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Proposing a Pareto-based Multi-Objective Evolutionary Algorithm to Flexible Job Shop Scheduling Problem

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Abstract**—**During last decades, developing multi-objective evolutionary algorithms for optimization problems has found considerable attention. Flexible job shop scheduling problem, as an important scheduling optimization problem, has found this attention too. However, most of the multi-objective algorithms that are developed for this problem use nonprofessional approaches. In another words, most of them combine their objectives and then solve multi-objective problem through single objective approaches. Of course, except some scarce researches that uses Pareto-based algorithms. Therefore, in this paper, a new Pareto-based algorithm called controlled elitism non-dominated sorting genetic algorithm (CENSGA) is proposed for the multi-objective FJSP (MOFJSP). Our considered objectives are makespan, critical machine work load, and total work load of machines. The proposed algorithm is also compared with one the best Pareto-based algorithms of the literature on some multi-objective criteria, statistically.

Keywords**—**Scheduling, Flexible job shop scheduling problem, controlled elitism non-dominated sorting genetic algorithm

I. Introduction

FLEXIBLE job shop scheduling problem (FJSP) is known as one the most important scheduling problems in both cases, theoretical and practical areas. During last decades, because of multi-objective nature of the real world problems, its multi-objective version, called multi-objective FJSP (MOFJSP), has also found more attention. This problem is a developed version of the job shop scheduling problem (JSP) in which operation can be operated by machines from their set of capable machines [1]. Consequently, in FJSP, there are two main obstacles, including 1) assignment of the operation to machine and 2) sequencing of the operations. Since JSP belongs to NP-Hard class of the optimization problem [2], FJSP is known as a NP-Hard problem too.

In the literature, some researchers considered the two mentioned obstacles separately and proposed a hierarchical approach. Brandimarte [2] and Barnes and Chambers [3] are two examples that used this approach in single objective environment and Xia and Wu [4] is an example of the users of this approach in multi-objective environment. For instance, Xia and Wu [4] proposed an algorithm in which SA is used for operation sequence and PSO is used for machine assignment sub problem.

On the other hand, most studies optimize these two sub-problems, simultaneously. Many of these studies had used tabu search (TS) algorithm ([5]-[6]-[7]-[8]) or genetic algorithm (GA) ([9]-[10]-[11]) in their single objective proposing algorithms. However, in recent years, new generation of the meta-heuristic algorithms like variable neighborhood search (VNS) [12] or biogeography-based optimization (BBO) algorithm [13] have been also introduced to the single objective area of the FJSP.

In the multi-objective literature of the integrated approach, Kacem et al. [14] developed a localization approach. Liu et al. [15] proposed an algorithm, called VNPSO, in which VNS and particle swarm optimization (PSO) algorithms are combined. Gao, et al. [16] used genetic algorithm to solve multi objective FJSP. Wang et al. [1] proposed a multi-objective genetic algorithm (MOGA) in which immune and entropy principles are used to guide the Pareto-based optimization process. It worth to be mentioned their algorithm dominate most of the famous algorithm of the literature.

As it mentioned, most of the studies of the literature are aggregated single-objective algorithms. But, recently, a new generation of the multi-objective algorithms has been introduced. These algorithms don't convert a multi-objective problem to a single objective one [17] and are more strength to guide multi-objective process. Non-dominated sorting genetic algorithm (NSGAII) is one the most famous algorithms of these category which was proposed by [17]. A controlled based version of the NSGAII is called controlled elitism non-dominated sorting genetic algorithm (CENSGA) [18]. The major difference of the CENSGA with NSGAII is in selection strategy that in CENSGA all fronts participate in the selection through a geometric distribution.

In this paper, CENSGA is developed for the MOFJSP. Then, the proposed algorithm is compared with one the best MOEAs of the literature called MOGA [1]. To do so, in the beginning, some multi-objective criteria are introduced. Then, by using a non-parametric statistical test, called Mann-Whitney test, algorithms are compared on Brandimarte library [2].

Rest of the paper is organized as follow. Next section introduces MOFJSP. Section 3 explains fundamental principles of MOEAs and our proposed algorithm. Section 4, by introducing some multi-objective metrics, compares proposed algorithm with the literature. Finally, Section 5 presents the conclusion and suggests some future work.

