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Traveling wave solutions for shallow water wave equation by $\left(\frac{G'}{G}\right)$-expansion method

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Abstract—This paper presents a new function expansion method for finding traveling wave solution of a non-linear equation and calls it the $\left(\frac{G'}{G}\right)$-expansion method. The shallow water wave equation is reduced to a non linear ordinary differential equation by using a simple transformation. As a result the traveling wave solutions of shallow water wave equation are expressed in three forms: hyperbolic solutions, trigonometric solutions and rational solutions.

Keywords—Shallow water wave equation, Exact solutions, $\left(\frac{G'}{G}\right)$-expansion method.

I. Introduction

THE general idea of dispersive waves originated from problems of water waves. It is well known that searching for exact solution of nonlinear evolution equation arising in mathematical physics plays an important role in study of nonlinear physical phenomena. The shallow water wave equations describe the evolution of incompressible flow, neglecting density change along the depth. Shallow water wave equations are applicable to cases where the horizontal scale of the flow is much bigger than the depth of fluid. An important class of solution of non-linear evolution equation is concerned with those of traveling waves that reduce the guiding partial differential equation of two variable namely x and t to an ordinary differential equation of one independent variable $u = x - ct$ where $c \in (R - \{0\})$ is a parameter signifying the speed with which the wave travels either to the right or to left. “The most incomprehensible thing about the word is that it is at all comprehensible” (Albert Einstein), but the question is how do we fully understand incomprehensible things? There are number of methods to find exact and numerical solutions of nonlinear equations have drawn a lot of interest by a diverse group of scientists. Some of these methods are the homogeneous balance method [10], Differential quadrature method [1], the tanh method [12], the Jacobi elliptic function expansion [4,6], the truncated Painlevé expansion [3], Lie Classical method [7]. Recently, Wang et al. [5,11] introduced a method called the $\left(\frac{G'}{G}\right)$-expansion method and obtain traveling solution for the four well established non-linear evolution equation. The performance of this method is reliable, simple and gives many new solution.

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Our aim in this paper is to present an application of the $\left(\frac{G'}{G}\right)$-expansion method to some non-linear problem to be solved by this method for first time [2,8,9].

In this paper, we pay attention to the analytical method for getting the exact solution of some NLEES. Among the possible exact solutions of NLEEs, certain solutions for special form may depend only on a single combination of variables such as traveling wave variables. Our main goal in this study is to present the $\left(\frac{G'}{G}\right)$-expansion method for constructing the traveling wave solutions.

$\left(\frac{G'}{G}\right)$-expansion method is described in section II. In section III, we applied this method to shallow water wave equation of fifth order and various exact solutions are obtained which included the hyperbolic functions, the trigonometric functions and rational functions. Finally, some conclusions are drawn.

II. Description of the $\left(\frac{G'}{G}\right)$-Expansion Method

In this section we describe the $\left(\frac{G'}{G}\right)$-Expansion Method for finding shallow water wave solution of non linear evolution equation. Suppose that a nonlinear equation say in two independent variable x and t is given by

$$P(u, u_x, u_t, u_{xx}, u_{tt}, u_{xt}, ...) = 0, \tag{1}$$

where $u = u(x,t)$ is an unknown function, P is a polynomial in $u = u(x,t)$ and its partial derivatives in which the highest order derivatives and the nonlinear terms are involved.In following we gives the main steps of improved $\left(\frac{G'}{G}\right)$-Expansion Method.

Step 1. Suppose that $u = u(x,t)$.The traveling wave variable allows us reducing to an ODE for u=u(ξ)

$$P(u, u', u'', ...) = 0, \tag{2}$$

where prime denotes the derivative with respect to ξ .

Step 2. Suppose the solution of equation can be expressed by a polynomial in $\left(\frac{G'}{G}\right)$ as follows.

$$u(\xi) = \sum_{i=0}^{N} c_i \left(\frac{G'(\xi)}{G(\xi)}\right), \tag{3}$$

where c_i are real constants with $c_i \neq 0$ to be determined, N is a positive integer to be determined. The function $G(\xi)$ is the

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solution of the auxiliary linear ordinary differential equation

$$G''(\xi)+\lambda G'(\xi)+\mu G(\xi)=0, \tag{4}$$

where λ and μ are real constants to be determined.

