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Effects of Thermal Radiation and Magnetic Field on Unsteady Stretching Permeable Sheet in Presence of Free Stream Velocity

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***Abstract*—**The aim of this paper is to investigate two-dimensional unsteady flow of a viscous incompressible fluid about stagnation point on permeable stretching sheet in presence of time dependent free stream velocity. Fluid is considered in the influence of transverse magnetic field in the presence of radiation effect. Rosseland approximation is use to model the radiative heat transfer. Using time-dependent stream function, partial differential equations corresponding to the momentum and energy equations are converted into non-linear ordinary differential equations. Numerical solutions of these equations are obtained by using Runge-Kutta Fehlberg method with the help of Newton-Raphson shooting technique. In the present work the effect of unsteadiness parameter, magnetic field parameter, radiation parameter, stretching parameter and the Prandtl number on flow and heat transfer characteristics have been discussed. Skin-friction coefficient and Nusselt number at the sheet are computed and discussed. The results reported in the paper are in good agreement with published work in literature by other researchers.

***Keywords*—**Magneto hydrodynamics, Stretching sheet, Thermal radiation, Unsteady flow.

I. INTRODUCTION

FLOW of an incompressible viscous fluid over a stretching surface is a classical problem in fluid dynamics and important in various process. It is used to create polymers of fixed cross-sectional profiles, cooling of metallic and glass plates. Aerodynamics shaping of plastic sheet by forcing through die and boundary layer along a liquid film in condensation processes are among the other areas of application. The production of sheeting material, which includes both metal and polymer sheets arises in a number of industrial manufacturing processes. In technical processes concerning polymer involves the drawing of strips. Strips which are extruded from a die with some prescribed velocity may become sometime stretched. The stretching surfaces undergo cooling or heating that causes surface velocity and temperature variations.

Aforementioned issue attracted many researchers in recent years due to its applications in many areas. Crane [1] reported an exact solution for the steady two-dimensional flow of a viscous and incompressible fluid induced in the stretching of an elastic flat sheet in its own plane with a velocity varying linearly with the distance from a fixed point due to the application of a uniform stress. The pioneering works of [1] are subsequently extended by many authors to explore various aspects of the flow and heat transfer occurring in an infinite domain of the fluid surrounding the stretching sheet. Most of the work carried out on fluid at rest but, in some practical applications fluid can have some prescribed velocity. Mahapatra and Gupta [2] analyzed stagnation-point flow towards a stretching surface in presence of free stream velocity. They reported that a boundary layer is formed when stretching velocity is less than the free stream velocity. As the stretching velocity exceeds the free stream velocity than an inverted boundary layer is formed. Singh et al. [3], [4] reported effect of porosity parameter and radiations on stretching sheet for orthogonal flow and oblique flow respectively.

There are many situations where the flow and heat transfer are unsteady due to sudden stretching of a sheet. Pop and Na [5] analyzed the unsteady flow past a wall which starts impulsively to stretch from rest. They found that the unsteady flow would approach the steady flow situation after long passage of time. Heat transfer of an unsteady boundary layer flow over stretching sheet has been studied by Elbashbeshy and Bazid [6]. They reported that thermal boundary layer thickness and momentum boundary layer thickness decrease with unsteadiness parameter. Ishak et al. [7] investigates boundary layer flow over a continuous stretching permeable surface. They found that the heat transfer rate at the surface increase with unsteadiness parameter.

The study of magneto hydrodynamic flow of an electrically conducting fluid caused by the deformation of the wall of a vessel containing a fluid is of considerable interest in a modern metallurgical and metal-working process. The boundary layer flow passing a stretching plane surface in presence of a uniform magnetic field has practical relevance in polymer processes. Takher et al. [8] investigated the unsteady magnetohydrodynamic flow due to the impulsive motion of a

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stretching sheet. They reported that the surface heat transfer increase upto a certain portion of time, beyond that it decreases. Recently, Ishak [9] investigated unsteady magnetohydrodynamic flow and heat transfer characteristic over a stretching plate. Viscous dissipation in presence of external magnetic field over an unsteady stretching surface has been studied by Abel et al. [10]. Effect of Hall's current on flow and heat transfer over an unsteady stretching surface in presence of strong magnetic field has been analyzed by El-Aziz [11]. The radiation effect takes place at high temperature. These radiations effects might play a significant role in controlling heat transfer process in polymer processing industry. The quality of the final product depends to a great extends on the heat controlling factors and knowledge of radiative heat transfer in the system can leads to a desired product with sought characteristic. Free convection heat transfer with radiation effect over the isothermal stretching sheet and a flat sheet near the stagnation point have been investigated respectively by Ghaly and Elbarbary [12], and Pop et al. [13]. They found that a boundary layer thickness increases with radiation. There are very few attempts in literature to consider the effect of thermal radiation on the flow and heat transfer in a viscous fluid over an unsteady stretching surface. El-Aziz [14] studied the thermal radiation effects over an unsteady stretching sheet. However, radiation and magnetic effects on a stretching sheet for time dependent moving fluid has been not considered so far in best of our knowledge.