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II. PROBLEM DEFINITION

In MOFJSP, as many other scheduling problems, n jobs $J\,(J_i, i \in \{1,2,...,n\})$ should be operated by means of m existing machine $M\,(M_k, k \in \{1,2,...,m\})$. In this model, Job J_i, is consisted of n_i operations. For each of these operations (O_{ij}) a predetermined set of capable machines is considered (M_{ij}). One of these capable machines should be selected to do the operation. The processing time and start time of operation $j\ (O_{ij})$ of job J_i on machine k are denoted by p_{jik} and t_{jik}, respectively.

Now, according to what mentioned, MOFJSP is going to optimize some objective functions simultaneously. In this paper, these objectives are maximal makespan $(C_{\max})$, critical machine work load (CWL), and total work load (TWL) of machines. In fact, MOFJSP is going to 1) assign each operation to a suitable machine and 2) determine the sequence of assigned operation on each machine in a way that mentioned objectives being optimized. These objective functions are optimized in Eq.1 to Eq.3. In these equations, x_{jik} denotes an assignment decision variable and C_k denotes complementation time of machine k. The assumptions of the MOFJSP are as follows.

- Fixed and predetermined order is assumed for the operations of each job.
- Among operations of different jobs priority restriction isn't assumed.
- Jobs priorities are the same.
- In the beginning (at time 0), jobs are released and machines are available.
- Move times between operations and setup times of machines are ignorable.
- Only one job can be processed on each machine at each specific moment and during the process, operations can't be broken off.

$$C_{max} = \max\{C_K \mid k = 1, \ldots, n\} \tag{1}$$

$$TWL = \sum_{i=1}^{n} \sum_{j=1}^{n_i} \sum_{k=1}^{m} P_{ijk} X_{ijk} \quad k: 1,2,3,\ldots,m \tag{2}$$

$$CWL = max\ \{\sum_{i=1}^{n} \sum_{j=1}^{n_i} P_{ijk} X_{ijk}\} \quad k: 1,2,3,\ldots,m \tag{3}$$

An example of a FJSP is shown in Table1 [13]. This example presents a FJSP with 3 jobs and 4 machines. Numbers of the table presents processing times of operations on different machines of their set of capable machines and symbol '-' shows an infeasible situation in which the operation cannot be processed on corresponding machine.

TABLE I
AN EXAMPLE OF FJSP WITH 3 JOBS AND 4 MACHINES [13]

FJSP		Processing Times			
		M1	M2	M3	M4
J1	O1,1	2	-	1	6
	O1,2	5	3	-	2
	O1,3	-	2	4	-
J2	O2,1	7	-	-	11
	O2,2	4	4	12	8
J3	O3,1	2	-	7	9
	O3,2	3	5	8	1
	O3,3	4	3	-	5

III. CONTROLLED ELITISM NON-DOMINATED SORTING GENETIC ALGORITHM

As mentioned, Pareto-based algorithms are the most professional approaches in which Pareto optimality is incorporated in the selection process. In these approaches, the optimization process is guided by considering all objective functions simultaneously. It means that they don't convert a multi-objective problem to single objective one. One of these algorithms is CENSGA which is a developed version of the popular NSGAII [17]. In this part of the paper operators of this algorithm are presented. However, since this paper want to compare CENSGA with MOGA of Wang et al. [1], most of the CENSGA's operator are designed like MOGA. In this way, we can minimize the effects of different operators on the performance of the algorithms. Therefore, different results of the algorithms are just according to their search ability. In the beginning, some fundamental concept of multi-objective algorithms is defined.

A. Fundamental concept of multi-objective algorithms

Assume a minimization model of a set of conflict objectives $f(\vec{x}) = \left[f_1(\vec{x}),..., f_m(\vec{x})\right]$ subject to, $g_i(\vec{x}) \le 0, i = 1,2,...,c, \vec{x} \in X$ ($\vec{x}$ denotes an n-dimensional vector that can gets real, integer, or even Boolean value and X is the feasible region). Now, solution $\vec{a}$ dominates solution $\vec{b}\ (\vec{a}, \vec{b} \in X)$ if:

1) $f_i(\vec{a}) \le f_i(\vec{b}),\ \ \forall i = 1, 2, ..., m$

2) $\exists i \in \{1, 2, ..., m\}: f_i(\vec{a}) < f_i(\vec{b})$

Under these circumstances, a set of solutions that cannot dominate each other is called Pareto solutions set or Pareto front. Then, the objective is to obtain Pareto optimal front. For obtaining Pareto optimal front two main characteristics should be achieved [17], including 1) good convergence of the Pareto front and 2) good diversity within the solutions of the Pareto front.

