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Analysis of a Fluid Behavior in a Rectangular Enclosure under the Effect of Magnetic Field

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***Abstract*—**In this research, a 2-D computational analysis of steady state free convection in a rectangular enclosure filled with an electrically conducting fluid under Effect of Magnetic Field has been performed. The governing equations (mass, momentum, and energy) are formulated and solved by a finite volume method (FVM) subjected to different boundary conditions. A parametric study has been conducted to consider the influence of Grashof number (Gr), Prantdl number (Pr) and the orientation of magnetic field on the flow and heat transfer characteristics. It is observed that Nusselt number (Nu) and heat flux will increase with increasing Grashof and Prandtl numbers and decreasing the slope of the orientation of magnetic field.

***Keywords*—**Rectangular Cavity, magneto-hydrodynamic, free convection, simulation

I. Introduction

THE effect of the magnetic field has many applications in engineering problems such as plasma studies, nuclear reactors, boundary layer control in the field of aerodynamics, geothermal energy extraction and electromagnetic launch technology. Semiconducting and superconducting materials are special types of materials that they are used in electromagnetic launch technology.

In industry the quality of crystal is affected adversely by instabilities in the melt phase because instabilities impose temperature fluctuations at the solidification front and lead to striations in the crystalline product. It is well known that applying magnetic field to the system leads to damping unavoidable hydrodynamic movement and consequently growing high quality crystals. In general, the quality and homogeneity of single crystals grown from dropped semiconductor melts are very important and interesting for manufactures of semi- and superconductors. Therefore, analysis of flow and heat transfer of liquid metals in cavities subjected to external magnetic field is interesting for researchers in this field.

[1] studied numerically the effect of a transverse magnetic field on the natural – convection flow inside a rectangular cavity with adiabatic horizontal walls and iso-thermal vertical walls. They found that a circulating flow is formed with a relatively weak magnetic field that the convection is suppressed and the role of convective heat transfer is decreased when the magnetic field strength increases. [4] have studied the effect of a magnetic field on the buoyancy driven convection in a differentially heated square cavity. The results showed that the flow characteristics inside the cavity and heat transfer mechanism depend strongly upon both the strength of the Rayleigh number and magnetic field. For a review of other numerical, experimental and analytical studies and interested reader may refer to [2-13].

Most of the previous studies apply magnetic field in perpendicular or parallel direction with gravity vector, no more existing studies apply magnetic field in the direction inclined with gravity vector. When the direction of a magnetic field is perpendicular to the gravity vector, the flow induced by the buoyant force crosses it. In that case, in the momentum equation, the vertical velocity component includes an additional term for the electromagnetic force appears. Therefore, the boundary layer approximation is applicable, and the equation is simplified as in [3-4]. However when the direction of magnetic field is parallel to gravity vector, a term for the electromagnetic force appear in the momentum equation for horizontal velocity component and the buoyancy force appears in it due to vertical velocity component. Therefore, the momentum equations for velocity components must be solved [7]. The main scope of the present paper is to study the effect of orientation of magnetic field on thermal and hydrodynamic behavior of typical a fluid in a rectangular cavity.

II. Governing Equation

The Fig. 1 shows a schematic diagram of the system considered in the present study. The system consists of a rectangular cavity with length of L and height of H.The flow in the rectangular cavity is subject to a uniform magnetic field of M_0 . The orientation of magnetic field forms an angle α with horizontal axis. The Boussinesq approximation of linear temperature dependence of density is utilized.

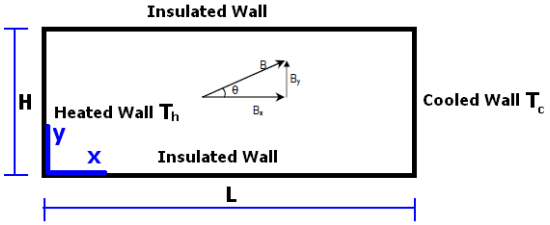

Fig.1. Schematic of computational domain

The magnetic current density is;

$$J = \sigma(V \times M) \quad (1)$$

And the electromagnetic force is;

$$F_{EM} = J \times M \quad (2)$$

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The two-dimensional, non dimensional governing equation for an incompressible, Newtonian liquid in laminar regime and in steady state condition with application the electromagnetic field is given by:

Continuity equation:

$$\frac{\partial \hat{u}}{\partial \hat{x}} + \frac{\partial \hat{v}}{\partial \hat{y}} = 0 \tag{3}$$

X-Momentum equation:

$$\hat{u}\frac{\partial \hat{u}}{\partial \hat{x}} + \hat{v}\frac{\partial \hat{u}}{\partial \hat{y}} = -\frac{\partial \hat{p}}{\partial \hat{x}} + \frac{\partial^2 \hat{u}}{\partial \hat{x}^2} + \frac{\partial^2 \hat{v}}{\partial \hat{y}^2} + Ha^2\left(\hat{v}\cos(\alpha)\sin(\alpha) - \hat{u}\sin^2(\alpha)\right) \tag{4}$$

