodicities in Fugitive Emission of Gases from Lyari Waterway

Rana Khalid Naeem, and Asif Mansoor

***Abstract*—**Periodicities in the environmetric time series can be idyllically assessed by utilizing periodic models. In this communication fugitive emission of gases from open sewer channel Lyari which follows periodic behaviour are approximated by employing periodic autoregressive model of order *p*. The orders of periodic model for each season are selected through the examination of periodic partial autocorrelation or information criteria. The parameters for the selected order of season are estimated individually for each emitted air toxin. Subsequently, adequacies of fitted models are established by examining the properties of the residual for each season. These models are beneficial for schemer and administrative bodies for the improvement of implemented policies to surmount future environmental problems.

***Keywords*—**Exchange of Gases, Goodness of Fit, Open Sewer Channel, PAR(*p*) Models, Periodicities, Season Wise Models.

I. Introduction

SOME environmetric processes generally have seasonal mean, variance, skewness and serial dependence structure. Normally, to model such procedures first of all seasonality is removed by initially subtracting the seasonal mean and then dividing with the seasonal standard deviation. By such procedure seasonality in the mean and variance is removed, however the seasonality in serial dependence structure remains. Such seasonality can be appropriately quantified by the use of periodic models.

In this communication a periodic autoregressive (PAR) model which extends a non-periodic autoregressive model by allowing the autoregressive parameter to vary with the season is employed to quantify the periodicities in the gases exchanges by the industrial and sewage waste flowing through river Lyari. The order of PAR model for each season are selected by the use of information criteria i.e. AIC propose by Akiake [1]-[2] or the BIC of Schwarz [3] and by finding the lowest lag for which the sample periodic partial autocorrelation function cuts off 95% confidence limits. The parameters of the selected order i.e. PAR(*p*) are estimated by using the periodic Yule-Walker equations for each season. Consequently, sufficiency of proposed model is ascertained by testing whiteness, heteroscedasticity and normality in the residual for each season.

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II. Periodic AR(*p*) Model

Consider a time series having *s* seasons per year (*s = 12* for monthly data) over a period of *n* years, let $z_{t,m}$, in which $t = 1,2,3,\ldots,n$ and $m = 1,2,3,\ldots,s$ represent a time series observation in the *t*th year and *m*th season. Then PAR model of order $(p_1, p_2, \ldots, p_s)$ for season *m* is represented as

$$z_{t,m} - \mu_m = \sum_{i=1}^{p_m} \phi_{i,m}(z_{t,m-1} - \mu_{m-i}) + \varepsilon_{t,m} \tag{1}$$

where μ_m is the mean for *m*th season of the series $z_{t,m}$ for the *m*th season, $\phi_{i,m}$ is the autoregressive coefficient for season *m* and *i*th lag, and $\varepsilon_{t,m}$ is the innovation disturbance. The innovation series $\varepsilon_{t,m}$ is assumed to have an expected value of zero and covariance defined by

$$\operatorname{cov}(\varepsilon_{t,m}, \varepsilon_{t,m-i}) = \begin{cases} \sigma_m^2, & i = 0 \\ 0, & i \neq 0 \quad \text{for } i = 1,2,\ldots,s \end{cases} \tag{2}$$

the disturbance $\varepsilon_{t,m}$ are distributed as IID($0, \sigma_m^2$).

III. Identification of PAR Order

The suitable PAR model can be selected either by examining the plots of sample periodic partial autocorrelation (*PePACF*) or by using Akiake (*AIC*) or Schwarz (*BIC*) information criterion. In this study both *PePACF* or information criterion has been used for the identification of PAR order. The sample periodic autocorrelation function (*PeACF*) at lag *k* for season *m*, AIC and BIC for the overall periodic autoregressive model are determined by utilizing (3), (4) and (5) respectively, whereas Sakai's [4] algorithms has been used for the computation of *PePACF*.

$$PeACF = \frac{\sum_{t=1}^{n}(z_{t,m} - \hat{\mu}_m)(z_{t,m-k} - \hat{\mu}_{m-k})}{\sqrt{\left[\sum_{t=1}^{n}(z_{t,m} - \hat{\mu}_m)^2\right]\left[\sum_{t=1}^{n}(z_{t,m-k} - \hat{\mu}_{m-k})^2\right]}} \tag{3}$$

$$AIC = \sum_{m=1}^{s} AIC_m + 2 \tag{4}$$

$$BIC = n \ln \hat{\sigma}_m^2 + \ln(n) p_m \tag{5}$$

where

$$\hat{\mu}_m = \frac{1}{n}\sum_{t=1}^{n} z_{t,m}$$

$$AIC_m = 2\left(n\ln(\hat{\sigma}_m) + \sum_{t=1}^{n} z_{t,m} + p_m + 2 \right)$$

p_m = Number of AR parameter in season *m*.