In this paper our concern is to investigate two dimensional unsteady flow of a viscous incompressible fluid about stagnation point on permeable stretching sheet in presence of time dependent free stream velocity. Fluid is considered in the influence of transverse magnetic field in the presence of thermal radiation. The mechanical and thermal characteristic of such an unsteady process is investigated in the boundary layer approximation. This problem arises in a large class of industrial manufacturing processes as polymer extrusion, wire drawing, drawing of plastic sheet, coloring of fabrics etc. The reported results are in good agreement with the available published work in the literature.

II. Formulation of Problem

The mathematical model considered here consists of a viscous, incompressible, unsteady two-dimensional flow of an electrically conducting fluid on a permeable stretching sheet. Fluid is considered in the influence of transverse magnetic field in the presence of thermal radiation effect. Stretching sheet is placed in the plane $y=0$ and $x-$axis is taken along the sheet as shown in Fig. 1. The fluid occupies the upper half plane i.e. $y>0$. The sheet has uniform temperature T_∞ and moving with non-uniform velocity $u_w(x,t)=\dfrac{cx}{1-\alpha t}$, where c and α are positive constants with dimension (time)$^{-1}$, c is the initial stretching rate and $\dfrac{c}{1-\alpha t}$ is the effective stretching rate which is increasing with time t (as discussed in [14]).

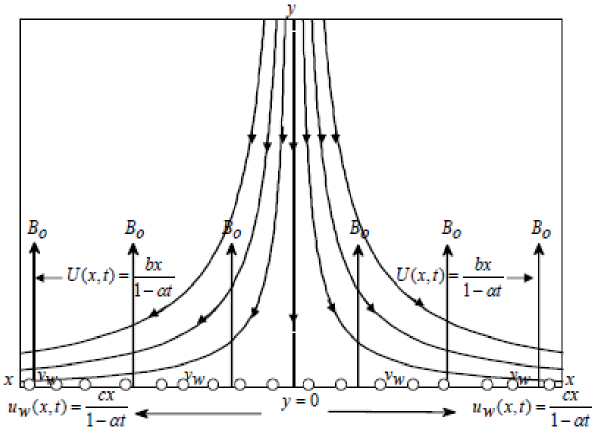

Fig. 1. Physical Model of the Problem

The viscous dissipation, Joule heating and induced magnetic field are neglected. The governing equations of continuity, momentum and energy under above assumptions are

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=U\frac{\partial U}{\partial x}+\frac{\partial U}{\partial t}+\nu\frac{\partial^2 u}{\partial y^2}-\frac{\sigma B_o^2}{\rho}(u-U) \tag{2}$$

and

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{K}{\rho c_p}\frac{\partial^2 T}{\partial y^2}-\frac{1}{\rho c_p}\frac{\partial q_r}{\partial y} \tag{3}$$

where u and v are velocity components along x and y axes respectively, $U(x,t)=\dfrac{bx}{1-\alpha t}$ is the free stream velocity of fluid, b is a positive constant with dimension (time)$^{-1}$. ν is kinematic viscosity, σ is electrical conductivity, T is the temperature, ρ is density of the fluid, K is thermal conductivity and c_p is specific heat at constant pressure.