Step 3. Substituting (3) into (2) and using second order LODE (4). Separate all terms with same order of $\left(\frac{G'}{G}\right)$ together,the left hand side of (2) is converted into another polynomial in $\left(\frac{G'}{G}\right)$. Equating each coefficient of polynomial to zero .Then we get algebraic equation for c_i,.....λ and μ.

Step 4. Since the following general solution of equation (4) has been well known for us,then substituting c_i, c and general solution of equation (4) into (3). We have more traveling wave solution of non linear partial differential equation (1).

III. Applications of $\left(\frac{G'}{G}\right)$ Method

In this section, we apply the $\left(\frac{G'}{G}\right)$-expansion method to solve the Shallow water wave equation.

Shallow water wave (SWW) Equation

The Shallow water wave Equation (SWW) equation is

$$(u_t+u_x+c_1uu_x+c_2u_{xxx}+c_3u_xu_{xx}+c_4uu_{xxx}+c_5u_{xxxxx}=0, \tag{5}$$

where u is function of x and t.

According to the method described above in section 2, we make the transformation $u(x,t)=u(\xi),\xi=x-ct$. Then we get

$$-cu'+u'+c_1uu'+c_2u'''+c_3u'u''+c_4u'u'''+c_5u'''''=0, \tag{6}$$

where prime denotes the derivative with respect to ξ.

Now, balancing uu''' with u''''' gives $N=2$. Therefore, we can write the solution of equation (6) in the form

$$u(\xi)=a_0+a_1\left(\frac{G'}{G}\right)+a_2\left(\frac{G'}{G}\right)^2, \tag{7}$$

where $a_2\neq 0$ and $G=G(\xi)$. From equations (4) and (7), we derive.

$$\begin{aligned}u'(\xi)&=-2a_2\left(\tfrac{G'}{G}\right)^3-(a_1+2a_2\lambda)\left(\tfrac{G'}{G}\right)^2\\&-(a_1\lambda+2a_2\mu)\left(\tfrac{G'}{G}\right)\\&-a_1\mu,\end{aligned} \tag{8}$$

$$\begin{aligned}u''(\xi)&=6a_2\left(\tfrac{G'}{G}\right)^4+(10a_2\lambda+2a_1)\left(\tfrac{G'}{G}\right)^3\\&+(4a_2\lambda^2+8a_2\mu+3a_1\lambda)\left(\tfrac{G'}{G}\right)^2\\&+(a_1\lambda^2+2a_1\mu+6a_2\lambda\mu)\left(\tfrac{G'}{G}\right)\\&+a_1\lambda\mu,\end{aligned} \tag{9}$$

$$\begin{aligned}u'''(\xi)&=-24a_2\left(\tfrac{G'}{G}\right)^5-(6a_1+54a_2\lambda)\left(\tfrac{G'}{G}\right)^4\\&-(40a_2\mu+38a_2\lambda^2+12a_1\lambda)\left(\tfrac{G'}{G}\right)^3\\&-(52a_2\lambda\mu+8a_2\lambda^3+7a_1\lambda^2+8a_1\mu)\left(\tfrac{G'}{G}\right)^2\\&-(8a_1\lambda\mu+14a_2\lambda^2\mu+16a_2\mu^2+a_1\lambda^3)\left(\tfrac{G'}{G}\right)\\&-6a_2\lambda\mu^2-2a_1\mu^2-a_1\lambda^2\mu,\end{aligned} \tag{10}$$

$$\begin{aligned}u''''(\xi)&=120a_2\left(\tfrac{G'}{G}\right)^6+(336a_2\lambda+24a_1)\left(\tfrac{G'}{G}\right)^5\\&+(330a_2\lambda^2+240a_2\mu+60a_1\lambda)\left(\tfrac{G'}{G}\right)^4\\&+(50a_1\lambda^2+130a_2\lambda^3+40a_1\mu+440a_2\lambda\mu)\left(\tfrac{G'}{G}\right)^3\\&+(15a_1\lambda^3+16a_2\lambda^4+60a_1\lambda\mu+232a_2\lambda^2\mu+136a_2\mu^2)\left(\tfrac{G'}{G}\right)^2\\&+(a_1\lambda^4+22a_1\lambda^2\mu+120a_2\lambda\mu^2+16a_1\mu^2+30a_2\lambda^3\mu)\left(\tfrac{G'}{G}\right)\\&+16a_2\mu^3+14a_2\lambda^2\mu^2+a_1\lambda^3\mu+8a_1\lambda\mu^2,\end{aligned} \tag{11}$$

