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Incorporation of Long-Term Redundancy in ECG Time Domain Compression Methods through Curve Simplification and Block-Sorting

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Abstract**—**We suggest a novel method to incorporate long-term redundancy (LTR) in signal time domain compression methods. The proposition is based on block-sorting and curve simplification. The proposition is illustrated on the ECG signal as a post-processor for the FAN method. Test applications on the new so-obtained FAN+ method using the MIT-BIH database show substantial improvement of the compression ratio-distortion behavior for a higher quality reconstructed signal.

Keywords**—**ECG compression, Long-term redundancy, Block-sorting, Curve Simplification.

I. Introduction

Digital signals compression is a requirement in modern applications for efficient storage and or transmission over the network. Basically, compression consists in the principle of eliminating redundancy. Ideally, we desire the smallest representation of the original data as a result of compression, i.e. a fully decorrelated form where 'no structure in the data is discernible' [1]. This objective is yet to be attained since all existing compression methods exploit partially the existing redundancy in the data. For instance in the case of ECG signal, Jalaleddine *et al* [2] classify compression methods in three categories: Direct data compression methods, like AZTEC [3] and FAN [4] ; transform methods , e.g. [5] and parameter extraction methods, e.g. [6]. From another point of view, signals compression can be achieved in the time domain or in the frequency domain. This study concentrates particularly on a specific class of time domain compression methods known as *piecewise linear approximation* (PLA). This approach is widely used in many computer areas including time series representation for analysis, compression and data mining and, although no assumption is made on the nature of the signal, we concentrate specifically on the ECG signal.

Time domain compression methods (PLA) have in common the principle of reducing redundancy through selection of a set of characteristic points (CP) on the signal trace. These points are selected based on wisely predetermined rules so as the most significant points are selected. The so-obtained set of CPs stand for the reduced form of representation of the original signal. The reconstruction process is achieved through interpolation between successive CPs in this set. This type of lossy compression is acceptable in many biological signals, including the ECG.

It can be noted that in PLA methods, the CPs are selected on the basis that all samples between two successive CPs are correlated. Therefore, in this type of compression, short-term redundancy (STR) only is considered. Yet, in biological signals (especially the quasi-periodic signals like ECG), in addition to short-term redundancy (within a period), there exists also long-term redundancy (LTR, between periods). This is the main reason for existing PLA methods low compression performances. But, since these methods consider few points only at a time, they have the advantage of possessing linear temporal complexity allowing effective real-time implementation.

The objective of the proposed method is to incorporate LTR in this category of methods. Our objective is clearly justified since the so-enhanced methods would have competing compression performances and at the same time would allow real-time implementation. Our proposition is based on two main tools: block-sorting through a variant of the quite recent Burrows-Wheeler algorithm (BWA) [7] and curve simplification through a variant of the Douglas-Peucker algorithm [8]. Our algorithm is coupled as a post-processing step to the FAN method in the specific case of ECG signal, as an illustration.

The rest of this work is organized as follows. In section 2, tools and methods used in the proposed approach are presented. In section 3 experiments on the novel method are illustrated on selected ECG records from the MIT-BIH

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database. In section 4 results of the previous section are discussed. At last, in section 5, a general conclusion is given.

II. Materials and Methods

A. Curve Simplification

Given a discrete curve, formally expressed by polyline $P=(p_i)$, $i=1..N$, where $p_i=(x_i,y_i)$, with x_i the horizontal coordinate and y_i the vertical coordinate of p_i, the simplification of P consists in the computation of another polyline $Q=(q_j)$, $j=1..M$, satisfying the following conditions:

a. $M<N$;

b. $q_1=p_1$ and $q_M=p_N$;

c. Let $\|. \quad , \quad .\|$ be a distance between P and any PLA of it Q, then $\|P \quad , \quad Q\| < \varepsilon$, with $\varepsilon>0$, a preset threshold on the tolerance of the simplification error.

