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Bayesian decision approach to protection on the flood event in upper Ayeyarwady River, Myanmar

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***Abstract*—**This paper introduces the foundations of Bayesian probability theory and Bayesian decision method. The main goal of Bayesian decision theory is to minimize the expected loss of a decision or minimize the expected risk. The purposes of this study are to review the decision process on the issue of flood occurrences and to suggest possible process for decision improvement. This study examines the problem structure of flood occurrences and theoretically explicates the decision-analytic approach based on Bayesian decision theory and application to flood occurrences in Environmental Engineering. In this study, we will discuss about the flood occurrences upon an annual maximum water level in cm, 43-year record available from 1965 to 2007 at the gauging station of Sagaing on the Ayeyarwady River with the drainage area - 120193 sq km by using Bayesian decision method. As a result, we will discuss the loss and risk of vast areas of agricultural land whether which will be inundated or not in the coming year based on the two standard maximum water levels during 43 years. And also we forecast about that lands will be safe from flood water during the next 10 years.

***Keywords*—** Bayesian decision method, conditional binomial distribution, minimax rules, prior beta distribution.

I. INTRODUCTION

THE flood is a natural disaster that people in the world have been facing usually. It has caused extensive damages in the world over the past years. In our country, Myanmar, we used to face flood occurrences yearly. Broadly, the factors affecting a flood can be grouped in the categories, excessive rainfall with combination of snowmelt, heavy rainfall, and some other factors. One of the main important facts is the heavy rain condition which causes continuously on during four or five days. However, for Ayeyarwady River, the falling heavy rain in the river at Putao can affect the main point of flood among many factors.

Our country is an agricultural country and most of our food and provision depend on the agricultures. So, it is our duty to learn about the truth of ruinous agricultural lands. So, we try to know the nature of the flood of the Ayeyarwady and also attempt the ways of flood protection strategy. One of the protections of flood events is the construction of embankments on the side of the Ayeyarwady River. So we investigate the Bayesian decision method, a useful tool for Civil and Environmental engineers, in decision making on a protection of flood event in upper Ayeyarwady River, Myanmar.

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Many decisions making can make them upon the different kinds of situations. Depending on what degree of knowledge is present, different decision models can be applied [3]. Decision making can be classified four classifications by means of degree of knowledge such as decision making under certainty, decision making under uncertainty, decision making under risk, and decision making under ignorance.

Engineers sometimes face situations that require knowledge of probabilities without the benefits of knowing repeated outcomes under similar conditions, so that the long run frequency approach is not possible. In such case, we need to apply subjective probabilities quantifying personal knowledge or belief. This activity, the inclusion of the subjective element, is called decision making under uncertainty. On the other hand, the activity, the inclusion of the objective elements, is called the decision making with objective probabilities and we can call this case as decision making under risk.

In this paper, our discussion focuses on decisions that are called for under conditions of unpredictability. Each choice must be logically based with the aim of meeting given objectives, which often have an economic basis. This paper commences with basic Bayes' rules for action by the engineer, followed by decision trees that show the available alternatives in terms of actions, states of nature, losses and risks. Although we have turned the spotlight on economic decision making, environmental benefits are of direct concern.

We also consider about minimax rules. These rules and Bayes rules are two basic applications of decision theory. For the Bayes method to be useful, we need a reliable estimate for the prior distribution of the state of nature. The minimax rule is adopted when there is no prior information or when the prior distribution is vaguely defined [7]. In the case of prior probabilities which can be vague or diffused (i.e., non informative) at times, or more definitive on other occasions, Bayes' theorem provides a method of revising the probabilities on receipt of additional data. The activity includes the subjective element and so-called decision making under uncertainty.

In analyzing the data, initially, we have collected the annual maximum water levels of the gauging station of Sagaing on the upper Ayeyarwady River in Myanmar. We can remark the specified three years among the 43 years. Firstly, in 1971, 1221cm, secondly, in 1974, 1227cm, and thirdly, in 2004, 1274cm are regarded as a documentation of maximum water levels [10].

Then, we consider four basis applications of decision theory to provide answers upon the following four situations:

- Minimax rule in protective embankments case
- Bayes rule in protective embankments case with prior information by the engineer's experience and using loss information
- Bayes rule action for protective embankments case using observed data
- Bayesian decision rule and minimum error rate classification.

II. Bayesian Decision Method

A. Bayesian Decision Procedure

Suppose that the decision is to take an action or set of actions **"*a*"** that belong to a set **"*A*"**, the action space. The state of nature is denoted by a parameter θ. This parameter indexes the probability distribution of a random variable *X*, the observation of which form the basis of our decisions. The model comprises the probability distributions that *X* can take.

We denote the set of all possible values of θ by Θ, the parameter space. The decision *d* is part of a decision space *D*. It is the link between *X* and *A* and maps the sample space of the basic random variable *X* on to the action space. In general, we expect that there will be a loss, proper wastage of resources. This is quantified by a loss function $l(\theta, a)$.