$$\begin{aligned}u'''''(\xi)&=-720a_2\left(\tfrac{G'}{G}\right)^7\\&-(2400a_2\lambda+120a_1)\left(\tfrac{G'}{G}\right)^6\\&-(360a_1\lambda+1680a_2\mu+3000a_2\lambda^2)\left(\tfrac{G'}{G}\right)^5\\&-(3960a_2\lambda\mu+1710a_2\lambda^3+390a_1\lambda^2+240a_1\mu)\left(\tfrac{G'}{G}\right)^4\\&-(480a_1\lambda\mu+180a_1\lambda^3+3104a_2\lambda^2\mu+1232a_2\mu^2+422a_2\lambda^4)\left(\tfrac{G'}{G}\right)^3\\&-(136a_1\mu^2+32a_2\lambda^5+884a_2\lambda^3\mu+1712a_2\lambda\mu^2+292a_1\lambda\mu\\&+31a_1\lambda^4)\left(\tfrac{G'}{G}\right)^2-(272a_2\mu^3+a_1\lambda^5+584a_2\lambda^2\ mu^2+52a_1\lambda^3\mu\\&+62a_2\lambda^4\mu+136a_1\lambda\mu^2)\left(\tfrac{G'}{G}\right)^1-(22a_1\lambda^2\mu^2+120a_2\lambda\mu-a_1\lambda^4\\&mu-30a_2\lambda^3\mu^2+16a_1\mu^3).\end{aligned} \tag{12}$$

Substituting equations (8-12) into equation (6), setting the coefficients of $\left(\frac{G'}{G}\right)^i,(i=0,1,2,3,4,5,6,7,)$ to zero, we obtain a system of algebraic equations for $a_0,a_1,a_2,c,c_1,c_2,c_3,c_4,c_5,\lambda$ and μ as follows:

$$\begin{aligned}&\left(\tfrac{G'}{G}\right)^7: 24c_4a_2^2+720c_5a_2+12c_3a_2^2,\\&\left(\tfrac{G'}{G}\right)^6: 32c_3a_2^2\lambda+10c_3a_2a_1+2400c_5a_2\lambda+30c_4a_2a_1\\&+120c_5a_1+54c_4a_2^2\lambda,\\&\left(\tfrac{G'}{G}\right)^5: 2c_3a_1^2+40c_4a_2^2\mu+24c_2a_2+24c_4a_0a_2\\&+1680c_5a_2\mu+3000c_5a_2\lambda^2+6c_4a_1^2+28c_3a_2^2\lambda^2\\&+28c_3a_2^2\mu+2c_1a_2^2+360c_5a_1\lambda+26c_3a_2a_1\lambda\\&+66c_4a_2a_1\lambda+38c_4a_2^2\lambda^2,\\&\left(\tfrac{G'}{G}\right)^4: 12c_4a_1^2\lambda+8c_4a_2^2\lambda^3+1710c_5a_2\lambda^3+240c_5a_1\mu\\&22c_3a_2a_1\mu+22c_3a_2a_1\lambda^2+54c_2a_2\lambda+3960c_5a_2\lambda\mu\\&+8c_3a_2^2\lambda^3+5c_3a_1^2\lambda+390c_5a_1\lambda^2+6c_4a_0a_1\\&+3c_1a_1a_2+54c_4a_0a_2\lambda+45c_4a_0a_2\lambda^2+52c_4a_2^2\lambda\mu\\&+48c_3a_2^2\lambda\mu+48c_4a_2a_1\mu+6c_2a_1+2c_1a_2^2\lambda,\end{aligned} \tag{13}$$