If Q satisfies conditions a-c, it is said to be an ε-Simplification of P. Fig. 1 illustrates a polyline $P=(p_1, p_2, ..., p_{100})$, simplified with eight points $Q=(q_1, q_2,..., q_8)$. It is clear that simplification is a compression tool too, with the additional condition that the essential characteristics of the initial curve be reconstructed with precision. These characteristics depend on the nature of the signal. For instance, in the case of ECG, the main features P, QRS and T are of clinical importance. It can be shown also that for a given curve, there exist more than one ε-Simplification. Therefore, a minimal set of rules must be established so as to ensure selection of the ε-Simplification with the minimal number of CPs, according to these rules. In our proposed method, we use a variant of the Douglas-Peucker line simplification algorithm (DPA)[8]. This algorithm uses a recursive selection strategy, reducing gradually the distance between P and Q by the maximal possible amount under norm ||.,.|| at each selection. Our choice for the recursive approach is motivated by the excellent performance of this strategy at selection of most perceptually important points on the initial curve. By contrast, the classical PLA methods use a sequential strategy leading to selection of locally only significant points. The DPA main steps are as follows. The initial curve endpoints are first selected (Q=[p_1, p_N]). The next selected point , say $q_3=(x_3,y_3)$, is s. t. :

$$\underset{q_3 \in [p_2 \; \cdots \; p_{N-1}]}{Max} \|q_3 \quad , \quad \hat{q}_3\| \tag{1}$$

where $\hat{q}_3$ is the vertical projection of point q_3 on polyline Q. Therefore, point q_3 is the most perceptible CP in the interval $]q_1,...,q_2[$. The process is then recursively repeated for the resulting sub-curves $[q_1, q_3]$ and $[q_3, q_2]$ until the precision of simplification ε is attained. In this study, we reduce the norm given by equation 2:

$$\|P,Q\| = \sqrt{\sum_{i=1}^{i=N} \left(y_i \quad - \quad \hat{y}_i\right)^2} \tag{2}$$

In the general case, for segment $[p_i,...,p_j]$ under process, the selected CP, say q_k, is s.t.:

$$q_k = Arg \underset{q_k \in]p_i,..., p_j[}{Max} \|q_k \quad , \quad \hat{q}_k\| \tag{3}$$

with

$$\|q_k \quad , \quad \hat{q}_k\| = |y_k \quad - \quad \hat{y}_k| \tag{4}$$

In Eqs. (2) and (4), y_i is the magnitude of point q_i and $\hat{y}_i$ that of $\hat{q}_i$. Note that the so computed CPs are selected according to a binary tree of segmentation where the most perceptible points are selected in the upper levels. Fig. 2 illustrates this property in the case reported in Fig. 1. It is easy to derive from the binary tree of segmentation that the process temporal complexity is of $\sim O(N.Log_2(N))$ order. The simplification algorithm is formally described in Fig. 3.

B. Compression Through Block-Sorting

Block sorting is quite a recent trend as fare as compression is concerned. The Burrows and Wheeler Algorithm (BWA) [7] is one of the first compression algorithms using this technique. The original BWA is a lossless compression method, reported to yield excellent results on images, text and sound [1]. The main idea behind the BWA is computation of a reversible permutation of the original data that creates concentrations. These concentrations of data are successively coded by RLE (run length encoding), MTF (move to front) techniques and finally, Huffman coding is applied. The decoder proceeds in reverse order, which allows reconstruction of the initial permutation. The permutation in question is the last column of the NxN matrix obtained by cyclic shifting of the initial data, $N-1$ times. The reconstruction of the original data from the permutation is achieved through a well-established process.