IV. Estimation of Par(p) Parameters

The autoregressive coefficients $\phi_{i,m}$ denote the vector of autoregressive parameters for season *m*. The asymptotically efficient estimates of $\hat{\phi}_{i,m}$ is obtained by utilizing Yule-Walker type equation [5].

$$\sum_{i=1}^{p_m} \hat{\phi}_{i,m} c_{k-i,m-i} = c_{k,m} \qquad k = 1,2,3,..., p_m \tag{6}$$

where $c_{k,m} = E[(z_{t,m} - \hat{\mu}_m)(z_{t,m-k} - \hat{\mu}_{m-k})]$. The residual variances are estimated by

$$\hat{\sigma}_m^2 = c_{0,m} - \hat{\phi}_{1,m} c_{1,m} - .... - \hat{\phi}_{p_m} c_{p_m,m} \quad m = 1,2,..., s \tag{7}$$

The data set obtained by $\sqrt{n(\hat{\phi}_{i,m} - \phi_{i,m})}$ is asymptotically normal with zero mean and covariance matrix $\frac{1}{n} I_m^{-1}$, where

$$I_m = \frac{1}{\sigma_m^2}\left(\gamma_{i-j,m-j}\right) \tag{8}$$

where $\gamma_{i-j,m-j} = E[(z_{t,m-j} - \mu_{m-j})(z_{t,m-i+j} - \mu_{m-i+j})]$ and j is the highest autoregressive order . The estimates of $\hat{I}_m$ is obtained by replacing $\gamma' s$ with $c' s$ .The PAR parameters are estimated independently for each seasons by utilizing this technique.

Diagnostic Checks

The analysis of residuals is carried out to check randomness, white noise and normality in the residual of fitted model of each season. Portmanteau test [6] is used to check that all the residual at lag 1,2,…,*L* are equal to zero for specified period *m*.

V. Application of Model to Emission of Gases From Lyari Waterways

Karachi, the provisional capital of Sindh is located at the extreme west end of the Indus delta between north latitude 24^{o}51$^/$ and east longitude 67^{o}4$^/$.

The river Lyari is one of the three rivers along with Malir and Hub river through the greater metropolitan area of Karachi. Lyari river becomes a perturbed and toxic channel when it enters into the metropolitan area. It carries the water that is purely combination of domestic sewage and industrial effluents. These effluents have very high load of pollutants which debauches into Arabian Sea [7]-[13]. This grimy water emits significant amount of polluted gases that are Chlorine (*Cl*), SOx($SO+SO_2$), COx($CO+CO_2$), NOx($NO+NO_2$), ammonia (NH_3) and volatile organic carbons (*VOCs*) in atmosphere which causes serious environmental and health impact on the nearby living organisms.

The data is obtained from daily exchange of gases at the end of Lyari river where it drains its effluents into costal water. The air exchange pollutants SOx, NOx, *VOCs* and NH_3 are measured in *ppb* whereas *Cl* and COx are measured in *ppm*. The measured pollutants are averaged over each of the respective month to obtain monthly series.

The classical descriptive statistics depicted in Table I are computed form the long term data for exchange of gases to acquire the prelude knowledge of air toxin. Table-I indicates that coefficients of variation signify the deviation of air pollutant over time which varies due to change in concentration of effluents in the watercourse or weather condition. The coefficient of skewness and kurtosis are also indicating the variation in symmetry and flatness of the probability density function of air exchange pollutants. These variations in the basic statistics are indicating non-stationarity of the air contaminant.

TABLE I
Descriptive Statistics of Emitted Gases from Lyari Channel

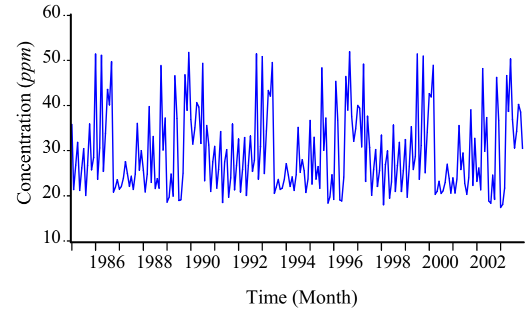
Descriptive Statistics	Pollutant					
	Cl (ppm)	CO_X (ppm)	NH_3 (ppb)	NO_X (ppb)	SO_X (ppb)	VOCS (ppb)
Mean	29.62	1.67	0.70	4.08	2.61	216.56
Median	26.51	1.66	0.70	4.19	2.57	219.81
Min	17.47	0.84	0.48	2.83	1.82	170.02
Max	51.91	2.72	0.94	5.07	3.53	253.01
S.d.	9.11	0.42	0.08	0.46	0.48	19.79
Coefficient of variation	0.31	0.25	0.12	0.11	0.18	0.09
Skew	0.89	0.13	0.07	-0.53	0.13	-0.53
Kurtosis	2.84	2.34	2.76	2.59	1.62	2.40
Jarque-bera	30.46	4.83	0.73	12.36	18.64	14.03

The correlation matrix appended in Table II signifies the time correlation between different pollutants. A high, positive correlation between chemically-similar pollutants is observed. It also shows synchronous time fluctuations of released poison gases from waste water.