$$
\begin{aligned}
&\left(\tfrac{G'}{G}\right)^3 : 2a_2 + 3104c_5a_2^2\lambda^2\mu + 3c_1a_1a_2\lambda + 9c_4a_2a_1\lambda^3 \\
&+20c_3a_2^2\lambda^2\mu + 6c_3a_2a_1\lambda^3 + 14c_4a_2^2\lambda^2\mu \\
&+38c_4a_0a_2\lambda^2 + 12c_4a_0a_1\lambda + 480c_5a_1\lambda\mu + 40c_4a_0a_2\mu \\
&+8c_4a_1^2\mu + 2c_1a_0a_2 + 7c_4a_1^2\lambda^2 + 422c_5a_2\lambda^4 \\
&+4c_3a_1^2\lambda^2 + 40c_2a_2\mu + 2c_1a_2^2\mu + 1232c_5a_2\mu^2 \\
&+16c_4a_2^2\mu^2 + 38c_2a_2\lambda^2 + 180c_5a_1\lambda^3 + 4c_3a_1^2\mu \\
&+20c_3a_2^2\mu^2 + 12c_2a_1\lambda + c_1a_1^2\lambda - 2ca_2 + 60c_4a_2a_1\lambda\mu \\
&+36c_3a_2a_1\lambda\mu, \\
&\left(\tfrac{G'}{G}\right)^2 : a_1 + 2c_1a_0a_2\lambda + 3c_1a_1a_2\mu + 52c_2a_2\lambda\mu \\
&-2ca_2\lambda + c_4a_1^2\lambda^3 + c_3a_1^2\lambda^3 + c_1a_0a_1 \\
&+7c_2a_1\lambda^2 + 31c_5a_1\lambda^4 + c_1a_1^2\lambda + 8c_2a_1\mu \\
&+8c_2a_2\lambda^3 + 32c_5a_2\lambda^5 + 136c_5a_1\mu^2 + 52c_4a_0a_2\lambda\mu \\
&+15c_4a_2a_1\lambda^2\mu + 14c_3a_2a_1\lambda^2\mu + 2a_2\lambda - ca_1 \\
&+6c_3a_1^2\lambda\mu + 14c_3a_1a_2\mu^2 + 16c_3a_2^2\lambda\mu^2 + 8c_4a_1^2\lambda\mu \\
&+18c_4a_1a_2\mu^2 + 6c_4a_2^2\lambda\mu^2 + 8c_4a_0a_1\mu + 7c_4a_0a_1\lambda^2 \\
&+8c_4a_0a_2\lambda^3 + 884c_5a_2\lambda^3\mu + 1712c_5a_2\lambda\mu^2 + 292c_5a_1\lambda^2\mu, \\
&\left(\tfrac{G'}{G}\right)^1 : -2ca_2\mu + 16c_2a_2\mu^2 + 4c_3a_2^2\mu^3 + c_2a_1\lambda^3 \\
&+c_1a_1^2\mu + 10c_3a_1a_2\lambda\mu^2 + 14c_4a_0a_2\lambda^2\mu + 8c_4a_0a_1\lambda\mu \\
&+6c_4a_1a_2\lambda\mu^2 + 2c_4a_1^2\mu^2 + c_5a_1\lambda^5 + 2c_3a_1^2\mu^2 \\
&-ca_1\lambda + 272c_5a_2\mu^3 + c_1a_0a_1\lambda + c_4a_1^2\lambda^2\mu \\
&+c_4a_0a_1\lambda^3 + 2c_1a_0a_2\mu + 8c_2a_1\lambda\mu + 14c_2a_2\lambda^2\mu \\
&+2c_3a_1^2\lambda^2\mu + 16c_4a_0a_2\mu^2 + 584c_5a_2\lambda^2\mu^2 + 52c_5a_1\lambda^3\mu \\
&+62c_5a_2\lambda^4\mu + 136c_5a_1\lambda\mu^2 + 2a_2\mu + a_1\lambda, \\
&\left(\tfrac{G'}{G}\right)^0 : 120c_5a_2\lambda\mu^3 + c_5a_1\lambda^4\mu + 30c_5a_2\lambda^3\mu^2 + a_1\mu \\
&+6c_4a_0a_2\lambda\mu^2 + c_2a_1\lambda^2\mu + 2c_2a_1\mu^2 - ca_1\mu \\
&+2c_4a_0a_1\mu^2 + 22c_5a_1\lambda^2\mu^2 + 6c_2a_2\lambda\mu^2 + c_1a_0a_1\mu \\
&+c_4a_0a_1\lambda^2\mu + 16c_5a_1\mu^3 + c_3a_1^2\lambda\mu^2 + 2c_3a_1a_2\mu^3.
\end{aligned} \tag{14}
$$

Solving these systems of algebraic equations by Maple gives

Case 1.