B. Initialization

This operator of the evolutionary algorithms can have a great deal in improving the performance of the meta-heuristic algorithms. As mentioned, in this paper Wang et al.'s [1] approach is utilized. In their method, first, operation sequence vector is generated randomly. Then, two machines are selected for each operation from the set of capable machines. Now, if a randomly generated number ($Rand \in [0,1]$) is less than 0.8, machine with shorter process time is chosen, otherwise, machine with longer process time is chosen.

C. Encoding and decoding scheme of the chromosome

As mentioned, to minimize the effect of different operators on the performance of the algorithms, another similar operator of our algorithm with MOGA of Wang et al. [1] is the chromosome representation. A scheme of this representation is shown is Fig.1.

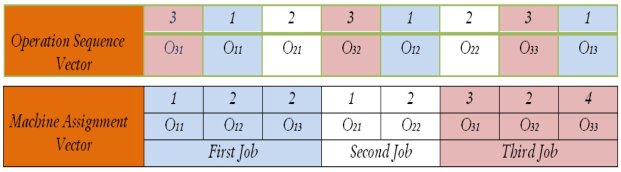

Fig. 1 A two vectors representation for a 3 jobs, 4 machines, 8 operations FJSP [13]

Considered decoding process of this paper produces active schedules. To do so, the decoding process starts from the left of the operation sequence vector and for each operation assigned machine is determined from machine assignment vector. Then, each operation is located on the first feasible available time of its assigned machine [1].

D. Selection method and elitism

In single objective algorithms, generally fitness value or objective function value is used to rank the solutions of the population. However, in Pareto-based multi-objective algorithms, domination concept is used for ranking. In NSGAII, for inserting dominance concept, an operator, called fast non-dominated sorting (FNDS) is developed [17]. The less value of FNDS means a better rank. In fact, this operator is used for searching the first objective of Pareto-based algorithms which is good convergence. To search the other objective which is good diversity, another operator called crowing distance (CD) is considered in NSGAII [17]. This operator is used for solutions of the same rank and estimates density of solutions which are laid surrounding a particular solution. More value of CD shows a better solution which is laid in a less crowded area [17]. Then, a binary tournament selection is performed according to these two operators.

CENSGA is a developed version of the NSGAII in which a specific selection is done such that all fronts participate in the selection strategy [17]. However, better fronts have more participation and affect on the next generation. This process is controlled by a geometric distribution. Different selection strategy of CENSGA vs. NSGAII is shown in Fig.2. Equation 4 formulates this distribution. In this equation, n_i denotes the maximum number of the allowed individuals in the i^{th} front and r (<1) denotes the reduction rate.

$$n_i = rn_{i-1} \tag{4}$$

It is also worth to be mentioned, in a population of size *N*, the maximum number of individuals which is allowed in each $i^{th}(i = 1, 2, ..., k)$ front is calculated as (5).

$$n_i = N \frac{1-r}{1-r^k} r^{i-1} \tag{5}$$

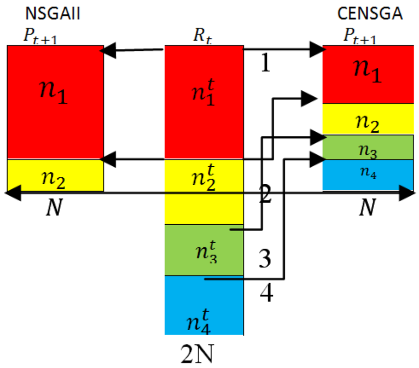

Fig. 2 Selection strategy of CENSGA vs. NSGAII [17]

E. Crossover operator

This operator is another similar operator with MOGA of Wang et al. [1]. In their method, they proposed improved precedence operation crossover (IPOX) and multipoint preservative crossover (MPX) for operation sequence vector and machine assignment vector, respectively.

