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An Analysis of Acoustic Function and Navier-Stokes Equations in Aerodynamic

Hnin Hnin Kyi, and Khaing Khaing Aye

Abstract**—**Acoustic function plays an important role in aerodynamic mechanical engineering. It can classify the kind of air-vehicle such as subsonic or supersonic. Acoustic velocity relates with velocity and Mach number. Mach number relates again acoustic stability or instability condition. Mach number plays an important role in growth or decay in energy system. Acoustic is a function of temperature and temperature is directly proportional to pressure. If we control the pressure, we can control acoustic function. To get pressure stability condition, we apply Navier-Stokes equations.

Keywords**—**Acoustic velocity, Irrotational, Mach number, Rotational.

I. Introduction

AERO-ACOUSTICS is a research area of growing interest and importance over the last decade. In the transportation sector, the interest for this field has emerged during the last few years, due to various reasons. Aircraft noise and emissions have been of concerned since the beginning of commercial aviation. While considerable progress has been made to reduce the noise signature of airliners, the public's perception of noise continues to grow, as illustrated by the ever-increasing number of public complaints. As a result, noise has become a major constraint to air traffic, with 60% of all airports considering it a major problem and the nation's fifty largest airports viewing it as their biggest issue.

Acoustic information can be used as boundary condition for various propagation equations. Prediction of the acoustic field at large distances from the sound source is enabled.

The direct solution of the Navier–Stokes equations is only possible for a limited number of engineering applications methods. Compressible Navier–Stokes equations describe both the flow field and the aerodynamically generated acoustic field. The governing equation based from Navier-Stokes equations can solve flow field (source region) or acoustic field (acoustic region) .They can be calculated with different equations, numerical techniques, and computational grids. But Navier-Stokes equations are the most fundamental solution.

II. Navier - Stokes Equations

Now we consider three dimensional flows of a compressible fluid, we will adopt the following conventions.

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- surface 1, 2 and 3 are opened and allowed fluxes of mass, momentum and energy,
- surface ω is a closed wall; no mass flux through the wall,
- external heat flux q_ω through the wall (q_ω is a fixed parameter),
- diffusive, longitudinal heat transfer ignored, q_x =0,
- wall shear τ_w allowed which is a known parameter,
- Cross-sectional area is a known fixed function, A_x ,

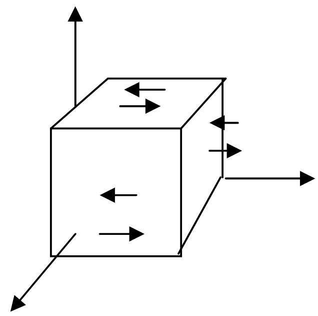

Fig. 1 Three dimensional flows of a compressible fluid

A. Mass

The amount of mass in a control volume after a time increment Δt is equal to the amount of mass plus that which came in minus that which left. The mass equation,

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho U) = 0 \quad (1)$$

B. Momentum

Newton's Second law says the time rate of change of linear momentum of a body equals the sum of forces acting on the body. The momentum equation,

$$\frac{\partial(\rho u)}{\partial t} + \nabla \cdot (\rho u U) = -\frac{\partial p}{\partial x} + \frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau zx}{\partial z} + \rho f_x \quad (2)$$

C. Energy

The first law of thermodynamics states that the change of total energy of a body equals the heat transferred to the body minus the work done by the body. The energy equation,

$$\frac{\partial}{\partial t}(\rho E)+\nabla\cdot(\rho EU)=\rho q+\frac{\partial}{\partial x}(q_x)+\frac{\partial}{\partial y}(q_y)+\frac{\partial}{\partial z}(q_z)-\frac{\partial(up)}{\partial x}-\frac{\partial(vp)}{\partial y}-\frac{\partial(wp)}{\partial z}+\frac{\partial(u\tau_{xx})}{\partial x}+\frac{\partial(u\tau_{yx})}{\partial y}+\frac{\partial(u\tau_{zx})}{\partial z}+\frac{\partial(v\tau_{xy})}{\partial x}+\frac{\partial(v\tau_{yy})}{\partial y}+\frac{\partial(v\tau_{zy})}{\partial z}+\frac{\partial(w\tau_{xz})}{\partial x}+\frac{\partial(w\tau_{yz})}{\partial y}+\frac{\partial(w\tau_{zz})}{\partial z}+\rho f\cdot U \quad (3)$$

where U is mean flow velocity component, ρ is density, p is pressure, t is time, λ is time rate of strain, τ_x is shear stress in x direction, f is force gravity, μ is molecular viscosity coefficient. We find in Navier-Stokes equations that mass and momentum uncouple from energy but energy couple to mass and momentum.