Here, q_r is approximated by Rosseland approximation, expressed as:

$$q_r=-\frac{4\sigma_s}{3k}\frac{\partial T^4}{\partial y} \tag{4}$$

where, k is mean absorption coefficient, σ_s is Stefan-Boltzmann constant. It is assumed that the temperature difference within the flow is so small that T^4 can be expressed as a linear function of T_∞. This can be obtained by expanding T^4 using Taylor series about T_∞ and neglecting the higher order terms. Thus we get:

$$T^4=4T_\infty^3\,T-3T_\infty^4. \tag{5}$$

Therefore, using above equation in (4), change in radiative flux with respect to y has been obtained as

$$\frac{\partial q_r}{\partial y} = -\frac{16\sigma_s T_\infty^3}{3k}\frac{\partial^2 T}{\partial y^2}. \quad (6)$$

Boundary conditions for the given model are:

$$u = u_w(x,t),\ v = v_w(t) \text{ and } T = T_w(x,t) \quad at\ y = 0$$
$$u = U(x,t) \text{ and } T \to T_\infty \quad as\ y \to \infty \quad (7)$$

where $v_w(x) = -v_o \frac{1}{\sqrt{1-\alpha t}}$ is the velocity of suction $(v_o > 0)$ at the wall, of the fluid. As discussed in [14], $T_w(x,t) = T_\infty + \frac{T_o \operatorname{Re}_x (1-\alpha t)^{1/2}}{2}$ is the wall temperature, where $\operatorname{Re}_x = \frac{u_w x}{\nu}$ is the local Reynolds number based on the stretching velocity $u_w(x), T_o$ is a reference temperature such that $0 \le T_o \le T_w$. The expression for $u_w(x,t)$, $T_w(x,t), U(x,t)$ and $v_o(t)$ are valid only for time $t < \alpha^{-1}$ unless α become zero.

III. Method of Solution

Introducing the stream function $\psi(x,y)$ as defined by $u = \frac{\partial \psi}{\partial y}$, $v = -\frac{\partial \psi}{\partial x}$, the dimensionless temperature $\theta = \frac{T - T_\infty}{T_w - T_\infty}$ and the similarity variable $\eta = \sqrt{\frac{c}{(1-\alpha t)\nu}}\,y$, $\psi = \sqrt{\frac{\nu c}{(1-\alpha t)}}\,xF(\eta)$ and $T = T_\infty + T_o\left[\frac{c\,x^2}{2\,\nu}\right](1-\alpha t)^{-3/2}\theta(\eta)$.

With the help of above relations the governing equations (2) and (3) finally reduces to

$$F'''(\eta) + F(\eta)F''(\eta) - F'(\eta)^2 - h\left(F'(\eta) + \frac{\eta}{2}F''(\eta)\right) - M(F'(\eta) - \lambda) + h\lambda + \lambda^2 = 0 \quad (8)$$

$$\theta''(\eta)(3R+4) - 3R\Pr\begin{pmatrix} \frac{h}{2}(3\theta(\eta) + \eta\theta'(\eta)) \\ +2F'(\eta)\theta(\eta) - F(\eta)\theta'(\eta) \end{pmatrix} = 0 \quad (9)$$

where $h = \frac{\alpha}{c}$ is the unsteadiness parameter, $M = \frac{\sigma B_o^2}{\rho c}(1-\alpha t)$ is magnetic parameter, $R = \frac{kK}{4\sigma_s T_\infty^3}$ is radiation parameter, $\Pr = \frac{\mu\, c_p}{K}$ is the Prandtl number, $\lambda = \frac{b}{c}$ is the ratio of free stream velocity parameter to stretching velocity parameter. It is noted that (1) also follows the same way.

The corresponding boundary conditions are

$$F(0) = s, \quad F'(0) = 1 \text{ and } \theta(0) = 1 \quad at\ \eta = 0$$
$$F'(\infty) \to \lambda, \text{ and } \theta(\infty) \to 0 \quad as\ \eta \to \infty \quad (10)$$

where $s = \frac{v_o}{\sqrt{\nu\, c}}$ is the suction parameter ($s > 0$ corresponds to suction).

The physical quantity of interest is the skin friction and Nusselt number. The skin-friction coefficient at the sheet is given by

$$C_f = 2\operatorname{Re}^{-1/2} F''(0) \quad (11)$$

where $\operatorname{Re} = \frac{u_w x}{\nu}$ is the local Reynolds number. The rate of heat transfer in terms of the Nusselt number at the sheets is given by,