If we adopt the action or set of actions $a = d(X)$ given the state of nature θ, then, we find expected or average loss. This is the risk function, $R(\theta, d)$. After expecting the risk, it gives the Bayes risks, $B(\pi, d)$.

Decision analysis is sometimes based on a prior distribution $\pi(\theta)$ to specify the probability distribution of parameters. To get the optimal solution, the average risk under each decision rule is evaluated in the application of Bayes risk through the prior distribution of the state of nature. By minimizing the Bayes risks, we get Bayes rule or Bayes decision [2] and [7].

In the absence of prior information; the minimax method provides an alternative summary of the risk function by considering maximum risk and then finding the minimum value of the maximum risks [2] and [7]. In the absence of loss data, Bayesian Decision Rule can provide an optimal decision by considering posterior with observed data. And then it can also classify the minimum error rate [3] and [6].

Needed to introduce about the useful terms can be defined as follows:

The *State of nature* (refer to its aspects of uncertainty) is quantified by probability, which is often evaluated subjectively when there is no practical alternative and it is a random variable.

$p(\theta_1) + p(\theta_2) = 1$ (exclusively and exhausitivity), (1)

where θ_1 and θ_2 are true state of nature.

Loss function is a generated function by taking a particular action *a* and true state of nature θ and it is denoted by

$$l(\theta, a), \forall (\theta, a) \in \Theta \times A. \tag{2}$$

The *risk function, R*, of a decision rule *d*(*x*) is the expected loss function

$$R(\theta, d) = E[l(\theta, d(x))]. \tag{3}$$

B. Bayesian Decision Theory

One of the main parts of this decision theory is Bayesian theorem. Bayes theorem is the best way to make consistent decision in the face of uncertainty. It is a result in probability theory which relates the conditional and marginal probability distribution of random variables. Bayes's theorem tells how to update or revise beliefs in illumination of new evidence so-called a posteriori. Bayesians can use all the available information, even if that information came from sources outside the experiment [2], [3], and [7].

To derive the theorem, we start the definition of conditional probability,

$$P(A|B) = \frac{P(A \cap B)}{P(B)} \quad \text{and} \quad P(B|A) = \frac{P(A \cap B)}{P(A)}. \tag{4}$$

After rearranging and combining above equations, we get

$$P(A|B)P(B) = P(A \cap B) = P(B|A)P(A). \tag{5}$$

This is sometimes called the product rule for probabilities. Dividing both sides by $Pr(B)$, providing that it is non-zero, we obtain Bayes' Theorem:

$$P(A|B) = \frac{p(B|A)P(A)}{P(B)}. \tag{6}$$

Each term in Bayes' theorem has a conventional name:

$P(A)$ is the prior probability or marginal probability of *A*.
$P(A|B)$ is the posterior probability.
$P(B|A)$ is the conditional probability of *B* given *A*.
$P(B)$ is the prior or marginal probability of *B*, and as a normalizing constant.

With this terminology, the theorem may be paraphrased as

$$\text{posterior} = \frac{\text{likelihood} \times \text{prior}}{\text{normalizing constant}}.$$

To approach Bayesian decision theory, we need to analyze the posterior. In this part, we develop a method for finding the Bayes rule. The Bayes risk for a prior distribution $\pi(\theta)$ is the expected loss of a decision rule *d*(*X*) when *X* is generated from the following probability model:

First, the state of nature is generated according to the prior distribution $\pi(\theta)$.

Then, the data *X* is generated according to the distribution $f(X, \theta)$, which we will denote by $f(x|\theta)$.

Under this probability model (call it is the Bayes model), the marginal distribution of *X* is (for the continuous case),

$$f_X(x) = \int f(x|\theta)\pi(\theta)\,d\theta \quad (7)$$

Applying Bayes theorem, the conditional distribution of θ given X is

$$h(\theta \mid x) = \frac{f_{X,\theta}(x,\theta)}{f_X(x)} = \frac{f(x \mid \theta)\pi(\theta)}{\int_{-\infty}^{\infty} f(x \mid \theta)\pi(\theta)\,d\theta}. \quad (8)$$

For the discrete case, the conditional distribution of θ given X is

$$h(\theta_j \mid x) = \frac{f(x \mid \theta_j)\pi(\theta_j)}{\sum_i f(x \mid \theta_i)\pi(\theta_i)} \quad (9)$$

The conditional distribution $h(\theta \mid x)$ is called the *posterior* for X=x of θ. The prior distribution $\pi(\theta)$ is specified before observation X and the posterior distribution $h(\theta \mid x)$ can be calculated after observing X=x.