III. Some Important Acoustic Functions

A. Pressure

Pressure oscillations during instability closely resemble the classical acoustic modes of the chamber, at least the frequencies measured in tests agree well with predictions based on classical formula. Now pressure is the most common experimental indicator of combustion instability. The Ideal Gas law [6],

$$P=\rho RT \quad (4)$$

where P is pressure, R is gas constant per unit weight, T is absolute temperature, ρ is density.

Acoustic stability of all vehicles depends on their pressure condition. Acoustic velocity is equal to the double amount of stagnation pressure condition. To get acoustic stability, we need to maintain pressure and temperature in balance condition. If the pressure increases suddenly in any condition, it will cause shock wave in chamber. Similarly the temperature increases at once, acoustic velocity will rise and it will happen noisy.

B. Acoustic Velocity

The people had recognized for several hundred years that sound is a variation of pressure. The ears sense the variations by frequency and magnitude which are transferred to the brain which translates to voice. Thus, it raises the question: what is the speed of the small disturbance travel in a "quiet" medium. This velocity is referred to as the speed of sound. The acoustic velocity equation [9],

$$a=\sqrt{\gamma RT} \quad (5)$$

where a is acoustic velocity, R is gas constant per unit weight, γ is specific heat ratio, T is absolute temperature.

C. Mach Number

The Mach number is a dimensionless flow parameter is defined as the ratio of the flow velocity to the local acoustic velocity. The Mach number equation [9],

$$M=\frac{v}{a} \quad (6)$$

where M is Mach Number, $M>0$ (positive), v is Velocity of air-vehicle, a is acoustic velocity of air-vehicle, according to Mach number, we can distinguish sonic conditions,

M<1 (subsonic),
M=1 (sonic),
M>1 (supersonic)

D. Vorticity

When the propellant is ignited, the combustion process generates high-temperature gaseous products which fall the hollow core of the motor. With a closed end at the top of the motor, the gases convert toward the nozzle and are accelerated to supersonic speeds. The hot gases fall the motor chamber which is characterized by co-existing acoustic and rotational flow fields. It is used to obtain more accurate results for the axial rotational velocity and the vorticity.

E. Growth Rate Factors

To predict the acoustic stability or instability function, researchers formulated Standard Stability Prediction method. In Standard Stability Prediction program, growth rate factors (α_1 to α_4) solution is irrotational flow and (α_1 to α_{10}) solution is rotational flow. These growth rate factors are derived from SSP program which based on Navier-Stokes equations especially from energy system equation. The explanation is shown in the next section.

α_1 =pressure coupling,	α_6 =mean vorticity
α_2 =dilatational,	α_7 =viscosity
α_3 =acoustic mean,	α_8 =pseudo acoustic
α_4 =flow turning,	α_9 =pseudo vorticity
α_5 =rotational flow,	α_{10} =unsteady nozzle

Now we get equations relation between Mach number and these factors. To get these equations, we solved step by step calculation such as from SSP classical irrotational form to rotational calculation form, improved general volume form, from volume to surface form, and asymptotic surface form calculations.

$$\alpha_1=\frac{4}{9}M(A+1-2\gamma) \quad (7)$$

$$\alpha_2=-\frac{16}{27}\xi M^3+O(M^4) \quad (8)$$

$$\alpha_4 = -\frac{4}{5}M \quad (9)$$

$$\alpha_5 = \frac{4}{5}M \quad (10)$$

$$\alpha_6 = \frac{2}{5}M \quad (11)$$

$$\alpha_7 = -\frac{2}{9}\xi M \quad (12)$$

$$\alpha_8 = \frac{2}{5}M^3 l^2 / m^2 \quad (13)$$

where M is Surface Mach number, A is burning surface admittance, γ is specific heat ratio, m is mode number, ξ is limit cycle amplitude and l is chamber length.