$$Nu = -\operatorname{Re}^{1/2}\theta'(0). \quad (12)$$

IV. Numerical Simulation

The set of non-linear coupled differential equations (8) and (9) subject to the boundary conditions (10) constitute a two-point boundary value problem. In order to solve these equations numerically, we follow most efficient numerical shooting technique with Runge–Kutta Fehlberg integration scheme. In this method it is most important to choose the appropriate finite values of $\eta \to \infty$. The solution process is repeated with another large value of η_∞ until two successive values of $F''(0)$ and $\theta'(0)$ same upto the desired significant value. The last value of η_∞ is chosen as appropriate value of the limit $\eta \to \infty$ for that particular set of parameters. The ordinary differential equation (8) and (9) were first converted into a set of five first-order simultaneous equations. To solve this system we require five initial conditions but we have only three initial conditions, $F(0)$ and $F'(0)$ on $F(\eta)$ and one initial condition $\theta(0)$ on $\theta(\eta)$. Still there are two initial conditions $F''(0)$ and $\theta'(0)$ are required, which are not prescribed. However the values of $F'(\eta)$ and $\theta(\eta)$ are known at $\eta \to \infty$. Shooting technique has been employed to find the two unknown initial values utilizing these two ending boundary conditions. After finding the required boundary conditions, the problem has been solved numerically using Runge-Kutta Fehlberg integration scheme.

V. Result and Discussion

In the absence of an analytical solution of a problem, a numerical solution is indeed an obvious and natural choice. Thus, the governing boundary layer and thermal boundary layer equations (8 and 9) with boundary conditions (10) are solved using Runge-Kutta Fehlberg method with shooting technique. Different values of magnetic parameter

M, radiation parameter R, unsteadiness parameter h and ratio of free stream velocity parameter to stretching velocity parameter λ taking step size 0.001 used for numerical simulation. While numerical simulation, step size 0.002 and 0.003 were also checked and values of $F''(0)$ and $\theta'(0)$ were found in each case correct up to six decimal places. To assess the accuracy of the present method, comparison with previously reported data available in the literature has been made. It is clear from Table I that the numerical values of $F''(0)$ in the present paper for different value of λ, when $M=0, h=0$ and $s=0$ are in good agreement with results published in [2] and [13]. To further validate the approach used in the paper comparison of local Nusselt number for different values of unsteadiness parameter has been carried out and shown in Table II, which are in good agreement with the El-Aziz [14]. From Table I and II it is clear that the scheme used in this paper is stable and accurate.

TABLE I
COMPARISON OF THE VALUES OF $F''(0)$ FOR DIFFERENT VALUES OF STRETCHING PARAMETER λ WHEN UNSTEADINESS PARAMETER $h=0$, MAGNETIC PARAMETER $M=0$, SUCTION PARAMETER $s=0$.

λ	Value of $F''(0)$		
	Pop et al. [13]	Mahapatra and Gupta[2]	Present Result
0.1	−0.9694	−0.9694	−0.969658
0.2	−0.9181	−0.9181	−0.918169
0.5	−0.6673	−0.6673	−0.667271
2.0	2.0174	2.0175	2.017393
3.0	4.7290	4.7293	4.729037

TABLE II
COMPARISON OF LOCAL NUSSELT NUMBER FOR DIFFERENT VALUES OF UNSTEADINESS PARAMETER h, PRANDTL NUMBER Pr WHEN STRETCHING PARAMETER $\lambda=0$, MAGNETIC PARAMETER $M=0$, SUCTION PARAMETER $s=0$ AND RADIATION PARAMETER $R\to\infty$.

h	Pr	Value of $-\theta'(0)$	
		El-Aziz [14]	Present Result
0.8	0.01	0.13810	0.13775
	0.1	0.45170	0.45168
	1.0	1.67280	1.67277
	10	5.70503	5.70502
1.2	0.01	0.16580	0.16621
	0.1	0.50870	0.50871
	1.0	1.81800	1.81806
	10	6.12067	6.12073

The skin-friction coefficient and Nusselt number are presented by (11) and (12) and are directly proportional $F''(0)$ and $-\theta(\eta)$, respectively. It is observed from Table III that skin friction decreases as unsteadiness parameter increase for $\lambda<1$. Here, the negative value of $F''(0)$ means the solid surface exerts a drag force on the fluid. This is due to the development of the velocity boundary layer is caused solely on the stretching plate. For $\lambda>1$, skin friction increases as unsteadiness parameter increase. This is due to fact that inverted boundary layer formed. Nusselt number increases with unsteadiness parameter.