$$
\begin{aligned}
&a_0 = a_0, a_1 = a_1, a_2 = a_2, \lambda = \tfrac{a_1}{a_2}, \mu = \mu, \\
&c_1 = \tfrac{c_4a_1^2 + 8c_4a_2^2\mu - 12c_2a_2 - 12c_4a_0a_2}{a_2^2}, \\
&c_2 = c_2, c_3 = \tfrac{-2(c_4a_2 + 30a_5)}{a_2}, c_4 = c_4, c_5 = c_5, \\
&c = \tfrac{\alpha}{a_4^2}
\end{aligned} \tag{15}
$$

where

$$
\begin{aligned}
\alpha = &-2a_2^3a_1^2c_4\mu - 2a_1^2a_0a_2^2c_4 + 8a_2^2a_1^2c_5\mu - a_1^2c_2a_2^2 \\
&-16a_2^2a_0c_4\mu - a_1^4c_5 - a_2^4 + 12a_0a_2^3c_2 + 12a_0a_2^3c_4 \\
&-8a_2^4c_2\mu + 4a_2^5c_4\mu^2 - 16c_5a_2^4\mu^2,
\end{aligned} \tag{16}
$$

and μ, λ and $a_0, a_1, a_2, c, c_1, c_2, c_3, c_4, c_5$ are arbitrary constants.

Case 2.

$$
\begin{aligned}
&a_0 = a_0, a_1 = a_1, a_2 = a_2, c_1 = \tfrac{-3c_4(a_1^2 - 2a_2a_1\lambda + a_2^2\lambda^2)}{2a_2^2}, \\
&c_2 = \tfrac{-c_4(a_1^2 - 26a_2a_1\lambda + 48a_0a_2 + 13a_2^2\lambda^2)}{48a_2}, \\
&c_3 = \tfrac{-13c_4}{4}, c_4 = c_4, c_5 = \tfrac{c_4a_2}{48}, \\
&\lambda = \lambda, \\
&c = \tfrac{\beta}{4a_2^2}, \\
&\mu = \tfrac{-a_1(a_1 - 2a_2\lambda)}{a_2^3}
\end{aligned} \tag{17}
$$

where

$$
\begin{aligned}
\beta = &-24a_2^3a_1c_4\lambda^3 + 33a_2^2a_1^2c_4\lambda - 18a_2a_1^2c_4\lambda + 12a_2^3c_4a_0\lambda \\
&+6a_2^4c_4\lambda^4 + 3a_1^4c_4 - 8a_2^3 - 24a_2^2c_4a_0\lambda + 12a_2a_1^2c_4a_0,
\end{aligned} \tag{18}
$$

and μ, λ and $a_0, a_1, a_2, c, c_1, c_2, c_3, c_4, c_5$ are arbitrary constants.

Case 3.

$$
\begin{aligned}
&a_0 = a_0, a_1 = a_1, a_2 = a_2, \lambda = \lambda, \mu = \tfrac{a_1^2 - 2a_2a_1\lambda + 10a_2\lambda^2}{36a_2^2}, \\
&c_1 = \tfrac{c_4(a_1^2 - 2a_2a_1\lambda + a_2^2\lambda)}{9a_2^2}, \\
&c_2 = \tfrac{-c_4(-13a_1^2 - 190a_2a_1\lambda + 432a_0a_2 + 95a_2^2\lambda^2)}{432a_2}, c_3 = \tfrac{-13c_4}{4}, \\
&c_4 = c_4, c_5 = \tfrac{c_4a_2}{48}, \\
&c = \tfrac{\gamma}{324a_2^3},
\end{aligned} \tag{19}
$$

where

$$
\begin{aligned}
\gamma = &-32a_2^3a_1c_4\lambda + 39a_2^2a_1^2c_4\lambda^2 - 14a_2a_1^3c_4\lambda + 36a_2^3c_4a_0\lambda^2 \\
&-8a_2^4c_4\lambda^4 - a_1^4c_4 + 32a_2^3 - 72a_2^2a_1c_4a_0\lambda + 36a_2a_1^2c_4a_0
\end{aligned} \tag{20}
$$

and μ, λ and $a_0, a_1, a_2, c, c_1, c_2, c_3, c_4, c_5$ are arbitrary constants.