F. Problem (1)

At constant velocity, the ratio of acoustic velocity and Mach number is here below

TABLE I
RATE OF CHANGE OF ACOUSTIC VELOCITY AND MACH NUMBER

	v =300 **m/s**	v =340 **m/s**	v =400 **m/s**
M	a	a	a
0.5	600	680	800
0.6	500	566	666
0.8	375	425	500
1.0	300	340	400
1.13	265	300	353
1.33	225	255	300
1.50	200	226	266

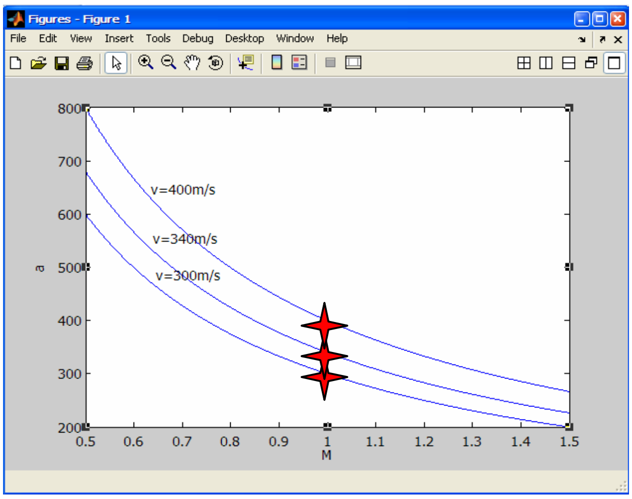

Fig. 2 Acoustic velocity and Mach number relation at constant velocity

Result and Discussion

The figure is done by MATLAB program with Mach number equation (6) and data form Table I. According to Fig. 1, we can easily find the condition of acoustic velocity and Mach number at constant velocity.

From mathematical point of view, when the acoustic velocity is equal to vehicle's velocity, we get Mach number one. It is called sonic condition. But the acoustic velocity is greater than vehicle's velocity; the Mach number is less than one that is called subsonic condition. In this condition, the acoustic velocity is at least 300m/s to larger than it. Similarly the Mach number increases over 1, it is called supersonic condition. So acoustic velocity is greater than its vehicle velocity, it will become noisy.

TABLE II
PROPERTIES OF INTERNATIONAL STANDARD ATMOSPHERE AT SEA LEVEL

Pressure	p^0	1.0133 bar
Velocity of sound	a^0	340.3 m/s
Temperature	T^0	288.2K
Density	ρ_0	1.225 kg/m^3

G. Problem (2)

Now we examine the pressure conditions in small motor, tactical rocket and space shuttle by using equation (7). We sketch a figure by using MATLAB program and use data from Table III. It is described that γ is 1.4 for all motors, A is 2.5 for small motor, 1.2 for tactical rocket and 1.0 for space shuttle according to physical parameters [7]

TABLE III
MACH NUMBER AND PRESSURE RELATION

M	small motor α_1	tactical rocket α_1	space shuttle α_1
0.5	0.155	-0.133	-0.622
0.8	0.248	-0.213	-0.995
1.0	0.311	-0.266	-1.244
1.2	0.373	-0.320	-1.490
1.5	0.466	-0.400	-1.866
2.0	0.622	-0.533	-2.488
2.5	0.777	-0.666	-2.711