TABLE III
VALUES OF $F''(0)$ AND $-\theta'(0)$ FOR DIFFERENT VALUES OF STRETCHING PARAMETER λ, UNSTEADINESS PARAMETER h WHEN MAGNETIC PARAMETER $M=0$, PRANDTL NUMBER $\Pr=0.71$, SUCTION PARAMETER $s=0$ AND RADIATION PARAMETER $R=1$.

h	$\lambda=0.5$		$\lambda=2$	
	$F''(0)$	$-\theta'(0)$	$F''(0)$	$-\theta'(0)$
0.0	−0.66727	0.76426	2.01739	1.07674
0.2	−0.69291	0.81275	2.05180	1.10852
0.5	−0.73045	0.88091	2.10278	1.15505
0.8	−0.76689	0.94443	2.15300	1.20026
1.0	−0.79057	0.98458	2.18607	1.22972
1.2	−0.81376	1.02318	2.18817	1.25865
1.4	−0.84765	1.07858	2.26733	1.30111

The magnetic parameter M represents the importance of magnetic field on the flow. The presence of transverse magnetic field sets in Lorentz force, which results in retarding force on the velocity field and therefore as M increases, so does the retarding force and hence the velocity decrease. This is shown in Fig. 2, when $\lambda<1$. In case $\lambda>1$, which is just opposite of $\lambda<1$, as expected the velocity profile increases with the increase in the M as shown in Fig. 3. This explain the results presented in Table IV, that is the shear stress at the sheet decreases due to increase in M when $\lambda<1$, while it increase with the increase in M for $\lambda>1$. Though the magntic parameter does not directly enter into the energy equation, it actuallty affects the velocity distribution and therefore affects the temperature profile indirectly. It is worth mentioning here that in the absence of magnetic field, the velocity profile is identically equal to the temperature profile i.e. $F'(\eta)=\theta(\eta)$ for Prandtl number equal to 1.

TABLE IV
VALUES OF $F''(0)$ AND $-\theta'(0)$ FOR DIFFERENT VALUES OF STRETCHING PARAMETER λ, MAGNETIC PARAMETER M WHEN PRANDTL NUMBER $\Pr=0.71$, UNSTEADINESS PARAMETER $h=0.8$, SUCTION PARAMETER $s=0$ AND RADIATION PARAMETER $R=1$.

M	$\lambda=0.5$		$\lambda=2$	
	$F''(0)$	$-\theta'(0)$	$F''(0)$	$-\theta'(0)$
0	−0.76689	0.94443	2.15300	1.20026
1	−0.91538	0.93505	2.37273	1.20688
2	−1.04299	0.92807	2.57400	1.21248
3	−1.15661	0.92256	2.76091	1.21732

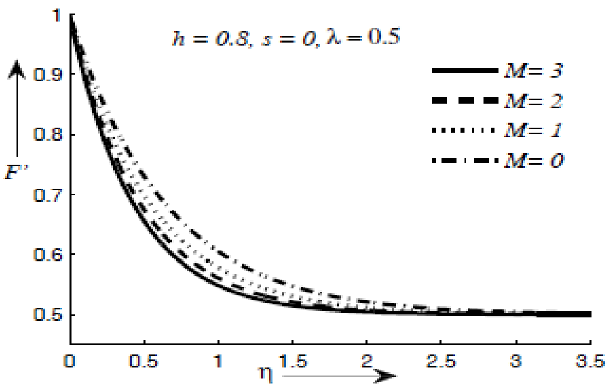

Fig. 2 Velocity profile $F'(\eta)$ versus η for different values M when $s=0, h=0.8, \lambda=0.5$.

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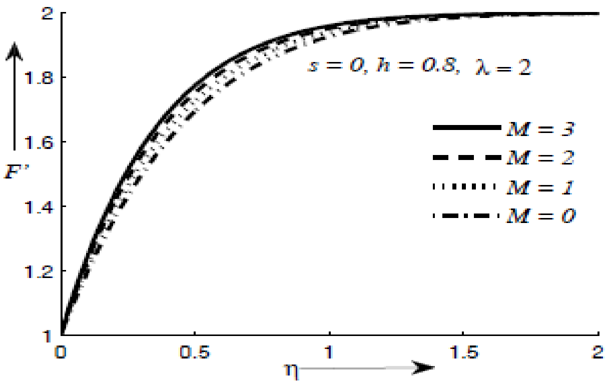

Fig. 3 Velocity profile $F'(\eta)$ versus η for different values M when $s=0, h=0.8, \lambda=2.$

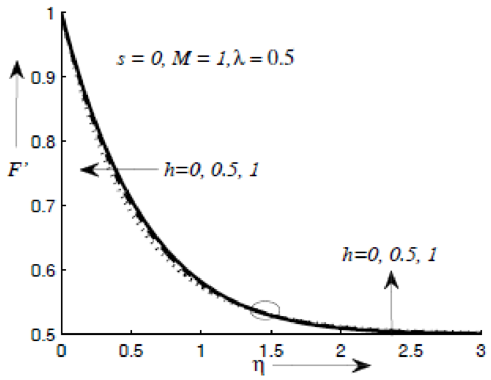

Fig. 4 Velocity profile $F'(\eta)$ versus η for different values h when $s=0, M=1, \lambda=0.5$.