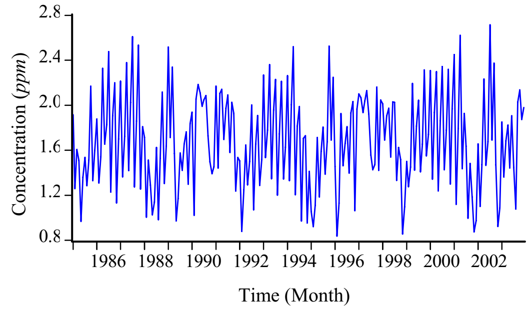
TABLE II
Correlation Matrix for Gases Exchange Data Set

Pollutant	*VOCs*	SO_X	NO_X	NH_3	CO_X	*Cl*
VOCs	1.000	0.060	0.011	0.143	0.033	0.036
SO_X		1.000	0.144	0.091	0.385	0.477
NO_X			1.000	0.553	0.197	0.191
NH_3				1.000	0.241	0.092
CO_X					1.000	0.218
Cl						1.000

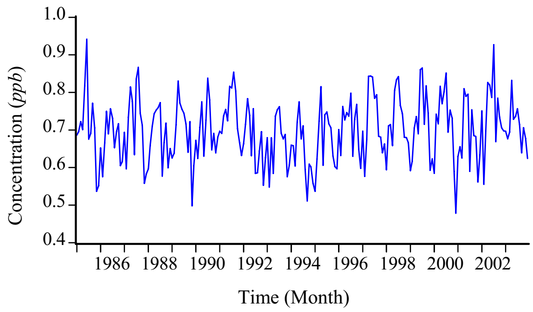
The time series trace plot of the air exchange gases are shown in Fig. 1. It shows the high seasonal inconsistency in the data sets.



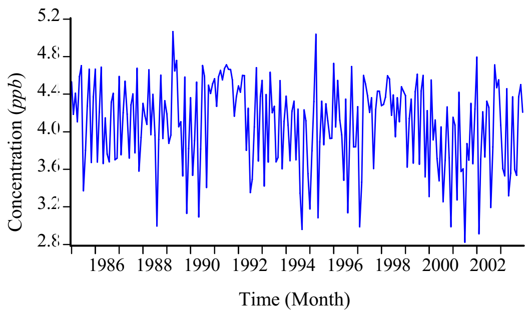
(a) Average monthly concentration plot of Cl.



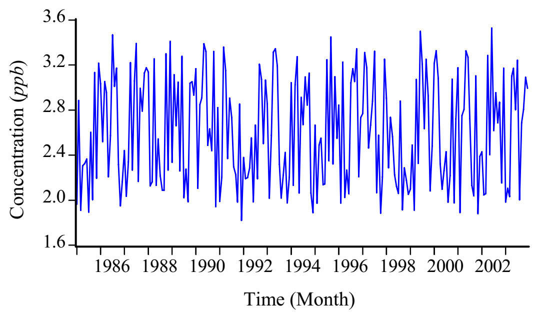
(b) Average monthly concentration plot of COx



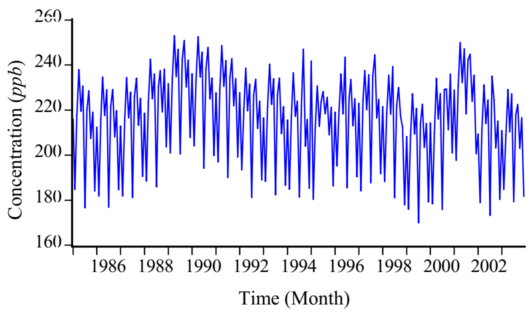
(c) Average monthly concentration plot of NH_3



(d) Average monthly concentration plot of NOx



(e) Average monthly concentration plot for SOx.



(f) Average monthly concentration plot of VOCs

Fig. 1 Time series trace plot of the air exchange gases.

The scatter plots presented in Fig. 2 shows weak and strong periodic correlation in air exchange data sets.

There are seventy two figures of scatter plots, and in here for the sake of completeness, only one for each pollutant is given. Here *season 1* corresponds to month of January and so on.