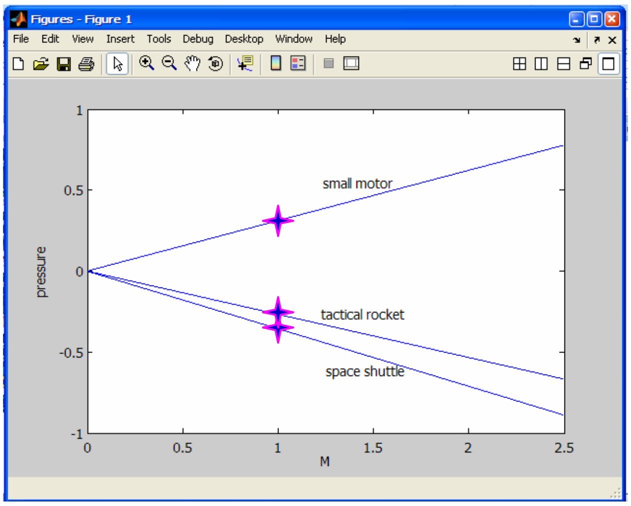

Fig. 3 Different Pressure condition of solid motor, tactical rocket and RSRM

Results and Discussion

According to air-vehicle's burning surface admittance and Mach number, pressure value will change. Acoustic stability can change due to its pressure. In this problem, burning surface admittance in small motor is greater than tactical and space shuttle. Heat source above the propellant surface is equivalent to the heat released by chemical reactions. So we have two different conditions for pressure and Mach number relation.

In figure, small motor's pressure condition is straightly forward from stagnation condition. The more the pressure is, the greater the Mach number. Because this motor's mass flow rate is greater than its velocity. Hence we can easily know that the temperature is less than its pressure according to equation (4).

But in other two motors, the relation lines show downward straight lines because their mass flow rates are less than their velocities. These conditions obey the Ideal gas law and pressure is inversely proportional to Mach number. The smaller the pressure, the larger the Mach number. Similarly the less the pressure, the more the acoustic stability function. The pressure is greater than its temperature. Hence we need to control pressure condition to get acoustic stability.

H. Problem (3)

TABLE IV
RATE OF CHANGE OF MACH NUMBER AND MEAN VORTICITY

Mach number	Mean vorticity
0.6	0.24
0.8	0.32
1	0.4
1.2	0.48
1.4	0.56
1.6	0.64
1.8	0.72

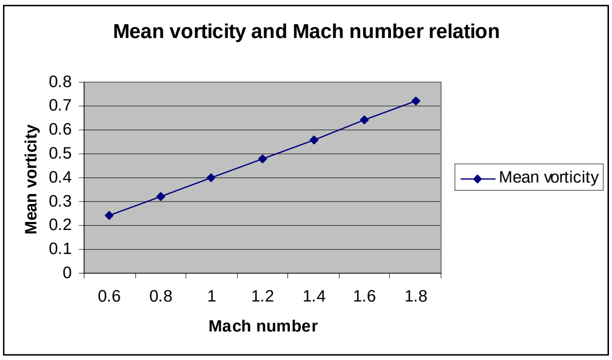

Fig. 4 Mach number and mean vorticity relation

We check up the vorticity and Mach number relation by using SSP result equation (11) for mean vorticity. We consider Mach number value between 0.6 to 1.8. This result shows the upward straight line depending on Mach number value. All these result will show the more the velocity, the more the vorticity. It can control the acoustic stability when the pressure increases in rectangular chamber. Hence the SSP method is consequence, accuracy and fundamental relation between physical, theoretical and mathematical relations. To control the acoustic function pressure stability is one of the most important portions in air-vehicle's engine.

NAVIER–STOKES EQUATIONS AND GROWTH RATE FACTORS

Solid-propellant rocket instability calculations (e.g., Standard Stability Prediction Program) account only for the evolution of acoustic energy with time. However, the acoustic component represents only part of the total unsteady system energy; additional kinetic energy resides in the shear waves that naturally accompany the acoustic oscillations boundaries. Modifications of the classical acoustic stability analysis have been proposed that partially correct this defect by incorporating energy source/sink terms arising from rotational flow effects. Stability prediction methods are based on three crucial assumptions:

1) small-amplitude pressure fluctuations superimposed on a low speed means flow;
2) Thin, chemically reacting surface layer with mass addition;
3) Oscillatory flow field represented by chamber acoustic modes.