Our idea is that, since PLA algorithms reduce STR only, introduction of a block-sorting algorithm as a post-processing step to these methods would take into account LTR too. Therefore, the output (x_i,y_i), i=1..M, of a specific STR method is sorted on the y_i coordinate. This yields a novel curve (x_j', y_j'), j=1.. M. This last curve is then simplified with the Douglas-Peucker algorithm of section II.A to reduce LTR. This yields another curve (x_k'', y_k''), k=1..L, L<M. The compression ratio associated with the STR reduction (first simplification), as expressed in terms of number of samples reduction is given by Eq. 5:

$$CR_0=N/(2.M) \tag{5}$$

The compression ratio related to the LTR reduction (second simplification) is given by Eq. 6:

$$CR_1=N/(M+2.L) \tag{6}$$

The 2 factor in Eqs. 5 and 6 is due to the fact that in this type of compression, both the magnitude y_i and its temporal index x_i are stocked/sent. Then, for $L<M/2$, $CR_1>CR_0$, hence, gain in compression. Yet, CR_1 is upper bounded by $2.CR_0$. Finally, for $L<M/2$: $CR_0 < CR_1 < 2.CR_0$.

The reconstruction is conducted as follows. Magnitude $\tilde{y}$ of the M CPs associated with the STR compression is first computed using the L CPs of the LTR compression through linear interpolation between successive CPs, using $(x_j')_{j=1..M}$ and $(x_k'', y_k'')_{k=1..L}$, according to Eq. 7.

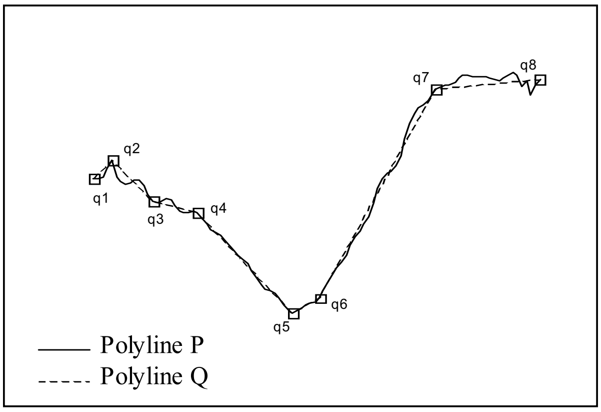

Fig. 1. An ε-Simplification of polyline $P=(p_1,...,p_{100})$ with polyline $Q=(q_1,...,q_8)$.

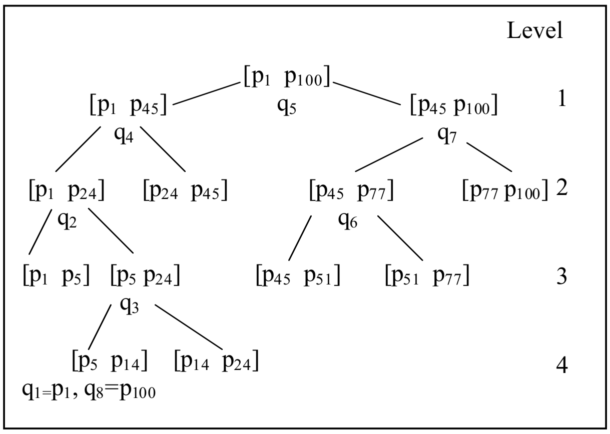

Fig. 2. CPs selection tree for the case in Fig. 1.

Step0 : $Q \leftarrow \{p_1, p_N\}$;
$DPA([p_1,...,p_N], \varepsilon)$;
$Q \leftarrow Sort(Q)$;
$M \leftarrow |Q|$;
Return(Q, M) ;
Procedure $DPA([p_i,...,p_j], \varepsilon)$ $\quad 1 \le i < j \le N$
if $\| [p_i,p_{i+1},...,p_j],[p_i,p_j] \| \geq \varepsilon$ ***then***
$q_k \leftarrow Arg \underset{q_l \in]p_i \cdots p_j[}{Max} d(q_l, \hat{q}_l)$;
$Q \leftarrow Q \cup \{q_k\}$;
$DPA([p_i,...,q_k],\varepsilon)$;
$DPA([q_k,...,p_j],\varepsilon)$;
endif ;
End.