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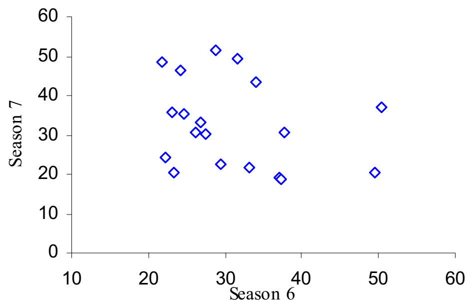
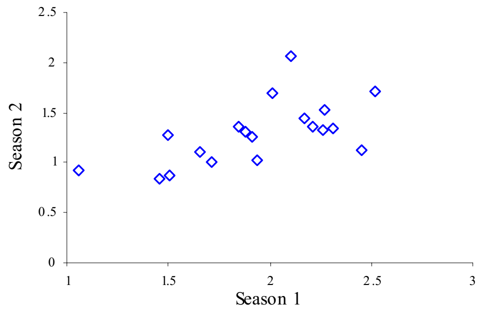
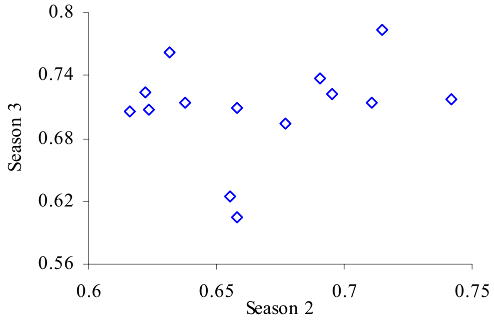


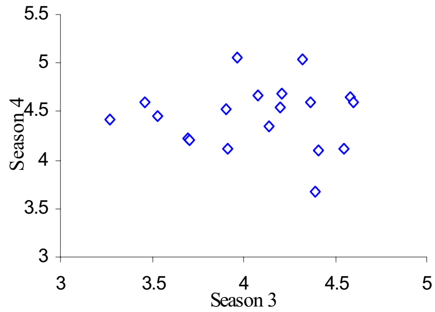
(a) Scatter plot for Cl (*ppm*) concentration.



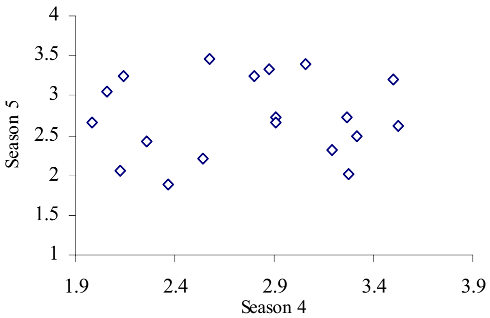
(b) Scatter plot for COx (*ppm*) concentration.



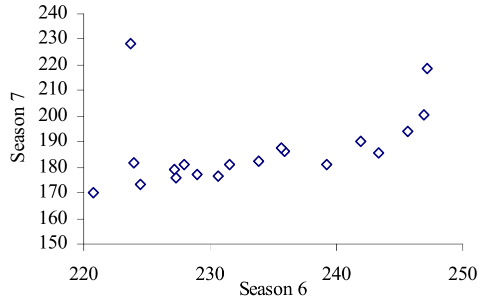
(c) Scatter plot for NH_3 (*ppb*) concentration.



(d) Scatter plot for NOx (*ppb*) concentration.



(e) Scatter plot for SOx (*ppb*) concentration.



(f) Scatter plot for VOCs (*ppb*) concentration.

Fig. 2 Season wise scatter plot for exchange of gases from Lyari channel.

The season wise schematic plots of periodic autocorrelation function (PeACF) of logarithmic mean monthly gases exchange by the river are depicted in Fig. 3 which also confirms the presence of periodic correlation. In these plots vertical pair of parallel lines are providing 5% significance limit.

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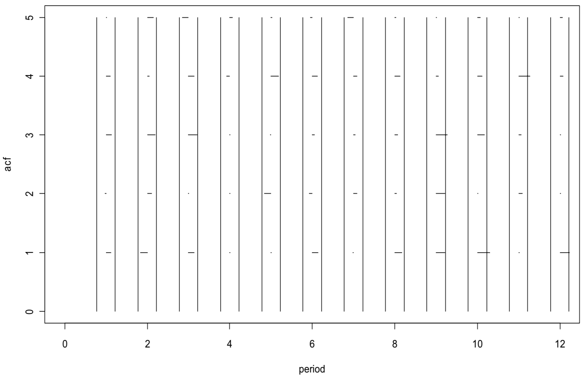
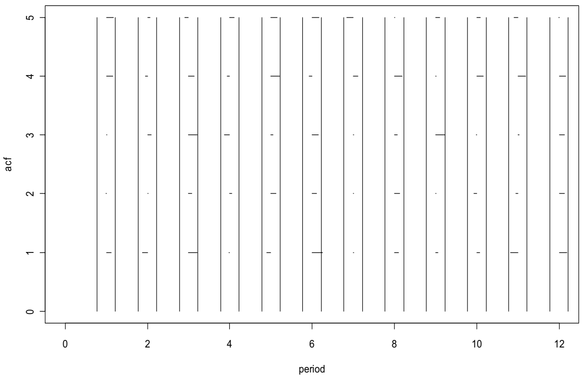
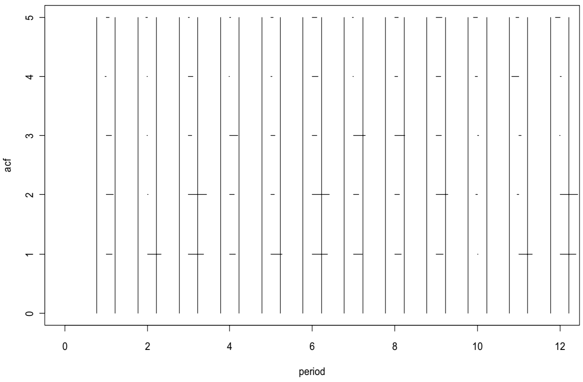