The first assumption allows linearization of the governing equations both in the wave amplitude and the surface. The second causes all surface reaction effects, including combustion, to collapse, to simple acoustic admittance boundary condition at the chamber surfaces. The last assumption over simplifies the oscillatory gas dynamics by suppressing all unsteady rotational flow effects; the acoustic representation is strictly irrotational. Stability terms must be identified that will establish the correct energy pathways between the rotational and the irrotational parts of the gas oscillations. This is best accomplished by applying the energy balance approach because this method enhances the physical understanding of the many gas dynamic interactions. The outcome is a stability algorithm that accounts for both acoustic and vortical flow interactions.

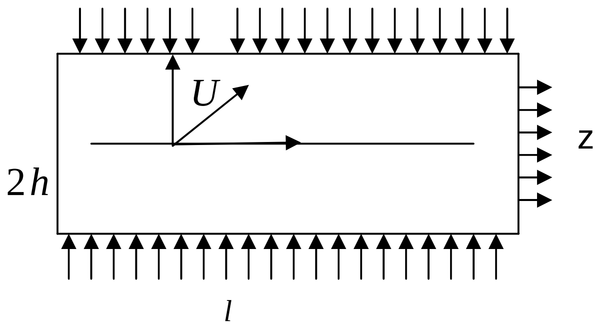

Fig. 5 Physical and geometric models of rectangular chamber

The analysis starts by assuming small-amplitude unsteady perturbations on a mean flow described by the vector MU. A more complete representation of the linearized motor aero-acoustic is utilized to determine the growth or decay of the system energy. With the increase in computational power, the direct computation of aerodynamic noise has become feasible. These growth factors are derived from energy system equation. Isentropic conditions are assumed (no heat transfer through the walls, no friction), but viscous forces, both shear and dilatational are retained. The linearized continuity and momentum equations (15) and (16),

$$\frac{\partial \rho_1}{\partial t} + \nabla \cdot u_1 = -MU \cdot \nabla \rho_1 \quad (15)$$

$$\frac{\partial u_1}{\partial t} + \nabla \rho_1 = M\{-\nabla[U \cdot u_1] + u_1 \times \nabla \times U + U \times \nabla \times u_1\}$$
$$+\delta^2 \left\{ (2 + \frac{\lambda}{\mu}) \nabla[\nabla \cdot u_1] - \nabla \times \nabla \times u_1 \right\} \quad (16)$$

They change from equation (2) and (3) when

$U = MU \cdot \rho_1 + u_1$, and

$$\rho f_x = \delta^2 \left[(2 + \frac{\lambda}{\mu}) \nabla (\nabla \cdot u_1) - \nabla \times \nabla \times u_1 \right].$$

The main variables will be represented as combinations of irrotational and rotational components:

$p_1 = \hat{p} + \tilde{p}$, $\quad p_1 = \hat{p}$
$u_1 = \hat{u} + \tilde{u}$,

The oscillatory energy defines as, $e = \frac{1}{2}\left[\hat{p}^2 + u_1 \cdot u_1\right]$

To get time rate of change of oscillatory energy equation, we use these factors in equation (15) and (16),
$\nabla \times \hat{u} = 0$, $\quad \nabla \cdot \tilde{u} = 0$

$\omega = \nabla \times \tilde{u}$, and $\quad \delta_d^2 \equiv \delta^2 (2 + \frac{\lambda}{\mu})$

$-\hat{u} \cdot (\tilde{u} \times \Omega) - \tilde{u} \cdot (\hat{u} \times \Omega) = 0$, then we get

$$\begin{aligned} \frac{\partial e}{\partial t} = &-\left[\hat{p} \nabla \cdot \hat{u} + u_1 \cdot \nabla (\hat{p} + \tilde{p})\right] - \\ &M \left\{ \frac{1}{2} U \cdot \nabla \hat{p}^2 + u_1 \cdot \nabla [U \cdot u_1] \right\} - \\ &M u_1 \cdot [u_1 \times \nabla \times U + U \times \nabla \times \tilde{u}] + \\ &\delta^2 u_1 \cdot \left[(2 + \frac{\lambda}{\mu}) \nabla (\nabla \cdot \hat{u}) - \nabla \times \nabla \times \tilde{u} \right] \end{aligned} \quad (17)$$