Fig. 3. Modified Douglas-Peucker Algorithm.

The result is a set of tuples $(x_j', \tilde{y}_j)_{j=1..M}$. Sorting of these tuples on the first coordinate in ascending order yields the approximation of the M CPs of the STR compression. Finally, the reconstructed magnitude $\hat{y}$ as an approximation to the original magnitude y is realized by linear interpolation between successive CPs. This step yields the output tuples $(x_i, \hat{y}_i)_{i=1..N}$, where $\hat{y}_i$ is computed by Eq. 8.

$$\tilde{y}_k = \begin{cases} y'_i \quad , \ k=x'_i;\ i=1..L \\ y'_i+(k-x'_i).\dfrac{y'_{i+1}-y'_i}{x'_{i+1}-x'_i} \ , \ k=x'_i+1..x'_{i+1}-1,\ i=1..L-1 \end{cases} \tag{7}$$

$$\hat{y}_k = \begin{cases} \tilde{y}_i \quad , \ k=x_i;\ i=1..M \\ \tilde{y}_i+(k-1).\dfrac{\tilde{y}_{i+1}-\tilde{y}_i}{x_{i+1}-x_i} \ , \ k=x_i+1..x_{i+1}-1 \ , i=1..M-1 \ ) \end{cases} \tag{8}$$

III. Application to ECG signal

We apply particularly the proposed method to the ECG signal. The ECG (Electrocardiogram) is a biological signal reflecting the heart activity. Samples y_i of this signal represent the difference in potential as measured at the temporal index x_i between two electrodes positioned at specific positions on the body skin. Due to its quasi-periodic nature, a typical ECG signal is composed of a sequence of cardiac cycles. A normal cycle is itself composed of three clinically significant features, in this order: P wave, QRS complex and T wave. It may be interesting to mention that compression of the ECG has been under way during the last four decades. As illustration of our method, we propose to enhance the compression capability of the classical FAN method [4]. FAN is a popular time domain-STR compression technique dedicated to the ECG signal, reported to yield quite high compression ratios in its category [2]. The FAN method uses a sequential strategy in selecting the CPs. Starting with the first input point as CP, FAN selects the next CP as the furthest point s.t. the maximal deviation between the original points and the points obtained by linear interpolation between successive CPs is below a preset threshold (ε). The deviation is measured by the absolute difference of magnitudes. Therefore, FAN can be viewed as a simplification process using a sequential strategy that reduces the distance $\|P,Q\| = \underset{i=1}{\overset{N}{Max}} \left| y_i - \tilde{y}_i \right|$. It can easily be shown that the FAN temporal complexity is of linear order, which allows effective real-time implementation of this method. Our proposed algorithm for LTR reduction is coupled as a post-processing step to this method. The so-enhanced FAN method is denoted herein FAN+. Evaluation of the FAN and FAN+ methods is performed on carefully selected records from the *Massachusetts Institute of Technology – Beth Israel Hospital* (MIT-BIH) ECG database. The MIT-BIH database is a collection of 48 records sampled at 360 Hz. Each record is 30 minutes long and each sample is coded on 12 bits. This base serves as a cross-reference for researchers. The evaluation is performed on the numerical level through the compression ratios CR_0 (Eq. 5) for FAN and CR_1 (Eq. 6) for FAN+ and computation of the respective distortions upon reconstruction expressed by the percent root difference PRD_0 for FAN and PRD_1 for FAN+ where PRD_0 is given by Eq. 9 and PRD_1 by Eq. 10 with $\bar{y}$ representing the mean original magnitude.