Fig. 4, represents variation of velocity profile of the fluid with unsteadiness parameter for $\lambda < 1$. It is observed that with the increase in unsteadiness parameter h, the fluid velocity first decrease and then a certain value of η is reached (marked in Fig. 4) it starts increasing. We notice that a crossing over point appears in the figure. This is a special point where all the velocity curves cross each other that is, velocity profile exhibit different behavior before and after this point. This is contrary to the result from the [14], where the flow is without any such point for all values of h is considered. At $\lambda = 1$, there would be no formulation of boundary layer, as the fluid velocity is equal to the surface velocity. For $\lambda > 1$, it is observed from Fig. 5 that with increase in h, fluid velocity increase which is due to fact that an inverted boundary is formed. Temperature is found to decrease with increasing unsteadiness parameter for all values of λ as shown in Fig. 6, and 7. We also notice that impact of unsteadiness parameter on temperature profile is more pronounces than on the velocity profile.

It is observed from Figs. 8 and 9 that the increase of the radiation parameter R leads to decrease of the temperature profile for all values of λ. This result can be explained by the fact that a increase in the value of R for a given value of T_∞ means a increase in the Rosseland radiation absorpitivity k.

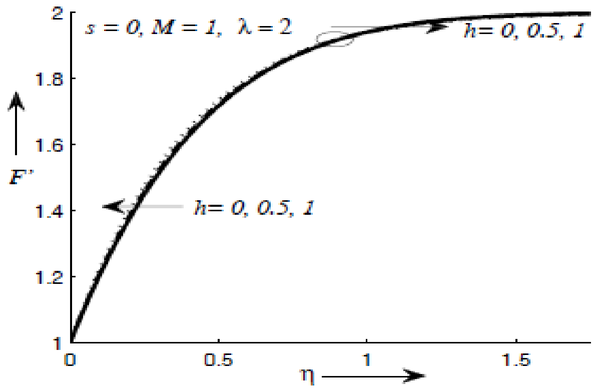

Fig. 5 Velocity profile $F'(\eta)$ versus η for different values h when $s=0, M=1, \lambda=2$.

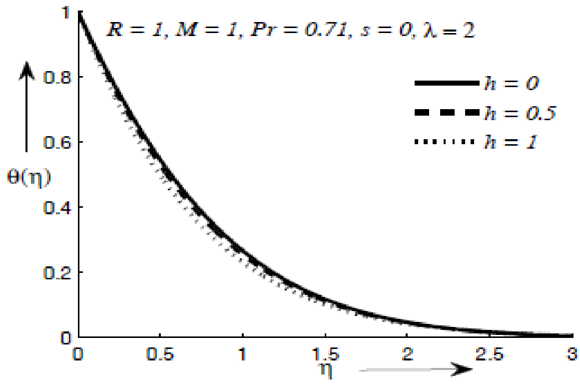

Fig. 6 Temperature profile $\theta(\eta)$ versus η for different values h when $M=1, R=1, \Pr=0.71, s=0$ and $\lambda=2$.

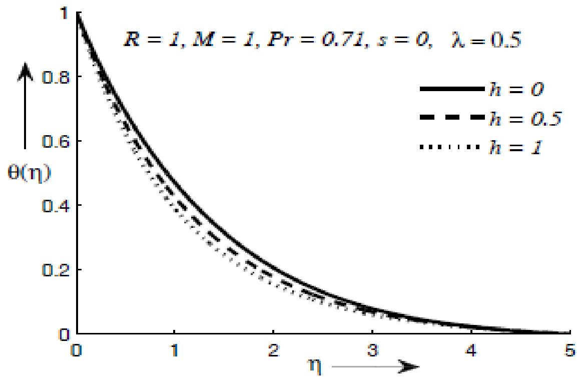

Fig. 7 Temperature profile $\theta(\eta)$ versus η for different values h when $M=1, R=1, \Pr=0.71, s=0$ and $\lambda=0.5$.