Suppose that we have observed X=x. We define the *posterior risk* (PR) of an action a=$d(X)$ as the expected loss, where the expectation is taken with respect to the posterior distribution of θ. For continuous random variables, we have

$$\begin{aligned} PR(a) &= E_{h(\theta|X=x)}[l(\theta, d(x))] \\ &= \int_{-\infty}^{\infty} l(\theta, d(x))\, h(\theta|X=x)\,d\theta . \end{aligned} \quad (10)$$

For discrete random variables,

$$PR(a) = \sum_j l(\theta_j, d(x_i))\, h(\theta_j|x_i). \quad (11)$$

These procedures are called *posterior Bayesian decision analysis*.

C. Minimax Rules

Decision theory is concerned with choosing a “good” decision function, that is, one has a small risk as in (3). We have to face the difficulty that R depends on θ, which is not known. The two widely used methods for confronting this difficulty are to use either a minimax rule or a Bayes rule.

The minimax method proceeds as follows:

First, for a given decision function d, consider the worst that the risk could be: $\max_{\theta\in\Theta}[R(\theta,d)]$.

Then choose a decision function, say d^*, that minimizes this maximum risk:

$$\max_{\theta\in\Theta}\{R(\theta,d^*)\} = \min_{d\in D}\{\max_{\theta\in\Theta}[R(\theta,d)]\}. \quad (12)$$

This minimax decision rule (also named minimax regret, minimax risk, or minimax loss) focuses on the possible regret of each alternative.

This decision rule requires criteria that the decision alternatives and their possible outcomes are known [2], [3], and [7].

D. Bayes Rule

Let $d_0(x)$ be a function that minimizes the posterior risk. For the case of a prior distribution which has been given, we can calculate the Bayes risk of a decision function d: $B(d) = E_{\pi(\theta)}[R(\theta,d)]$, where the expectation is taken with respect to the distribution $\pi(\theta)$. The Bayes risk is the average of the risk function with respect to the prior distribution of θ. A function d that minimizes the Bayes risk is called a *Bayes rule* [2] and [7].

For the continuous case, the Bayes risk of a decision function d is

$$B(d) = \int\left[\int l(\theta, d(x))\, f(\theta \mid x)\, d\theta\right] f_X(x)\, dx \quad (13)$$

When discrete values of θ are considered, we should replace the integral on the right hand side by a summation as follows:

$$B(d) = \sum_i \left\{\sum_j l(\theta_j, d(x_i)) f(\theta_j|x_i)\right\} f_x(x_i) \quad (14)$$

E. Bayesian Decision Rule and Minimum Error Rate Classification

For the problem which has been updated or inferred with the light of new information, when we want to know the probability error for observed data, *Bayesian decision rule* and *minimum error rate classification* can be applied. They can decide a true state of nature and describe the probability error for the problem to get a terminal decision.

First, we calculate the posterior distributions of θ by using (9) for a given value of X.

Second, we must compare them which is smaller to decide the true state of nature.

If $h(\theta_1 \mid x) \rangle\, h(\theta_2 \mid x)$, we will decide θ_1.
If $h(\theta_1 \mid x) \langle\, h(\theta_2 \mid x)$, we will decide θ_2. (15)

Then, we classified the minimum error rate based on the posterior distributions which have been decided.

$$P(error \mid x) = \begin{cases} h(\theta_1 \mid x) & \text{if we decide } \theta_2. \\ h(\theta_2 \mid x) & \text{if we decide } \theta_1. \end{cases} \quad (16)$$

$$P(error \mid x) = \min[h(\theta_1 \mid x), h(\theta_2 \mid x)] \quad (17)$$

From this point, we choose the least probability error to get the good accuracy for action with observations as a terminal decision.

F. Inference with Conditional Binomial and Prior Beta

Let the prior distribution of the state of nature θ be beta

(α, β) , that is,

$$\pi(\theta) = \begin{cases} \dfrac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} \theta^{\alpha-1}(1-\theta)^{\beta-1}, & 0<\theta<1, \alpha>0, \beta>0. \\ 0 & , \quad \text{otherwise}. \end{cases} \quad (18)$$

This distribution is appropriate if θ represents a probability, for example θ =1-$F(y)$, where the cdf $F(y)=Pr(y<Y)$ and Y is a random variable.

If X is the number of exceedances of Y in n independent trails, the sample likelihood function of X given θ is binomial. That is,

$$f(x|\theta) = \binom{n}{x} \theta^{x}(1-\theta)^{n-x}, \; x = 0,1,2,...,n \quad .$$

(19)

Hence the joint distribution X and θ becomes:

$$\begin{aligned} f(x,\theta) &= f(x|\theta)\pi(\theta) \\ &= \binom{n}{x} \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} \theta^{\alpha+x-1}(1-\theta)^{\beta+n-x-1} \quad .(20) \end{aligned}$$

The marginal distribution of X is obtained by integrating out θ as follows:

$$\begin{aligned} f_x(x) &= \binom{n}{x} \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} \int_0^1 \theta^{\alpha+x-1}(1-\theta)^{\beta+n-x-1} d\theta \\ &= \binom{n}{x} \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} \frac{\Gamma(\alpha+x)\Gamma(\beta+n-x)}{\Gamma(\alpha+\beta+n)} \quad . \end{aligned}$$