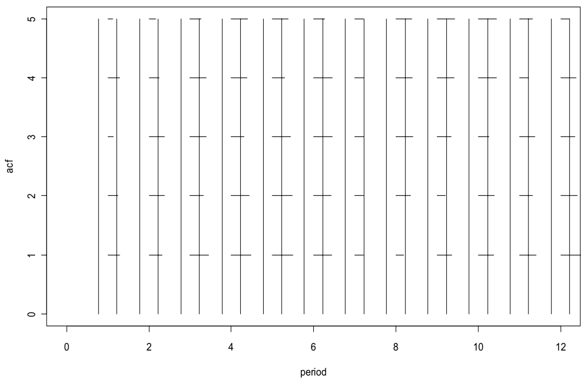
(a) Sample periodic ACF for logarithmic NH_3 exchange exchange



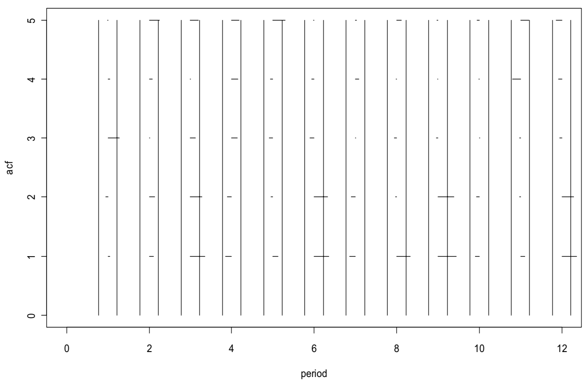
(b) Sample periodic ACF for logarithmic NOx



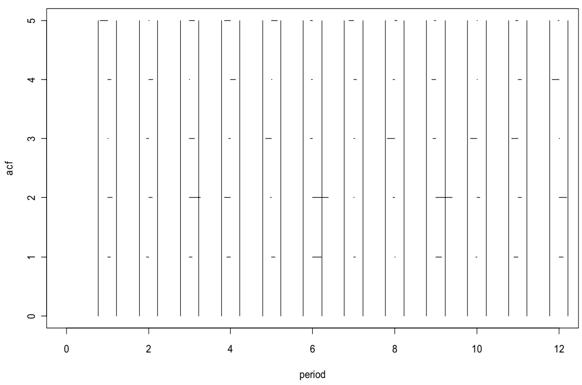
(c) Sample periodic ACF for logarithmic COx exchange.



(d) Sample periodic ACF for logarithmic VOCs



(d) Sample periodic ACF for logarithmic Cl exchange.



(d) Sample periodic ACF for logarithmic SOx exchange.

Fig. 3 Sample periodic ACF of emitted gases for each season.

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The time trace, scatter and partial autocorrelation plots authenticate appropriateness of periodic model. In this study suitable periodic models for air exchange gases are selected by examining sample periodic partial autocorrelation functions (*PePACF*) in addition to information criterias. The PAR order for each season of toxin gases exchange from Lyari river to nearby atmosphere on the basis of above mentioned techniques are depicted in Table III.

TABLE III
PAR ORDER FOR GASES EXCHANGED DATA SET OF LYARI RIVER.

Season	Toxic Gases					
	VOCs	SO_X	NO_X	NH_3	CO_X	*Cl*
1	3	0	0	0	0	0
2	3	0	0	0	1	0
3	1	2	1	0	2	2
4	1	0	0	0	0	0
5	1	0	0	0	0	0
6	4	2	2	0	2	2
7	1	0	0	0	1	0
8	3	0	0	0	0	1
9	4	2	0	0	2	2
10	4	0	0	1	0	0
11	1	0	0	0	4	0
12	3	0	5	1	2	2

The estimated PAR parameters for specified order of fugitive emitted gases with their standard errors in parenthesis are displayed in Tables IV through IX.