The divergence of the radiative heat flux $\partial q_r / \partial y$ decrease as k increase which in turn decrease the rate of radiative heat transferred to the fluid and hence fluid temperature decrease. In view of this explanation the effect of radiation become more significant as $R \to \infty$, and can be neglected when $R \to 0$. Further, it is seen from figures, that for the small values of R, the thermal boundary layer thickness is small. Therefore higher value of radiation parameter implies higher surface heat flux. Skin friction and Nusselt number increases as radiation parameter as shown in Table V.

TABLE V
VALUES OF $-\theta'(0)$ FOR DIFFERENT VALUES OF STRETCHING PARAMETER λ, MAGNETIC PARAMETER M, RADIATION PARAMETER R WHEN PRANDTL NUMBER $\Pr = 0.71$, UNSTEADINESS PARAMETER $h = 0.8$ AND SUCTION PARAMETER $s = 0$.

R	$\lambda = 0.5$		$\lambda = 2$	
	$M = 0$	$M = 2$	$M = 0$	$M = 2$
1	0.94443	0.92807	1.15505	1.21248
3	1.21599	1.19567	1.50157	1.51728
6	1.32774	1.30599	1.62343	1.64047
10	1.38151	1.35912	1.68173	1.69939

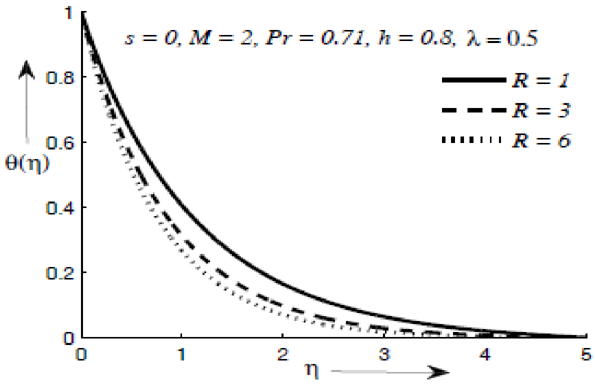

Fig. 8 Temperature profile $\theta(\eta)$ versus η for different values R when $M = 1, h = 0.8, \Pr = 0.71, s = 0$ and $\lambda = 0.5$.

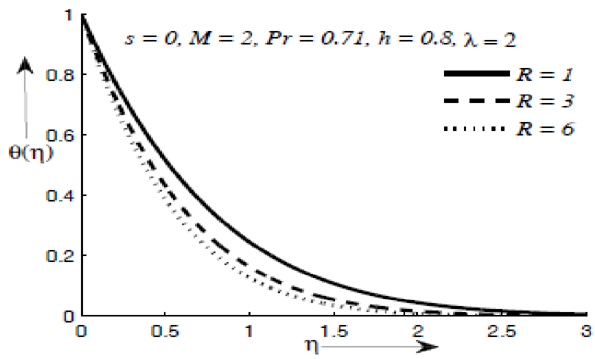

Fig. 9 Temperature profile $\theta(\eta)$ versus η for different values R when $M = 1, h = 0.8, \Pr = 0.71, s = 0$ and $\lambda = 2$.

Now we concentrate on the velocity and temperature distribution for the variation of the suction parameter s in the presence of unsteadiness parameter h. It is observed form Fig. 10 that with increase in suction parameter $s\ (s > 0)$, fluid velocity is found to decrease for $\lambda < 1$ that is suction causes to decrease the velocity of the fluid in the boundary layer region. This effect acts to decrease the wall shear stress. Increase in suction cause progressive thinning of the boundary layer. For $\lambda > 1$, it is observed from Fig. 11 that with increase in s, fluid velocity increase which is due to fact that an inverted boundary is formed. Fig. 12, and 13 exhibits that the temperature $\theta(\eta)$ in boundary layer also decreases with the increase $s\ (s > 0)$. The thermal boundary layer thickness decreases with suction parameter (s) which cause an increase in the rate of heat transfer.

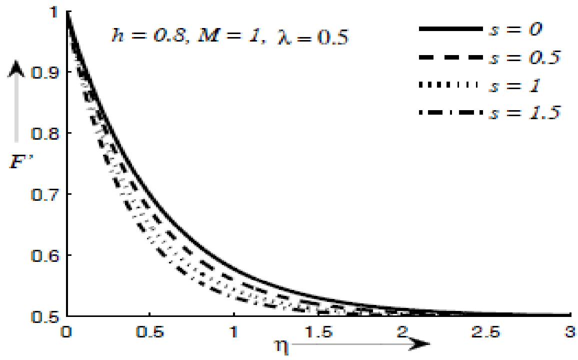

Fig. 10 Velocity profile $F'(\eta)$ versus η for different values s when $M = 1, h = 0.8, \lambda = 0.5$.