(21)

The posterior distribution of θ for a given value of X is obtained by following the Bayes' theorem and by using (7) as follows:

$$\begin{aligned} f(\theta|x) &= \frac{f(x|\theta)\pi(\theta)}{\int_0^1 f(x|\theta)\pi(\theta)d\theta} \quad (22) \\ &= \frac{\Gamma(\alpha+\beta+n)}{\Gamma(\alpha+x)\Gamma(\beta+n-x)} \theta^{\alpha+x-1}(1-\theta)^{\beta+n-x-1}. \end{aligned}$$

The updating rule for betas: α becomes $\alpha + x$ and β becomes $\beta + n - x$.

From (18) it follows that (22) is the pdf of a beta $(\alpha + x, \beta + n - x)$ distribution [1] and [2].

III. Experimental Works

A. Protective Embankments Cases

In this case, we will discuss about the flood event and protective embankments on the largest river basin in Sagaing Division. The upper Myanmar has been doing observation, data processing, and forecasting, annual maintenance for the Ayeyarwady and Chindwin, the two main channels since 1964

As Ayeyarwady is a main channel, the people from the towns and villages along the river have been suffering from the effect of plain flood. Whenever the flood occurs, the various damages happen more or less.

In analyzing the data, we have collected the annual maximum water levels of the main stations along Ayeyarwady River in upper Myanmar at first, and we emphasised the gauging station of Sagaing.

Then, we analyzed them based on the collective data in the above danger levels (Above D/L), the nearly danger levels (Nearly D/L) and the ordinary levels (Ordinary D/L).

TABLE I
The Record of The Flood Occurrence in Ayeyarwady River at Sagaing Station

Nearly D/L	Above D/L	Ordinary L	Total Flood	Frequency Year	Total Year
19	20	4	39	2	43

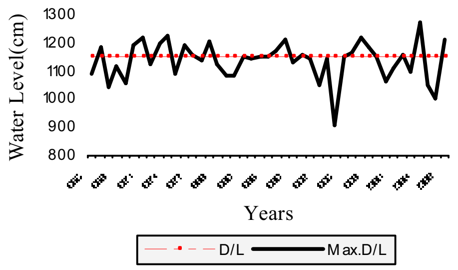

Fig. 1 Water levels of Ayeyarwady River at Sagaing in 43 years (1965-2007)

Figure 1 illustrates the conditions of danger level, above danger level and ordinary danger level of flood event in Sagaing Division which occurred during 43 years. In this figure, two standard maximum water levels of Ayeyarwady River which occurred in 1974 and in 2004 can be marked. We analyze about the prediction of the flood occurrence of Ayeyarwady river in next year and the protection of embankments on the largest river basin in Sagaing based on the data of forty-three years (from 1965 to 2007) which has been taken from Sagaing Station and gained the remarkable two standard maximum water levels of Ayeyarwady River.

B. Inference with Conditional Binomial and Prior Beta in Embankments Case

To protect vast areas of agricultural land in Sagaing, we need the largest river basin in Sagaing. If the water level reaches the lower basin, vast areas of agricultural land will be protected by embankments along both sides of the river. There is concern about the adequacy of the flood protection scheme. Consider over a period of 43 years it is known that agricultural lands have been extensively flooded 20 times, that is, in 43 different years. Assuming the overtopping of the embankments constitutes a series of independent events.

Consider about X is the number of exceedances over a future period of n years. The marginal pdf of X is obtained from (21) as follows, with $\alpha = t + 1$ and $\beta = m - t + 1$:

$$f_x(x)=\frac{n!}{x!\,(n-x)!}\,\frac{(m+1)!\,(x+t)!\,(n+m-t-x)!}{t!\,(m-t)!\,(n+m+1)!},$$

where $m = 43$, and $t = 20$.

Put n=1 for the next year, x=1 for the inundated event in the next year.

$$f_x(x=1)=\frac{1!}{1!0!}\,\frac{44!21!23!}{20!23!45!}=0.49\,.$$

Put n=10 for the next 10- years, x=0 for low-lying lands which safe from flood water during the next 10 year.

$$f_x(x=0)=\frac{10!}{0!10!}\,\frac{44!20!33!}{20!23!54!}=0.004\,.$$

After we have determined the probability that the adjoining lands whether which will be inundated or not during the next years, we will check the past experience of the flood event in Ayeyarwady River.

There will be gained remarkable the flood event of the probability that the adjoining lands which had been safe and inundated from flood water after checking the marginal probability distribution of the flood event based on the final data from year to year.

Then, Fig. 2 will demonstrate about the probability of low-lying lands which are inundated or safe conditions from flood water in each year based on the previous year.