F. Mutation operator

This operator is the final similar operator with Wang et al.'s MOGA [1]. In this operator, swap method is used for operation sequence vector and machine changing is used for machine assignment vector [1].

G. CENSGA flow chart

Figure 3 summarizes CENSGA algorithm schematically.

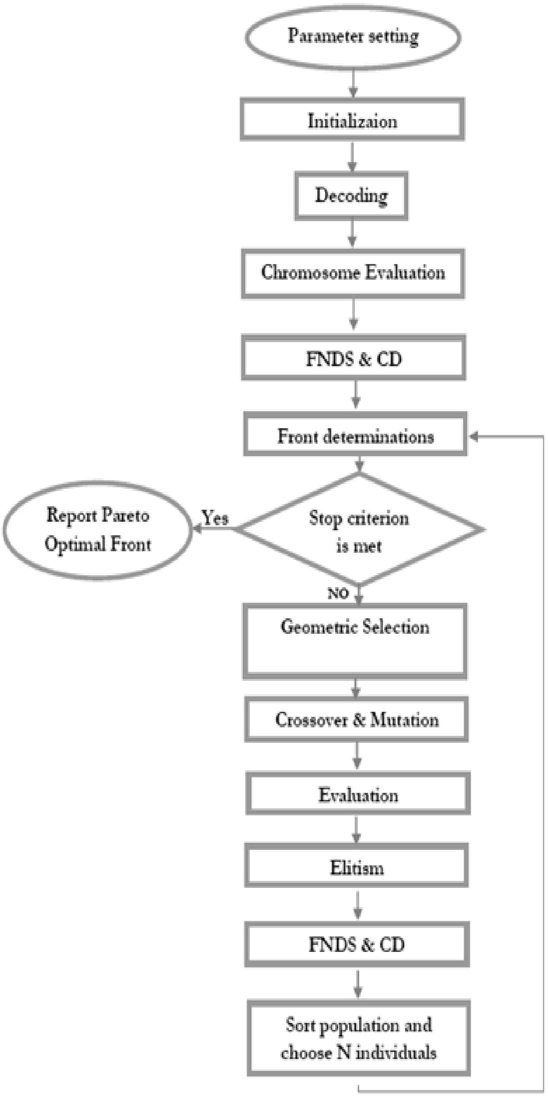

Fig. 3 Flow chart of CENSGA algorithm

IV. Computational Experience

This section compares proposed algorithm with Wang et al.'s algorithm [1]. To do so, the impact of different operators on the results of the algorithms by designing our genetic operators similar to Wang's operators has been minimized. In this way, the comparisons of the proposed algorithm with MOGA of Wang et al. [1] will be more sensible. In this comparison Brandimarte' library [2] with 10 test problems is used. Algorithms are written by Matlab software on a PC with 4 GB RAM and 2.4 GHz CPU. Population and iteration size, in all test problems, are set as 200, Pc=%85(Crossover Rate), and Pm=%10(Mutation rate). In following sub-sections of this section, computational results of the algorithm on some multi-objective performance metrics, are presented.

A. Performance measures

To compare proposed algorithm with the literatures MOGA four common metrics of multi-objective literature are implemented as follows.

- Diversity: measures the extension of the Pareto front [18].
- Spacing: measures the standard deviation of the distances among solutions of the Pareto front [19].
- Mean ideal distance (MID): measures the convergence of Pareto fronts to a certain point $(0,0)$ [20].
- Number of found solutions (NOS): measures number of the Pareto solutions in Pareto optimal front.

B. Computational results

The outputs of the mentioned metrics are shown in Table 2. Then, they metrics are evaluated statistically by means of Mann-Whitney test [21]. Output of this statistical test is shown on Table 3.

Before describing the metrics, it should be noticed that for *Diversity* and *NOS* the more value is better, while as for *Spacing* and *MID* less value is better. Now, in a total view to summarized results presented in last row of Table 2, CENSGA has a better value of the *Spacing* and *NOS*, while as MOGA has a better value of the *Diversity* and *MID*. The outputs of these metrics on different test problems for each metric are also shown in Fig.4.