TABLE IV
PARAMETER ESTIMATES WITH STANDARD ERRORS FOR VOCS.

Season	Lags			
	1	*2*	*3*	*4*
1	4.839 (0.089)	-4.757 (0.105)	-0.438 (0.233)	-
2	0.866 (0.177)	-4.379 (0.113)	5.431 (0.125)	-
3	0.763 (0.086)	-	-	-
4	0.846 (0.053)	-	-	-
5	1.079 (0.018)	-	-	-
6	3.956 (0.101)	-4.319 (0.106)	0.187 (0.283)	0.687 (0.136)
7	0.880 (0.043)	-	-	-
8	0.004 (0.071)	0.016 (0.267)	0.844 (0.284)	-
9	0.137 (0.222)	0.015 (0.069)	-0.182 (0.259)	0.962 (0.333)
10	0.163 (0.131)	0.184 (0.142)	0.072 (0.047)	0.603 (0.136)
11	0.871 (0.143)	-	-	-
12	1.143 (0.023)	-0.147 (0.029)	0.209 (0.021)	-

TABLE V
PARAMETER ESTIMATES WITH STANDARD ERRORS FOR SOx.

Season	Lags			
	1	*2*	*3*	*4*
3	0.197 (0.196)	0.582 (0.206)	-	-
6	0.348 (0.152)	0.881 (0.168)	-	-
9	0.313 (0.147)	0.828 (0.127)	-	-

TABLE VI
PAR PARAMETER ESTIMATES WITH STANDARD ERRORS FOR NOx.

Season	Lags				
	1	*2*	*3*	*4*	*5*
3	0.343 (0.172)	-	-	-	-
6	0.462 (0.142)	0.501 (0.251)	-	-	-
12	0.729 (0.169)	1.213 (0.350)	0.217 (0.136)	0.313 (0.119)	-0.265 (0.126)

TABLE VII
PARAMETER ESTIMATES WITH STANDARD ERRORS FOR NH_3.

Season	Lags			
	1	*2*	*3*	*4*
10	0.302 (0.030)	-	-	-
12	0.442 (0.044	-	-	-

TABLE VIII
PARAMETER ESTIMATES WITH STANDARD ERRORS FOR COx.

Season	Lags			
	1	*2*	*3*	*4*
2	0.723 (0.181)			
3	0.234 (0.110)	0.672 (0.120)		
6	0.343 (0.093)	0.558 (0.106)		
7	0.453 (0.220)			
9	0.116 (0.122)	0.332 (0.128)		
11	0.758 (0.132)	0.210 0.272)	0.315 (0.149)	-0.607 (0.177)
12	0.386 (0.145)	0.678 (0.151)		

TABLE IX
PARAMETER ESTIMATES WITH STANDARD ERRORS FOR Cl.

Season	Lags			
	1	*2*	*3*	*4*
3	0.480 (0.099)	0.289 (0.085)	-	-
6	0.517 (0.095)	0.411 (0.094)	-	-
8	0.427 (0.115)	-	-	-
9	0.724 (0.122)	0.259 (0.080)	-	-
12	0.555 (0.109)	0.293 (0.083)	-	-

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VI. Adequacy of Fitted Models

The adequacy of the fitted models is established by examining the properties of residuals for each season. The following analysis is performed to check the significance of proposed models.

(i) The schematic plots of periodic residual ACF for specified PAR order of emitted toxic gases depicted in Fig. 4 through Fig. 9 are well within 95% confidence limit. These plots suggest the departure of autocorrelation between residuals at different time lags.

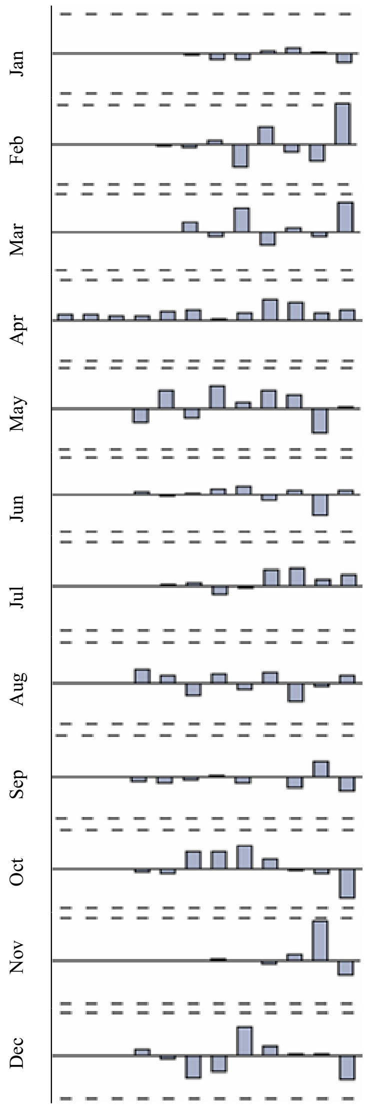

Fig. 4 Periodic residual ACF plots for VOCs.