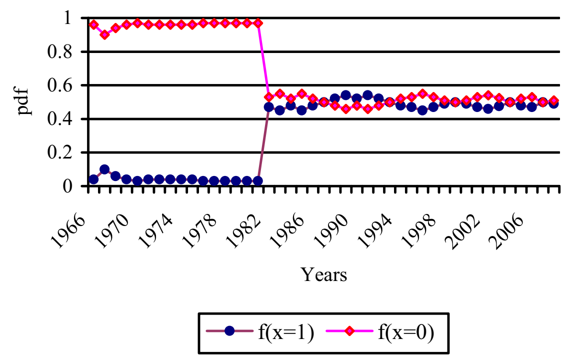

Fig. 2 Probability of low-lying lands for the inundation or safe events in each year and the next year

To demonstrate the model of the probability of inundation of adjacent lands with m-file, we can perform as a model as in Fig. 3. The approaching to construct this model can easily be done by using MATLAB program, as in the following;

```
%the whole solution for three graphs
m=43;
n=1;
x=1;
t=20;
Alpha=t+1;
Beta=m-t+1;
theta=0:0.05:1;
A=factorial(m+n+1).*((theta).^(Alpha+x-1)).*((1-theta).^(Beta+n-x-1));
B=factorial(x+t).*factorial(n+m-t-x);
Ans=A/B
n=10;
x=0;
Alpha=t+1;
Beta=m-t+1;
theta=0:0.05:1;
A1=factorial(m+n+1).*((theta).^(Alpha+x-1)).*((1-theta).^(Beta+n-x-1));
B1=factorial(x+t).*factorial(n+m-t-x);
Ans1=A1/B1
A2=factorial(m+1).*theta.^(t).*(1-theta).^(m-t);
B2=factorial(t).*factorial(m-t);
Ans2=A2/B2
plot(theta,Ans,theta,Ans1,theta,Ans2)
title('Protective Embankments on the Large Basin along Ayeyarwady River at Sagaing')
xlabel('theta')
ylabel('Pi(theta);Fn(theta/x=0);Fn(theta/x=1)')
legend('Fn=(theta/x=1)','Fn(theta/x=0)',' Pi(theta)');
```

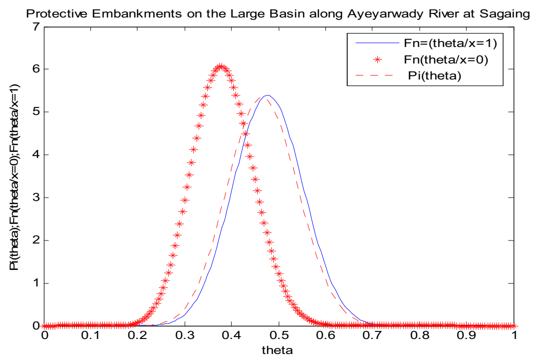

Fig. 3 Prior and posterior beta distributions of probability of a flood event

It illustrates that the posterior pdf is taller and narrower than the prior pdf. This reflects a reduction in the uncertainty.

C. Minimax Rules in Protective Embankments Case (Without Prior Information)

In one of the parts of the consideration of the largest river basin in Sagaing, embankments section is to be protected to the both sides of the Ayeyarwady River. We consider this problem based on the data of forty-three years (from 1965 to 2007) which has been taken from Sagaing Station.

During these 43 years, there are two standard maximum water levels: 1227 *cm* in 1974 and 1274 *cm* in 2004. So, we have two choices (actions):

a_1 : selects a 1230 *cm* section for 1227 *cm*

a_2 : selects a 1280 *cm* section for 1274 *cm*

First, consider two possible states of nature such that the height of embankments section is 1230 *cm* and the height of embankments section is 1280 *cm*.

Suppose that the damage cost of vast areas of agricultural lands must be in wasted resources at a cost of $13,000 if the 1230 *cm* section is incorrectly chosen. If the 1280 *cm* section is incorrectly chosen, we must face extraordinary cost at a cost of $10,000. The loss function is therefore represented in the following table:

TABLE II
LOSS TABLE

Action (a)	State of nature θ_1	θ_2
a_1	\$0	\$13,000
a_2	\$10,000	\$0

A height sounding is taken by means of forecasting. Suppose that the measured height, X, has three possible values, x_1 =1230, x_2 =1255, and x_3 =1280, and that the probability distribution of X depends on θ in table III.