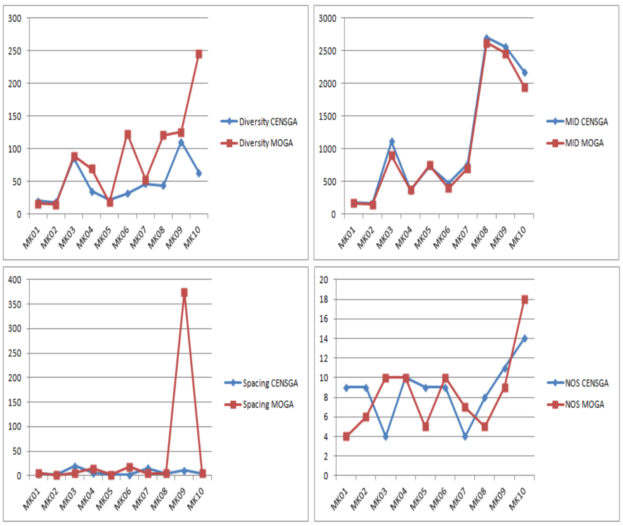

Fig. 4 Outputs of metrics on different test problems

TABLE II
COMPARISON OF CENSGA WITH MOGA ON MULTI-OBJECTIVE METRICS

Problem	$n \times m$	T_0	Flex.	Proposed CENSGA				MOGA			
				Diversity	*Spacing*	*MID*	*NOS*	*Diversity*	*Spacing*	*MID*	*NOS*
MK01	10*6	58	2.09	20.49	4.03	175.36	9	16.03	5.76	169.06	4
MK02	10*6	150	4.10	18.05	1.74	170.8	9	14.79	1.38	150.00	6
MK03	15*8	90	3.01	85.66	20.13	1114.9	4	89.59	5.74	903.00	10
MK04	15*8	106	1.91	35.51	5.19	364.3	10	69.74	14.16	371.00	10
MK05	15*4	150	1.71	21.65	2.3	737.28	9	18.35	2.00	752.00	5
MK06	10*15	100	3.27	32.06	1.64	476.83	9	123.35	18.40	398.08	10
MK07	20*5	225	2.83	46.91	15.44	772.7	4	53.00	4.69	705.00	7
MK08	20*10	240	1.43	43.82	4.63	2700.8	8	121.00	5.07	2625.60	5
MK09	20*10	240	2.53	110.7	11.4	2563.59	11	125.52	374.67	2457.04	9
MK10	20*15	58	2.98	63.7	4.96	2174.9	14	246.10	5.00	1946.90	18
	Total value			478.55	**71.46**	11251.46	**87**	**877.47**	436.87	**10477.68**	84

However, mentioned total results cannot be proved statistically. According to Table 3, there is no statistically significant difference among our proposed algorithm and MOGA as one the best algorithms of the literature. Figure 4 is another witness for this similarity. Therefore, according to both casual and statistical looking at the outputs proposed algorithm is at least as good as one the best algorithms of the literature.

TABLE III
STATISTICAL COMPARISON OF PROPOSED ALGORITHMS WITH MOGA [1] ON BR-DATA

	Mann-Whitney test	
	P-value	Result
Diversity	**0.27**	Null hypothesis isn't not rejected
Spacing	**0.47**	Null hypothesis isn't not rejected
MID	**0.73**	Null hypothesis isn't not rejected
NOS	**0.82**	Null hypothesis isn't not rejected

V. CONCLUSION AND FUTURE WORKS

In this paper, a new Pareto-based algorithm called CENSGA was introduced to the library of the MOFJSP. Then, this algorithm was compared with one of the best algorithms of the literature on some multi-objective metrics. These metrics were also analyzed statistically by means of a non-parametric test called Mann-Whitney test. According to these statistical results, proposed algorithm and MOGA don't have a significant difference. However, by casual and non-statistical looking at results CENSGA has a better performance on *Spacing* and *NOS* metric and MOGA has a better performance on *Diversity* and *MID*. Therefore, it can be proved that our proposed algorithm is at least as good as one of the best algorithms of the literature. Future work of this paper can introduced other multi-objective algorithms to this area or used this algorithm in other scheduling environments.

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