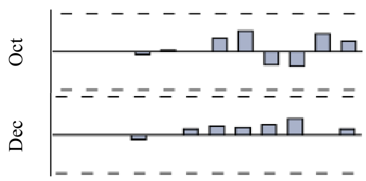

Fig. 5 Periodic residual ACF plots for NH_3.

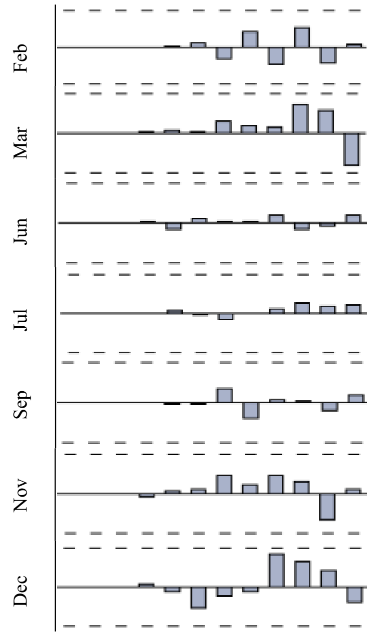

Fig. 6 Periodic residual ACF plots for

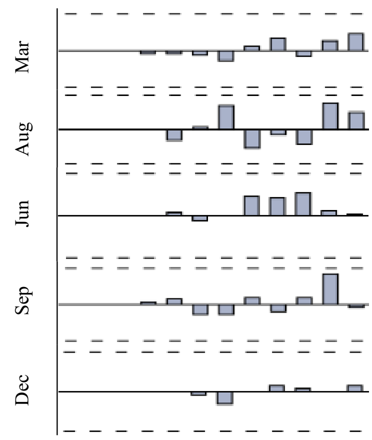

Fig. 7 Periodic residual ACF plots for Cl.

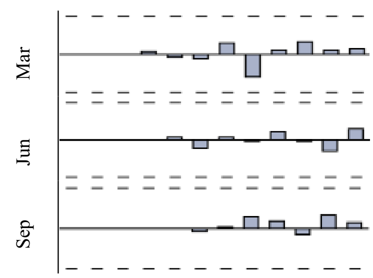

Fig. 8 Periodic residual ACF plots for SOx.

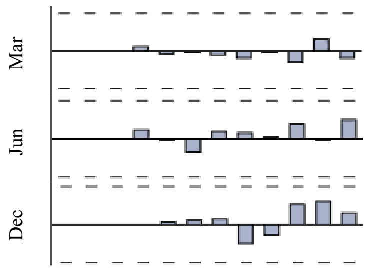

Fig. 9 Periodic residual ACF plots for NOx.

(ii) The residuals of the fitted models were assessed for whiteness by utilizing Doornik-Hansen, Shapiro-Wilk, Lilliefors and Jarque-Bera test. The output of these tests statistics with their probability values in parenthesis are depicted in Tables X through XV.

TABLE X
NORMALITY TESTS FOR SOx.

Season	Test Statistics			
	Doornik-Hansen	Shapiro-Wilk (W)	Lilliefors	Jarque-Bera
3	6.373 (0.041)	0.920 (0.114)	0.156 (0.025)	5.453 (0.065)
6	1.320 (0.050)	0.935 (0.210)	0.122 (0.063)	1.137 (0.056)
9	5.160 (0.075)	0.928 (0.162)	0.175 (0.012)	2.013 (0.036)

TABLE XI
NORMALITY TESTS FOR NOx.

Season	Test Statistics			
	Doornik-Hansen	Shapiro-Wilk (W)	Lilliefors	Jarque-Bera
3	4.008 (0.134)	0.918 (0.107)	0.184 (0.090)	2.905 (0.233)
6	0.973 (0.061)	0.938 (0.244)	0.122 (0.063)	1.015 (0.060)
12	3.957 (0.138)	0.927 (0.157)	0.137 (0.044)	3.414 (0.181)

TABLE XII
NORMALITY TESTS FOR Cl.

Season	Test Statistics			
	Doornik-Hansen	Shapiro-Wilk (W)	Lilliefors	Jarque-Bera
3	1.636 (0.044)	0.921 (0.119)	0.187 (0.080)	1.142 (0.056)
8	1.527 (0.046)	0.932 (0.190)	0.145 (0.035)	1.112 (0.057)
6	1.861 (0.039)	0.939 (0.258)	0.109 (0.079)	1.241 (0.053)
9	0.302 (0.085)	0.958 (0.546)	0.133 (0.049)	0.141 (0.093)
12	5.277 (0.071)	0.889 (0.031)	0.218 (0.020)	1.940 (0.037)

TABLE XIII
NORMALITY TESTS FOR COx.