$$PRD_0 = \sqrt{\frac{\sum_{i=1}^{N}\left[y_i - \tilde{y}_i\right]^2}{\sum_{i=1}^{N}\left[y_i - \bar{y}\right]^2}} \text{x}100\% \quad (9)$$

$$PRD_1 = \sqrt{\frac{\sum_{i=1}^{N}\left[y_i - \hat{y}_i\right]^2}{\sum_{i=1}^{N}\left[y_i - \bar{y}\right]^2}} \text{x}100\% \quad (10)$$

The first application is a detailed illustration of the proposed method and is reported in Fig. 4. This figure shows in (a) a 1000 samples ECG from the beginning of rec. 105. This segment has been simplified with FAN to precision ε=10. Obtained results are as follows: M=71, then CR_0=1000/(2x71)=7.04:1, with a reconstruction distortion of PRD_0=7.18%. The 71 CPs were processed with our proposed algorithm yielding L=10 CPs at the same precision. The new compression ratio is then CR_1=1000/(71+2x10)=11:1 and the new distortion is PRD_1=7.90%. The same figure shows in (a) the 71 CPs obtained by FAN plotted on the original segment as small squares. Fig. 4 (b) shows the reconstructed signal using the 71 CPs of FAN. Fig. 4 (d) shows the sorted magnitudes of the 71 CPs. Fig. 4 (e) shows the simplification of this last curve with the 10 CPs reported as small squares on Fig. 4 d. Fig. 4 (c) shows the reconstructed signal by the FAN+ method.

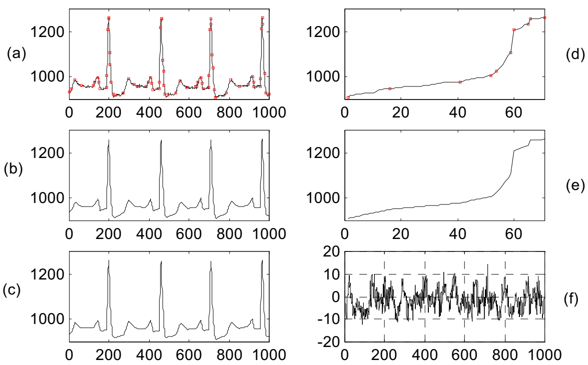

Fig. 4 Illustration of STR and LTR compression on a small ECG segment.

Fig. 4 (f) shows the overall reconstruction error.

Next, we have evaluated the compression ratio-distortion behavior using segments of 4094 samples from the beginning of rec. 108, 105 and 119. This behavior is in direct relation with the *rate-distortion* (RD) behavior. The size of the block B=4094=2^{12}-2 has been so determined because the MIT-BIH samples are coded on 12 bits. Therefore, any block size greater than 4096 would require more bits to be addressed and the compression ratio CR_1 (Eq. 6) would no longer reflect the true compression capability of FAN+. The RD behaviors are reported in Fig. 5 (*Upper*) for rec. 108, (*Middle*) for rec. 105 and (*Lower*) for rec. 119.