TABLE III
PROBABILITY DISTRIBUTION TABLE

Measured height (X)	State of nature θ_1	θ_2
x_1	0.6	0.1
x_2	0.3	0.2
x_3	0.1	0.7

TABLE IV
DECISION TABLE

Decision (D)	Measured height (X) x_1	x_2	x_3
d_1	a_1	a_1	a_1
d_2	a_1	a_1	a_2
d_3	a_1	a_2	a_2
d_4	a_2	a_2	a_2

To consider the above four alternative decision rules, the minimax rule must be applied at first. To do so, we need to compute the risk of each of the decision function in the case where $\theta = \theta_1$ and in the case where $\theta = \theta_2$. To do such computations for $\theta = \theta_1$, each risk function is computed by using following equation;

$$R(\theta_1, d_i) = E_{\theta_1}\{l(\theta_1, d_i(X))\}$$
$$= \sum_{j=1}^{3} l(\theta_1, d_i(x_j)) P(X = x_j | \theta = \theta_1) \quad .$$
(23)

$$R(\theta_1, d_1) = 0 \times 0.6 + 0 \times 0.3 + 0 \times 0.1 = 0$$
$$R(\theta_1, d_2) = 0 \times 0.6 + 0 \times 0.3 + 10{,}000 \times 0.1 = 1{,}000$$
$$R(\theta_1, d_3) = 0 \times 0.6 + 10{,}000 \times 0.3 + 10{,}000 \times 0.1 = 4{,}000$$
$$R(\theta_1, d_4) = 10{,}000 \times 0.6 + 10{,}000 \times 0.3 + 10{,}000 \times 0.1 = 10{,}000.$$

Similarly, in this case where $\theta = \theta_2$ we have,

$$R(\theta_2, d_1) = 13{,}000,$$
$$R(\theta_2, d_2) = 3{,}900$$
$$R(\theta_2, d_3) = 1{,}300$$
$$R(\theta_2, d_4) = 0.$$

To find the minimax rule, we note that the maximum values of the risk of d_1, d_2, d_3, and d_4 are 13,000, 3,900, 4,000, and 10,000 respectively. Thus, we choose the branch with smallest expected \$ loss. It is the branch corresponding to decision d_2 as the minimax rule. All other branches are pruned (two parallel vertical lines on the branch).

In Fig. 4, the height of embankments level, 1230 *cm* is denoted by θ_1 and θ_2 denotes the height of embankments level, 1280 *cm*. d_1, d_2, d_3, and d_4 are alternative decisions. Expected \$ losses are shown on right hand sight. Boxes represent the maximum risk. The rectangular marked a the left most position is the right choose decision for minimax rule.

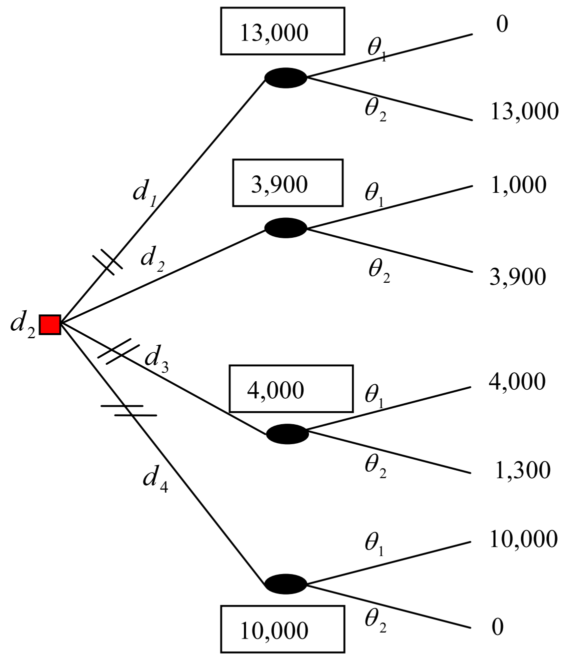

Fig. 4 Decision tree for minimax rule

D. *Bayes Rules in Protective Embankments Case* (*with Prior Information*)

Now, we consider about the computation of the Bayes Rule. On the basis of previous experience and maximum water levels of flood occurrences, we take $\pi(\theta_1) = 0.5$ and $\pi(\theta_2) = 0.5$ as the prior distribution.

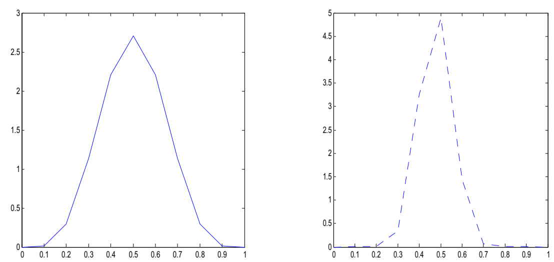

Fig. 5 Prior for θ_1 and θ_2

Using this prior distribution and the risk functions computed above, we find for each decision function its Bayes risk:

$$B(d) = E_{\pi(\theta)}[R(\theta, d)]$$
$$= R(\theta_1, d)\pi(\theta_1) + R(\theta_2, d)\pi(\theta_2).$$

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$$B(d_1) = 0 \times 0.5 + 13{,}000 \times 0.5 = 6{,}500$$
$$B(d_2) = 1{,}000 \times 0.5 + 3{,}900 \times 0.5 = 2{,}450$$
$$B(d_3) = 4{,}000 \times 0.5 + 1{,}300 \times 0.5 = 2{,}650$$
$$B(d_4) = 10{,}000 \times 0.5 + 0 \times 0.5 = 5{,}000.$$
(24)

We started from the right by computation of Bayes risks (using prior distribution $\pi(\theta)$) for all chance nodes (the nodes were evaluated). Then, we choose the branch with the smallest Bayes risks. It is the branch corresponding to decision d_2 and all others branches are pruned. In our problem, d_2 is the Bayes rule (among these four rules) and d_2 is Bayes action.