Season	Test Statistics			
	Doornik-Hansen	Shapiro-Wilk (W)	Lilliefors	Jarque-Bera
2	1.339 (0.051)	0.977 (0.898)	0.109 (0.079)	0.485 (0.078)
3	4.882 (0.087)	0.924 (0.138)	0.130 (0.052)	4.755 (0.092)
6	0.409 (0.081)	0.963 (0.644)	0.106 (0.083)	0.526 (0.076)
7	1.252 0.053)	0.968 (0.741)	0.161 (0.210)	0.026 (0.198)
9	10.927 (0.004)	0.829 (0.003)	0.221 (0.020)	7.669 (0.021)
11	1.689 (0.042)	0.967 (0.722)	0.147 (0.033)	0.159 (0.092)
12	4.437 (0.108)	0.914 (0.087)	0.144 (0.036)	2.649 (0.026)

TABLE XIV
NORMALITY TESTS FOR VOCs.

Season	Test Statistics			
	Doornik-Hansen	Shapiro-Wilk (W)	Lilliefors	Jarque-Bera
1	8.107 (0.017)	0.884 (0.025)	0.173 (0.014)	12.473 (0.001)
2	0.651 (0.072)	0.965 (0.675)	0.136 (0.045)	0.581 (0.074)
3	5.930 (0.051)	0.936 (0.226)	0.193 (0.060)	0.941 (0.062)
4	10.186 (0.006)	0.896 (0.041)	0.197 (0.050)	4.641 (0.098)
5	1.941 (0.037)	0.935 (0.218)	0.182 (0.090)	1.364 (0.050)
6	1.159 (0.056)	0.962 (0.615)	0.121 (0.064)	1.088 (0.058)
7	36.742 (0.000)	0.634 (0.000)	0.293 (0.000)	55.341 (0.000)
8	6.839 (0.032)	0.877 (0.019)	0.235 (0.010)	6.677 (0.035)
9	12.405 (0.002)	0.793 (0.001)	0.274 (0.000)	16.285 (0.000)
10	3.286 (0.020)	0.922 (0.126)	0.142 (0.039)	1.561 (0.046)
11	24.747 (0.000)	0.744 (0.0002)	0.279 (0.000)	17.279 (0.000)
12	1.110 (0.057)	0.938 (0.247)	0.158 (0.023)	1.05446 (0.059)

TABLE XV
NORMALITY TESTS FOR NH_3.

Season	Test Statistics			
	Doornik-Hansen	Shapiro-Wilk (W)	Lilliefors	Jarque-Bera
10	3.198 (0.202)	0.972 (0.816)	0.115 (0.072)	1.036 (0.059)
12	9.001 (0.011)	0.841 (0.004)	0.145 (0.036)	14.878 (0.000)

The test output rejects the non-normality in the residuals.

(iii) The overall significance for whiteness in residual autocorrelation function values from lag one to L in order is confirmed by applying Portmanteau test. The computed test statistics is depicted in Table XVI.

TABLE XVI
COMPUTED VALUES OF PORTMANTEAU TESTS

Season	Air pollutant					
	VOCs	SOx	NOx	NH_3	COx	Cl
1	7.680	-	-	-	-	-
2	1.212	-	-	-	2.859	-
3	4.533	1.855	5.1612	-	4.135	0.133
4	1.931	-	-	-	-	-
5	8.623	-	-	-	-	-
6	1.737	3.727	5.412	-	1.610	1.837
7	1.553	-	-	-	3.675	-
8	3.236	-	-	-	-	0.980
9	5.755	3.378	-	-	0.151	2.774
10	1.623	-	-	4.195	-	-
11	1.792	-	-	-	0.761	-
12	6.313	-	1.979	1.123	2.850	2.784

The comparison of calculated χ^2 to the actual χ^2 values for $L\text{-}p_m$ degree of freedom from tables rejects the correlation problem.

The entire above performed test implies that the estimated innovations are uncorrelated and verify the adequacy of the suggested models.

VII. CONCLUSION

In this communication periodicities present in the exchange of gases from open sewer channel to the atmosphere has been assessed by means of periodic models. The orders of parsimonious periodic model for each season are selected by utilizing information criterias as well as the plot of periodic partial autocorrelation function. The coefficients of suitable PAR(p) models are estimated. The goodness of fit is achieved through the analysis of residuals. All the diagnostics checks confirm that residual in the model appeared to fluctuate randomly around zero with no obvious trend and confirms the adequacy of the projected models. Finding of this study will serve as a basis for the improvement of implemented policies to overcome future environmental issues.

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