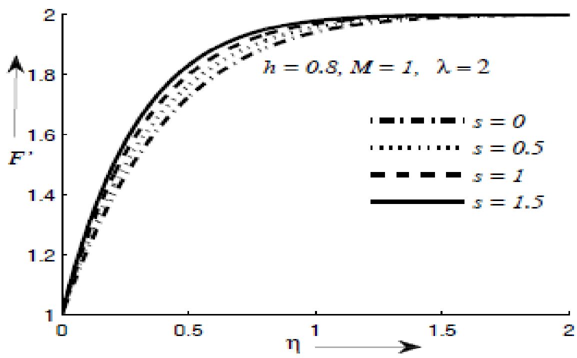

Fig. 11 Velocity profile $F'(\eta)$ versus η for different values s when $M = 1, h = 0.8, \lambda = 2$.

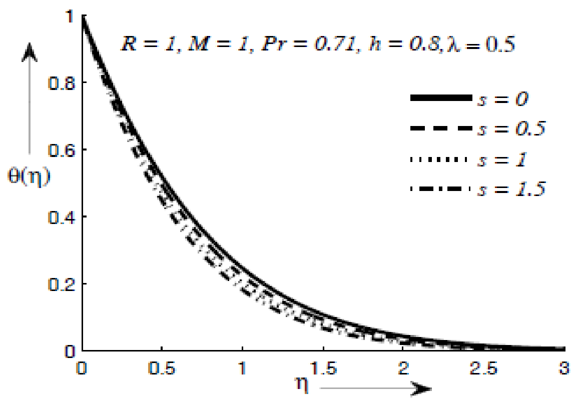

Fig. 12 Temperature profile $\theta(\eta)$ versus η for different values S when $M = 1, R = 1, \Pr = 0.71, h = 0.8$ and $\lambda = 0.5$.

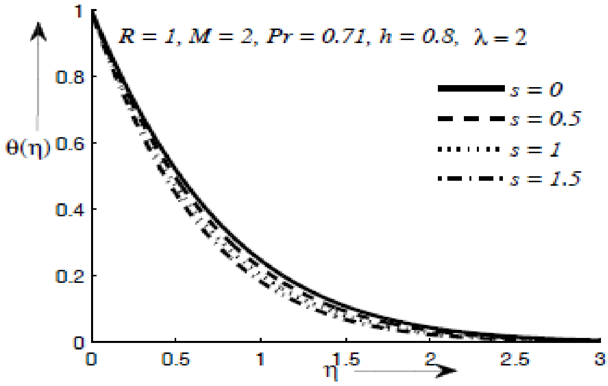

Fig. 13 Temperature profile $\theta(\eta)$ versus η for different values s when $M = 2$, $R = 1$, $\Pr = 0.71$, $h = 0.8$ and $\lambda = 2$.

VI. Conclusion

The two-dimensional viscous, incompressible, unsteady flow of an electrically conducting fluid on permeable stretching sheet in the presence to time dependent free stream velocity has been investigated. Fluid is considered in the influence of transverse magnetic field in the presence of radiation effect. Numerical solution for the governing equations has been obtained which allows the computation of the flow and heat transfer characteristics for various values of the magnetic parameter, unsteadiness parameter, radiation parameter, suction parameter and stretching parameter. The main results of the paper can be summarized as follows:

1. Nusselt number increases with increase in unsteadiness parameter. However, Skin friction increases with unsteadiness parameter for stretching parameter greater than one while it decrease with unsteadiness parameter less than one.
2. Velocity decrease with increases as magnetic field for stretching parameter less than one but, it decrease for stretching parameter greater than one.
3. Velocity profile first decreases with increase in unsteadiness parameter for stretching parameter upto one than opposite trends has been observed for stretching parameter greater than one.
4. Temperature profile decreases with the increase of radiation parameter. However, Nusselt number and Skin friction increase with radiation parameter.
5. Velocity profile decrease as suction parameter increase with stretching parameter less than one and it increase when stretching parameter greater than one.
6. Temperature profile decreases as suction parameter increase.

These results have possible technological applications in liquid-based systems, and are expected to be very useful for practical applications.

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