IV. Discussion

Fig. 4 illustrates the enhancement of the FAN method through incorporation of LTR reduction. For instance, for a distortion of PRD_0=7.18% FAN simplified the 1000 samples with 71 CPs, thus yielding a CR_0=7.04:1. Post-processing of the 71 CPs by the proposed method gave 10 CPs on the sorted magnitudes curve with 71 temporal indexes, thus a new CR_1=11:1 and a new distortion of PRD_1=7.90%. The gain in compression ratio is more than 56% for a small additional distortion. Note that the block size has great impact on LTR in the proposed method. In the specific case of the MIT-BIH database, the samples of which are coded on 12 bits, the maximal block size is 2^{12} samples, when expressing the compression ratio in terms of samples reduction. This constitutes no barrier, since the compression ratio can be expressed in terms of bits reduction (bit rate). The advantage of expressing the compression ratio in terms of samples reduction is to appreciate the true compression capabilities of a given method regardless of coding considerations. It is also of major importance to link the compression capability to a metric of distortion, for it is established that the highest compression ratios are obtained at higher distortion prices. We would then be interested in methods yielding higher compression ratios for lower distortions, ideally, in the operational rate-distortion sense (ORD). Fig. 5 clearly shows the enhancement of the RD behavior for all used records. For example, for a distortion of 10%, the compression ratios for the three records are as follows : 4.5:1, 7.75:1 and 10:1 for FAN and 8.5:1, 13.25:1 and 16.5:1 for FAN+, which represents gains in compression ratio for the same distortion respectively as follows : 88%, 71% and 65%. It can be checked that this gain varies for the different records as follows: Rec. 108 : 79%-87%, rec. 105 : 77%-79% and rec. 119 : 65%-71%. It can also be checked that the highest gains are obtained for the lowest distortions, i.e. for *near-lossless compression*. This is due to the fact that the more the FAN method goes to near-lossless compression, the more there are CPs to be post-processed, thus more LTR. In the case of higher compression ratios, it is all the way around. Nevertheless, the compression ratio is at most doubled. It is essential that the numerical evaluation be accompanied with visual inspection of the reconstructed signals in this type of applications. For this purpose, Fig. 6 shows the original signals (a), the reconstructed signals by FAN (b) and the reconstructed signals by FAN+ (c) for the three records. Note that in this case all plots are obtained around CR_1 =10:1. The plots clearly show the enhancement of the reconstructed signals due to taking into account LTR in the reconstruction process. This is an important property for our method with regard to classically used coding techniques to take into account LTR where the reconstructed signal is unchanged.

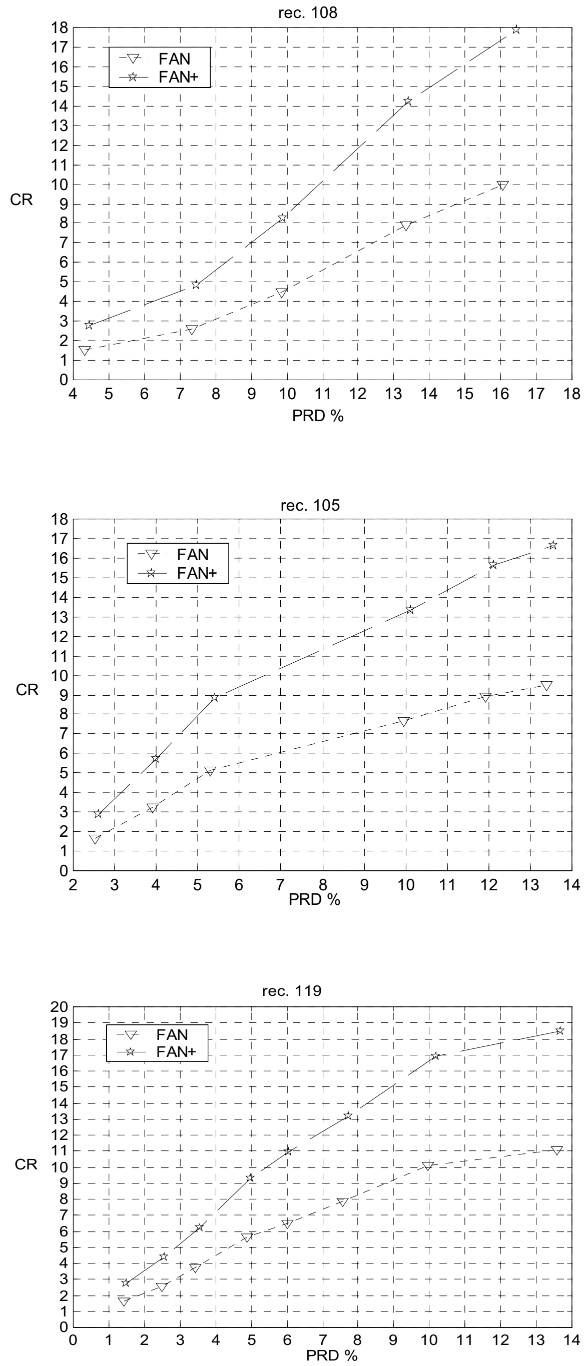

Fig. 5. RD behavior of FAN (triangles) and FAN+ (stars).