For Case 1, Substituting the solution set (15) and the corresponding solutions of (4) into (7), we have the solutions of equation (6) as follows:

When $\lambda^2 - 4\mu > 0$, we obtain the hyperbolic function shallow water wave solutions

$$
\begin{aligned}
u_{11}(\xi) = 1 + 4 &\left(\frac{\sqrt{\lambda^2 - 4\mu}}{2} \left(\frac{C_1 \sinh\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\xi\right) + C_2 \cosh\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\xi\right)}{C_1 \cosh\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\xi\right) + C_2 \sinh\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\xi\right)} - \frac{\lambda}{2} \right) \right) \\
&+ \left(\frac{\sqrt{\lambda^2 - 4\mu}}{2} \left(\frac{C_1 \sinh\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\xi\right) + C_2 \cosh\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\xi\right)}{C_1 \cosh\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\xi\right) + C_2 \sinh\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\xi\right)} - \frac{\lambda}{2} \right) \right)^2 .
\end{aligned} \tag{21}
$$

When $\lambda^2 - 4\mu < 0$, we obtain the trigonometric function shallow water wave solutions

$$
\begin{aligned}
u_{12}(\xi) = 1 + 2 &\left(\frac{\sqrt{4\mu - \lambda^2}}{2} \left(\frac{-C_1 \sin\left(\frac{\sqrt{4\mu-\lambda^2}}{2}\xi\right) + C_2 \cos\left(\frac{\sqrt{4\mu-\lambda^2}}{2}\xi\right)}{C_1 \cos\left(\frac{\sqrt{4\mu-\lambda^2}}{2}\xi\right) + C_2 \sin\left(\frac{\sqrt{4\mu-\lambda^2}}{2}\xi\right)} - \frac{\lambda}{2} \right) \right) \\
&+ \left(\frac{\sqrt{4\mu - \lambda^2}}{2} \left(\frac{-C_1 \sin\left(\frac{\sqrt{4\mu-\lambda^2}}{2}\xi\right) + C_2 \cos\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\xi\right)}{C_1 \cos\left(\frac{\sqrt{4\mu-\lambda^2}}{2}\xi\right) + C_2 \sin\left(\frac{\sqrt{4\mu-\lambda^2}}{2}\xi\right)} - \frac{\lambda}{2} \right) \right)^2 .
\end{aligned} \tag{22}
$$

When $\lambda^2 - 4\mu = 0$, we obtain the rational function solutions

$$
u_{13}(\xi) = \frac{C_2}{C_1 + C_2\xi} - \frac{\lambda}{2}, \tag{23}
$$

where $\xi = x - ct$, where c is given by equation (15).

For Case 2, Substituting the solution set (17) and the corresponding solutions of (4) into (7), we have the solutions of equation (6) as follows:

When $\lambda^2 - 4\mu > 0$, we obtain the hyperbolic function shallow

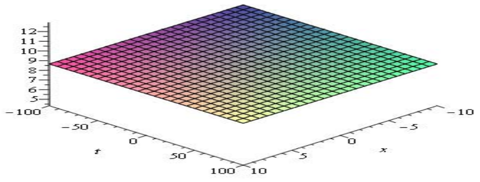

Fig. 1. Graphical representation of solution (21), when $\lambda = 4, \mu = 2, a_0 = 1, a_1 = 4, a_2 = 1, c = 175, c_1 = 8, c_2 = 1, c_3 = -62, c_4 = 1, c_5 = 1$

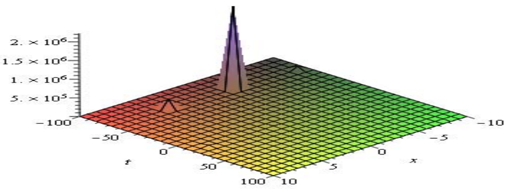

Fig. 2. Graphical representation of solution (22), when $\lambda = 2, \mu = 2, a_0 = 1, a_1 = 2, a_2 = 1, c = -123, c_1 = -4, c_2 = 1, c_3 = -62, c_4 = 1, c_5 = 1$

water wave solutions

$$u_{21}(\xi) = 1 + 2\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\left(\frac{C_1\sinh\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\xi\right)+C_2\cosh\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\xi\right)}{C_1\cosh\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\xi\right)+C_2\sinh\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\xi\right)}-\frac{\lambda}{2}\right)\right) + 1\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\left(\frac{C_1\sinh\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\xi\right)+C_2\cosh\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\xi\right)}{C_1\cosh\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\xi\right)+C_2\sinh\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\xi\right)}-\frac{\lambda}{2}\right)\right)^2 \tag{24}$$