The time-averaged oscillatory energy residing in the chamber at any instant as

$$E = \iiint_V e\, dV,$$

then we get energy equation,

$$\frac{\partial E}{\partial t} = \iiint_V \left\langle \begin{aligned} &-\nabla \cdot (\hat{\rho}\hat{u}) - \frac{1}{2} M (U \cdot \hat{\rho}^2) - \\ &M[\hat{u} \cdot \nabla (U \cdot \hat{u})] + \delta_d^2 \hat{u} \cdot \nabla (\nabla \cdot \hat{u}) + \\ &M[\hat{u} \cdot (\hat{u} \times \Omega) + \hat{u} \cdot (U \times \omega)] - \tilde{u} \cdot \nabla \hat{\rho} - \\ &M \begin{bmatrix} \tilde{u} \cdot \nabla (U \cdot \hat{u}) + \hat{u} \cdot \nabla (U \cdot \tilde{u}) + \\ \tilde{u} \cdot \nabla (U \cdot \tilde{u}) - \tilde{u} \cdot (U \times \omega) - \tilde{u} \cdot (U \times \omega) \end{bmatrix} \\ &+ \delta_d^2 \tilde{u} \cdot \nabla (\nabla \cdot \hat{u}) - \delta^2 [\hat{u} \cdot (\nabla \times \omega) + \tilde{u} \cdot (\nabla \times \omega)] \\ &- u \cdot \nabla \tilde{\rho} - \tilde{u} \cdot \nabla \tilde{\rho} \end{aligned} \right\rangle dV$$

where U is mean flow velocity component, ρ is density, m is mode number, δ acoustic Renold number, M is Mach number at burning surface, Ω is mean vorticity amplitude, ω is unsteady vorticity, $\hat{u}$ is irrotational acoustic velocity and $\tilde{u}$ is rotational acoustic velocity. Time and retaining only terms are linear in the Mach number M, the subscript (1) denotes the first-order terms in a perturbation parameter proportional to the amplitude of the unsteady pressure fluctuation. We decomposed these energy system equation, it becomes growth rate factors equations.

V. Navier-Stokes Equations and Control Parameters for Acoustic Stability

We consider a laminar, rotational, and compressible condition, a long and slender rectangular channel of length *L* and width *w*, bounded by plane porous walls that are 2h apart. Through this wall, a perfect gas is injected with constant uniform velocity. Stream-wise, transverse, and spanwise coordinates can be denoted by x, y and z. The analytical expressions for laminar flow variables that can help explain the acoustic character established in such a physical configuration. In fact, the presence of sidewall injection inside long and slender rectangular channels can lead to strong acoustic waves that are decreed by the system geometry. These waves can interact with the channel's solid boundaries to generate time dependent vorticity waves. The inevitable coupling between acoustic and vortical waves results in complex flow patterns that depend on the pressure oscillation mode shape. The current analysis will attempt to characterize these flow patterns and unravel the main link between pressure oscillation mode shapes and vorticity production.

The organized vorticity waves that fill the entire chamber in the laminar case reduce to a thin acoustic boundary layer. From the standpoint of vorticity production, this can be viewed as a transition from organized vorticity associated with the periodic wave structure to the wideband vorticity.

A. Pressure Oscillation

Acoustic pressure oscillations are induced in a porous channel of the closed-open type, the linearized Navier-Stokes equations can be solved analytically to obtain an accurate description of the temporal flow field corresponding to laminar conditions. Analytical expressions for laminar flow variables that can help explain the acoustic character established in such a physical configuration.

$$\frac{\partial \rho_1}{\partial t} + \nabla \cdot u_1 = -M\nabla \cdot (\rho_1 u_0) \qquad (19)$$

$$\frac{\partial u_1}{\partial t} = -M\left[\nabla(u_0 \cdot u_1) - u_1 \times (\nabla \times u_0) - u_0 \times (\nabla \times u_1)\right]$$
$$-\nabla p_1 + \overline{R}^{-1}\left[4\nabla(\nabla \cdot u_1)/3 - \nabla \times (\nabla \times u_1)\right] \quad (20)$$