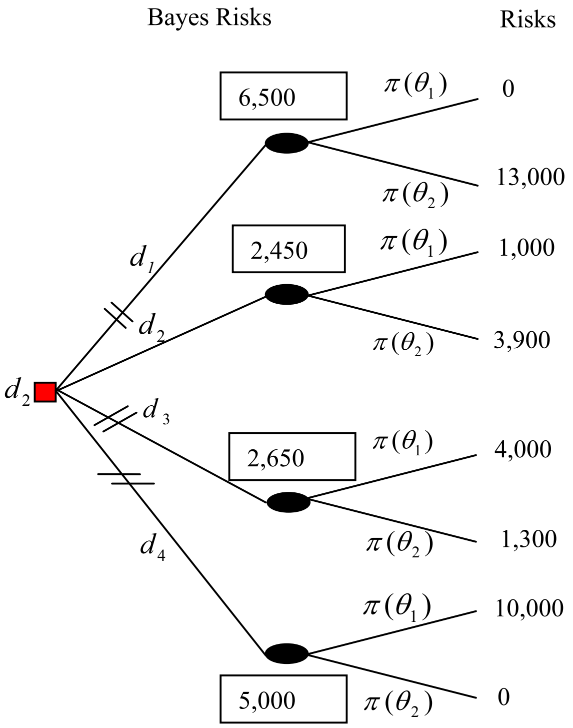

Fig. 6 Decision tree for Bayes rule of embankments case

In Fig. 6, θ_1 denotes the height of embankments, 1230 *cm* and θ_2 denotes the height of embankments, 1280 *cm*. d_1, d_2, d_3, and d_4 are decisions making. Expected \$ losses are shown on right-hand side. Expected \$ risks are given in boxes.

In this decision tree, the value of primary risk in the right most position is described and Bayes risk will be the second most and marked ovals. These sticks say that they cannot be chosen such as Bayes rule and the rectangular marked in the left most position is the right choose decision for Bayes rule.

E. Bayes Rule Action for Protective Embankment Cases (with Observed Data)

To consider again the protective embankments case for the Ayeyarwady River, we suppose that $X = x_2 = 1255\,cm$ is observed. We first calculate the posterior distribution;

$$h(\theta_1 | x_2) = \frac{f(x_2 | \theta_1)\pi(\theta_1)}{\sum_{i=1}^{3} f(x_2 | \theta_i)\pi(\theta_i)} = 0.6,$$

$$h(\theta_2 | x_2) = \frac{f(x_2 | \theta_2)\pi(\theta_2)}{\sum_{i=1}^{3} f(x_2 | \theta_i)\pi(\theta_i)} = 0.4.$$

Next, the posterior risk (PR) for a_1 and a_2 can be calculated as follows:

$$PR(a_1) = l(\theta_1, a_1)h(\theta_1 | x_2) + l(\theta_2, a_1)h(\theta_2 | x_2) = \$5{,}200$$
$$PR(a_2) = l(\theta_1, a_2)h(\theta_1 | x_2) + l(\theta_2, a_2)h(\theta_2 | x_2) = \$6{,}000.$$

By comparing these two functions, we can see that a_1 has the smaller posterior risk. Thus, choose a_1 as the Bayes rule action.

F. Bayesian Decision Rule and Minimum Error Rate Classification for Protective Embankments Case

In our application problem of the protection of flood event, we will decide θ_1 as a true state of nature because $h(\theta_1|x_1) \rangle\, h(\theta_2|x_1)$ (by using (9)) for $X = x_1 = 1230\,cm$. See Fig. 7. A Similar pattern is also noted on the observed, $X = x_2 = 1255\,cm$, we will decide θ_1 because $h(\theta_1|x_2) \rangle\, h(\theta_2|x_2)$. However, when $X = x_3 = 1280\,cm$ is observed, we can see that $h(\theta_1|x_3) < h(\theta_2|x_3)$ and θ_2 will be decided.

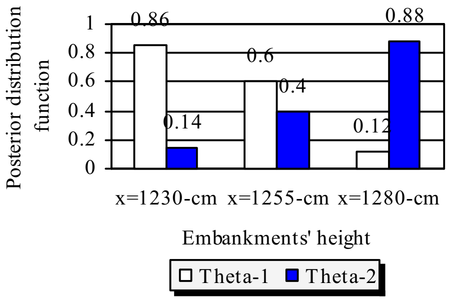

Fig. 7 Classification for θ_1 and θ_2

Thus, in our problem, after being calculated the probability error by using (15), (16), and (17), we get, the value of probability error is 0.12.