When $\lambda^2 - 4\mu < 0$, we obtain the trigonometric function

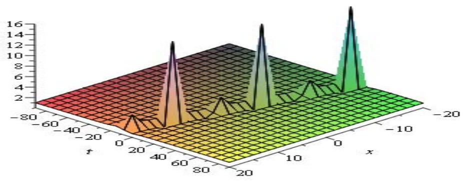

Fig. 3. Graphical representation of solution (23), when $\lambda = 2, \mu = 1, a_0 = 1, a_1 = 2, a_2 = 1, c = -1, c_1 = -12, c_2 = 1, c_3 = -62, c_4 = 1, c_5 = 1$

shallow water wave solutions

$$u_{22}(\xi) = a_0 + a_1\left(\frac{\sqrt{4\mu-\lambda^2}}{2}\left(\frac{-C_1\sin\left(\frac{\sqrt{4\mu-\lambda^2}}{2}\xi\right)+C_2\cos\left(\frac{\sqrt{4\mu-\lambda^2}}{2}\xi\right)}{C_1\cos\left(\frac{\sqrt{4\mu-\lambda^2}}{2}\xi\right)+C_2\sin\left(\frac{\sqrt{4\mu-\lambda^2}}{2}\xi\right)}-\frac{\lambda}{2}\right)\right) + a_2\left(\frac{\sqrt{4\mu-\lambda^2}}{2}\left(\frac{-C_1\sin\left(\frac{\sqrt{4\mu-\lambda^2}}{2}\xi\right)+C_2\cos\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\xi\right)}{C_1\cos\left(\frac{\sqrt{4\mu-\lambda^2}}{2}\xi\right)+C_2\sin\left(\frac{\sqrt{4\mu-\lambda^2}}{2}\xi\right)}-\frac{\lambda}{2}\right)\right)^2. \tag{25}$$

When $\lambda^2 - 4\mu = 0$, we obtain the rational function solutions

$$u_{23}(\xi) = \frac{C_2}{C_1 + C_2\xi} - \frac{\lambda}{2}, \tag{26}$$

where $\lambda = \lambda, \mu = \frac{-a_1(a_1-2a_2\lambda)}{4a_2^2}$,
$\xi = x - ct$, where c is given by equation (17).

For Case 3, Substituting the solution set (19) and the corresponding solutions of (4) into (7), we have the solutions of equation (6) as follows:
When $\lambda^2 - 4\mu > 0$, we obtain the hyperbolic function shallow water wave solutions

$$u_{31}(\xi) = a_0 + a_1\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\left(\frac{C_1\sinh\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\xi\right)+C_2\cosh\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\xi\right)}{C_1\cosh\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\xi\right)+C_2\sinh\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\xi\right)}-\frac{\lambda}{2}\right)\right) + a_2 \cdot \left(\frac{\sqrt{\lambda^2-4\mu}}{2}\left(\frac{C_1\sinh\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\xi\right)+C_2\cosh\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\xi\right)}{C_1\cosh\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\xi\right)+C_2\sinh\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\xi\right)}-\frac{\lambda}{2}\right)\right)^2. \tag{27}$$

When $\lambda^2 - 4\mu < 0$, we obtain the trigonometric function

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shallow water wave solutions

$$\begin{aligned} u_{32}(\xi) &= a_0 \\ &+a_1 \left(\frac{\sqrt{4\mu-\lambda^2}}{2} \left(\frac{-C_1 \sin\left(\frac{\sqrt{4\mu-\lambda^2}}{2}\xi\right)+C_2\cos\left(\frac{\sqrt{4\mu-\lambda^2}}{2}\xi\right)}{C_1\cos\left(\frac{\sqrt{4\mu-\lambda^2}}{2}\xi\right)+C_2\sin\left(\frac{\sqrt{4\mu-\lambda^2}}{2}\xi\right)} - \frac{\lambda}{2} \right) \right) \\ &+a_2 \left(\frac{\sqrt{4\mu-\lambda^2}}{2} \left(\frac{-C_1 \sin\left(\frac{\sqrt{4\mu-\lambda^2}}{2}\xi\right)+C_2\cos\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\xi\right)}{C_1\cos\left(\frac{\sqrt{4\mu-\lambda^2}}{2}\xi\right)+C_2\sin\left(\frac{\sqrt{4\mu-\lambda^2}}{2}\xi\right)} - \frac{\lambda}{2} \right) \right)^2. \end{aligned} \tag{28}$$