These pressure oscillation equations are derived from Navier-Stokes equations (1) and (2) when

$$V = M \cdot \rho_1 u_0 + u_1$$
$$\rho f_x = \overline{R}^{-1}\left[4\nabla(\nabla \cdot u_1)/3 - \nabla \times (\nabla \times u_1)\right].$$

B. Acoustic Wave

The presence of sidewall injection inside long and slender rectangular channels can lead to strong acoustic waves that are decreed by the system geometry. Due to the large, disparity in energy and length scales between the acoustic variables and the flow variables generate the acoustic field, and acoustic waves propagate over large distances. From equation (19) and (20), we derive acoustic wave equation and vorticity equation. They also based from mass and momentum equations. The time dependent velocity component,

$$u_1 = \hat{u} + \tilde{u}\,,\ p_1 = \hat{p}, \quad \rho_1 = \hat{\rho}$$

where $\hat{u}$ is acoustic part and $\tilde{u}$ is vortical part.

$$\frac{\partial \hat{\rho}}{\partial t} + \nabla \cdot \hat{u} = -M\nabla \cdot (\hat{\rho} u_0) \qquad (21)$$

$$\frac{\partial \hat{u}}{\partial t} = -\nabla \hat{p} + 4\overline{R}^{-1}\nabla(\nabla \cdot \hat{u})/3 -$$
$$M\left[\nabla(\hat{u} \cdot u_0) - \hat{u} \times (\nabla \times u_0)\right] \qquad (22)$$

These equations (21) and (22) condenses the set into a single hyperbolic partial differential equation,

$$\frac{\partial^2 \hat{p}}{\partial t^2} - \nabla^2 \hat{p} = -4\overline{R}^{-1}\nabla^2(\nabla \cdot \hat{u})/3 - M\nabla \cdot (u_0 \frac{\partial \hat{p}}{\partial t}) -$$
$$\nabla^2(\hat{u} \cdot u_0) + \nabla \cdot \left[\hat{u} \times (\nabla \times u_0)\right] \qquad (23)$$

Then we use these factors, we get acoustic wave solution.

$$k_m = \frac{kh}{a} = \frac{(m - 1/2)\pi}{l}$$

Where m is oscillation mode number, k is radian frequency.

C. Vorticity

Fluid is injected steadily through the sidewall with a characteristic velocity. When we calculate vorticity condition, density and pressure can't affect it.

$$\nabla \cdot \tilde{u} = 0 \qquad (24)$$

$$\frac{\partial \tilde{u}}{\partial t} = -\overline{R}^{-1}\nabla \times (\nabla \times \tilde{u}) -$$
$$M\left[\nabla(\tilde{u} \cdot u_0) - \tilde{u} \times (\nabla \times u_0) - u_0 \times (\nabla \times \tilde{u})\right] \qquad (25)$$

Then $\tilde{u} = \overline{u}\,\mathbf{exp}(-ik_m t)$, we get solution for vorticity equation. According to mode number, acoustic wave and vorticity change other conditions. Similarly acoustic stability will change as their situation. So we need to observe these conditions to control acoustic functions. This study focused on elucidating the nature of acoustico-vortical interactions in the porous channel with permeable walls. The scope was limited to laminar conditions in order to manage explicit formulations. The procedure was very similar to that used in analyzing the closed-closed channel configuration. Some of the main results included closed-form expressions for the time-dependent velocity and vorticity fields. The solution was arrived at using successive approximations that applied to the vorticity transport equation.

VI. Conclusion

Navier-Stokes equation is very important in aerodynamic. Also acoustic stability is one of the most critical functions in air-vehicles. Aero-acoustic can classify the air-vehicle's type such as subsonic, sonic, supersonic and hypersonic. To estimate the acoustic function, we use Navier-Stokes equations as based. Acoustic velocity is a function of temperature and it is directly proportional to pressure in ideal gas. So to predict or control the pressure, we derive pressure equation from Navier-Stokes equations. Navier-Stokes equation is fundamental for all problems in aerodynamic.

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