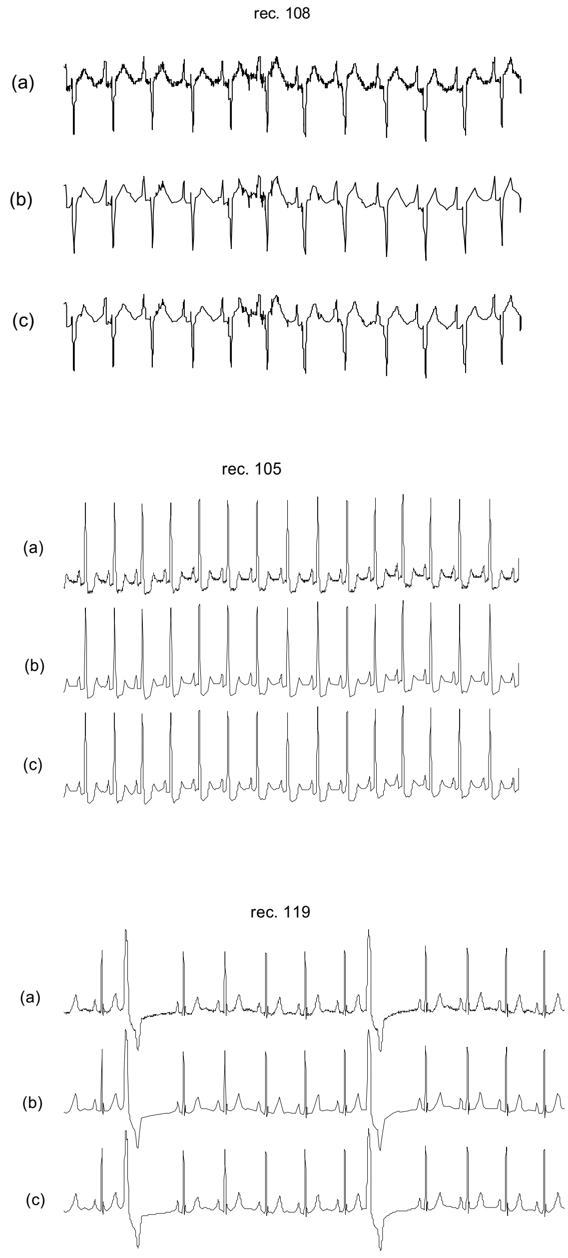

Fig. 6. Original (a) and reconstructed signals by FAN (b) and by FAN+ (c) of MIT-BIH records 108, 105 and 119.

V. CONCLUSION

A novel method for incorporation of long-term redundancy (LTR) in signal time domain compression methods has been proposed. The novel method is based on curve simplification and block sorting. The method is quite a general-purpose one-dimensional signals compression method, with more efficiency for quasi-periodic signals. It was implemented as a post-processing step for the FAN method in the specific case of the ECG signal. Results of the enhanced FAN+ method confirm the substantial improvement of the compression ratio-distortion behavior with respect to that of the FAN method for a better reconstructed quality signal. Note that the proposed method takes into account LTR on the magnitude axis only. It also needs no specific segmentation like R wave detection in the ECG signal. These two properties grant our method to be a general-purpose signal compression tool. Yet, for quasi-periodic signals, like ECG for instance, we believe that LTR reduction can be incorporated on the temporal axis too. Note also that the authors used the Douglas-Peucker algorithm as a unified tool for ECG baseline correction, features detection and STR compression in previous works [9]-[12].

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