When $\lambda^2 - 4\mu = 0$, we obtain the rational function solutions

$$u_{33}(\xi) = \frac{C_2}{C_1 + C_2\xi} - \frac{\lambda}{2}, \tag{29}$$

where $\lambda = \lambda, \mu = \frac{a_1^2 - 2a_2^2 a_1^2 \lambda + 10a_2^2\lambda^2}{36a_2^2}$,
$\xi = x - ct$, where c is derived in equation (19).

IV. Discussion and Concluding Remarks

On comparing between the $\left(\frac{G'}{G}\right)$-expansion method and the other methods, we come to the conclusion that $\left(\frac{G'}{G}\right)$-expansion method is more powerful, effective and convenient. We have shown graphical representation of solution (21), (22) and (23) in figure1, figure2, figure3 respectively. In the similar way we can represent graphically the behaviour of other derived solutions. As a result exact shallow water wave solution hyperbolic function solution, trigonometric function and rational solutions. As we can use the MATHEMATICA or MAPLE to find out a useful solution of the algebraic equation, so we can avoids difficult calculations. The authenticity of solutions has been checked by aid of software MAPLE.

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References

[1] R. Jiwari, S. Pandit, R. C. Mittal. *Numerical Simulation Of two - dimensional Sine-Gorden solitons by differential quadrature method*, Computer Physics Communciation, vol. 183,2012, pp. 600-616.

[2] K. A. Gapreel, *A Generalized* $\left(\frac{G'}{G}\right)$*-expansion method to find the traveling wave solutions of nonlinear evolution equations*, Journal of Partial Differential Equations, vol. 24, 2011, pp. 55-69.

[3] A. A. Mohammad and M. Can, *Painlevé Analysis and Symmetries of the HirotaSatsuma Equation*, Nonlinear Mathematical Physics, vol. 3, 1996, pp. 152-155.

[4] S. Zhang and H. Q. Zhang, *Discrete Jacobi elliptic function expansion method for nonlinear differential-difference equations*, Physica Scripta, vol. 80, 2009, pp. 045002.

[5] M. Wang, X. Li, J. Zhang, *The* $\left(\frac{G'}{G}\right)$*-expansion method and travelling wave solutions of nonlinear evolution equations in mathematical physics*, Physics Letters A, vol. 372, 2008, pp. 417-428.

[6] J. P. Yu and Y. L. Sun, *Weierstrass Elliptic Function Solutions to Nonlinear Evolution Equations*, Communication in Theoretical Physics, vol. 50, 2008, pp. 295-298.

[7] N. Goyal and R. K. Gupta, *The On symmetries and exact solution of Einstein vacuum equations for Axially symmetric Gravitional Fields*, International Journal of Mathematical and Computational Science, vol. 6,2012 pp. 104-107.

[8] B. Han, *Promotion* $\left(\frac{G'}{G}\right)$*-expansion method for solving nonlinear equation*. chifeng college Letters: Natural Science Press (2010).

[9] Sheng Zhang, Ling Dong, Jin-Mei Ba, Ying-Na Sun. *The* $\left(\frac{G'}{G}\right)$*-expansion method for solving nonlinear differential difference equation*, Physics Letters A, vol. 373,2009 pp. 905-910.

[10] Ming Liang Wang, ZhiBing Li,Yu Bing Zhou. *The Homogenous Balance Principal and its Application*. Physics Letters of LanZhou University, vol. 35, 1999, pp. 8-16.

[11] Wang M. , X. Li and J. Zang, $\left(\frac{G'}{G}\right)$*-expansion method and travelling wave solution equation of nonlinear evolution equation* in mathematical physics. Physics.Lett.A, vol. 372, 2008, pp. 417-423.

[12] W. Malfielt and W. Hereman, *The tanh method: I. Exact solutions of nonlinear evolution and wave equations*, Physica Scripta, vol. 54, 1996, pp. 563-568.