According to these computations, we can choose $x_3 = 1280\,cm$ as a terminal decision because it has the least value of probability error among three possible heights of embankments that has been considered in our problem.

This decision whether which will be good accuracy or not can be checked it by means of the minimum risk decision rules. By using the loss table (Table II) and the posterior distributions, we can compute the conditional risk;

$$R(a_j | x) = \sum_{i,j=1}^{2} l(a_j | \theta_i) h(\theta_i | x).$$
$$R(a_1 | x_3) = 0 \times 0.12 + 13{,}000 \times 0.88 = \$11{,}440.$$
$$R(a_2 | x_3) = 10{,}000 \times 0.12 + 0 \times 0.88 = \$1{,}200.$$

Thereafter applying the minimum risk decision rule, we can carry out that the optimal decision for θ_2, 1280 *cm* must be chosen since $R(a_1 \mid x_3) \rangle R(a_2 \mid x_3)$.

IV. Results and Discussions

We have introduced briefly some aspects of Bayesian decision theory that should serve as useful tools for engineers and the decision maker who may decide a Bayesian depending on the subject investigated and data available. These methods seem to have found favor in the economic and Business communities, partly because loss functions seem more straightforward to them. In Bayesian methods, whereas there is the problem of prior probabilities (i.e., unknown or vaguely defined, and the use of subjectivity), these probabilities can revise on receipt of additional data and consequent decision making is being implemented in one way or another in spite of the many uncertainties faced. The decision maker learns from the analysis and design, makes additional experiments, and gathers more information. According to this procedure, the decision maker can do less and less subject to error.

In this paper, Bayes and minimax rule have been considered and when we compute the minimax rule, we get d_2 is the minimax solution. Then we compute the Bayes rule on the basis of previous events and from actual-lengthening data by using the risk functions which have been calculated and the prior distribution from the forecasting of expert engineer's degree of belief.

From (24), we can see that d_2 is the Bayes rule by comparing the Bayes risk numbers. This Bayes rule is less conservative than the minimax rule in that it chooses action a_1 (1230 *cm* height) based on observation x_2 (1255 *cm* height sounding) because the prior distribution for this Bayes rule puts more weights on θ_1. If the prior distribution were changed sufficiently the Bayes rule would change.

To get the required optimal solution, after have been calculated the posterior risk with observed data $X = x_2 = 1255cm$ height (the additional information or sampling information); we choose the smaller risk function among these two (PR) functions (for two actions).

To do this, Bayesian decision method leads to an optimal decision considering the expected loss of all possible values of the random parameters values by posterior distribution. The object in decision making is to choose an action that minimizes the expected values of the loss function with respect to the posterior distribution, if data are available. However, if data are not available, the expected loss should be minimized with respect to the prior distribution.

Whenever we have the updated or new information and we want to check the probability error of observed data in their relative resource, Bayesian decision rule and the minimum error rate classification can be applied to have a terminal decision. These rule and classification are able to get the least value of probability error to be awareness of the good accuracy.

V. Conclusion and Further Extensions

In this study, we can see that Bayesian probability is an interpretation of probability suggested by Bayesian theory, which holds that the concept of probability can be defined as the degree to which a person believes a proposition and also suggests that Bayes' theorem can be used as a rule to infer or update the degree of belief in light of new information.

We can also see that the weakness of the minimax rule is intuitively apparent. It is very conservative procedure that places all its emphasis on guarding against the worst possible case. In fact, this worst case might not be very likely to occur. To make this idea more precise, we can assign a probability distribution to the state of nature; this distribution is called the prior distribution. We extend Bayes rule to situations in which a posterior distribution is estimated on the basis of observations or new data in addition to a known (or sometimes assumed). In such cases it can be justifiable to apply subjective probabilities quantifying personal knowledge or belief.

The posterior Bayes rule is to build an embankment of height 1230 *cm* because of its smaller posterior risk. This reflects a reduction in the uncertainty. According to this way, we get the Bayes optimal decision and when this is adopted, the agricultural expects to make a profit of $2,450 per embankment on average. If we want to check the height of maximum water level of Ayeyarwady River, it can be estimated with distribution and return period of their event.

In overall, this paper is presented on practical experiences in the gauging Sagaing station (with reference data, which is an actual-lengthening water levels and flood events) to be insights into the flood occurrence of Ayeyarwady River and protective embankments events. After have been studying about these events by Bayesian, these trends can concern about the environmental impacts and can describe some aspects of this theory to serve as useful tools for Civil and Environmental engineers.

This paper mentioned only a part of the author's research and approaching of her studies in protection of the flood events based on two standard maximum water levels in which emphasis only on the gauging Sagaing station. The real application of this research paper is the protection of flood events as mentioned in above. Therefore, further extensions of this paper are based on the collective information of maximum water levels during the monsoons to be more precise. After that calculating the optimal designing procedures under risk management that are expected to work well of sample risk decision problems, we get the optimal design which can be performed in the Environment.

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