Y-Momentum equation:

$$\hat{u}\frac{\partial \hat{v}}{\partial \hat{x}} + \hat{v}\frac{\partial \hat{v}}{\partial \hat{y}} = -\frac{\partial \hat{p}}{\partial \hat{y}} + \frac{\partial^2 \hat{v}}{\partial \hat{x}^2} + \frac{\partial^2 \hat{u}}{\partial \hat{y}^2} + Gr\theta + Ha^2\left(\hat{u}\cos(\alpha)\sin(\alpha) - \hat{v}\cos^2(\alpha)\right) \tag{5}$$

Energy equation:

$$\hat{u}\frac{\partial \theta}{\partial \hat{x}} + \hat{v}\frac{\partial \theta}{\partial \hat{y}} = \frac{1}{\Pr}\left(\frac{\partial^2 \theta}{\partial \hat{x}^2} + \frac{\partial^2 \theta}{\partial \hat{y}^2}\right) \tag{6}$$

III. Numerical Solutions

The governing equations (1)-(6) with their associated boundary conditions were solved numerically using an in-home developed finite volume code based on collocated grid. In order to correctly capture the viscous layer, the grids near the solid walls were refined. Figure 2 shows the generated grid used in the present simulation. The convective terms are calculated with using the QUICK [15] scheme and a second-order centered scheme was used to calculated diffusive terms in the governing equations. The SIMPLE [16] (Semi-implicit method for pressure-linked equations) algorithm was used to accomplish the pressure- velocity coupling.

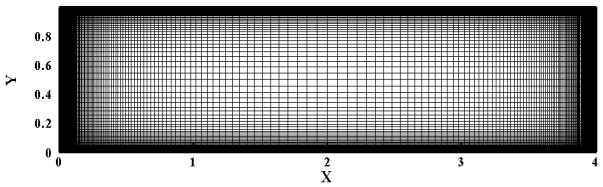


Fig. 2 Computational grid

To obtain better accuracy in the simulations, four quadrilateral grids with total sizes of 30×20 (coarse), 40×30 (medium), 50×40 (fine), 60×50 (very fine) were generated by discretizing the computational domain, for the grid sensitivity study. Table I shows the comparison between the calculated Nusselt number and maximum value of stream function in the computational domain in each case.

TABLE I
Girds Independency Study

Symbol	Number of grid	Nu	ψ_{max}
$\alpha = 0$ $Gr = 10^5$	30*20	.1926	5.69
	40*30	.2156	7.56
	50*40	.2499	8.19
	60*50	.2515	8.30
$\alpha = 90$ $Gr = 10^7$	30*20	.1926	5.69
	40*30	.2156	7.56
	50*40	.2499	8.19
	60*50	.2515	8.30

The comparison of results (Table I) shows, deviations among third and fourth grid were very small, hence the solution becomes independent of grid size in case3. Therefore, based on aforementioned parameters for grid independency test, the case3 with total number of 50×40 cells seemed to be adequate to accurately capture fluid flow and heat transfer behaviors in the cavity and further increasing the grids will have negligible effect on the solution and results.

IV. Results And Discussions

In order to validate the results, we applied our code a system composed of fluid in an enclosure with different Ra numbers and Pr= 0.7 which has been studied by other researchers, [17], [18] and [19]. Table II shows a comparison of calculated average Nusselt number with available data in the literature. Comparison of the present numerical results with available data indicates that the results of our numerical code are in good agreement with them.

TABLE II
Comparison of Present Numerical Results With Available Data In An Enclosure For Pr=0.7 At Different Rayleigh Numbers

Symbol	Present	Davis	Hadjisophocleous et al.[19]	Markatos and Pericleous.[18]
$Ra = 10^4$				
$N\bar{u}$	2.241	2.243	2.29	2.201
$Ra = 10^5$				
$N\bar{u}$	4.513	4.519	4.964	4.430
$Ra = 10^6$				
$N\bar{u}$	8.756	8.799	10.39	8.754

Effects of the parameters such as Grashof number (Gr), Hartmann number (Ha), orientation of magnetic field and Prandtl number of fluid (Pr) on heat transfer and fluid flow inside the cavity have been studied. The first section has

focused on flow and temperature fields, which contains streamlines and isotherms for the different cases. Heat transfer including average Nusselt number at the heated wall has been discussed in the following section. The ranges of Gr, Ha, and α for this investigation vary from 10^5 to 10^7, 0 to 80 and 0 to 90 ,respectively while the prandtl number is kept fixed at 0.015 and 0.15 .

The influence of Grashof number Gr (from $Gr=10^5$ to $Gr=10^7$) on streamlines and isotherms at Ha = 0, Pr = 0.015, $\alpha=0$ has been shown in Fig. 3. The flow with $Gr=10^5$ has been affected by the buoyancy force, thus creating a vortex at the center of cavity. The speed of vortex increases with increasing Grashof number as shown and for higher Gr number, the size of the existing recirculation region becomes smaller while two other vortex are beginning to develop at the right and left of the cavity. The size of these vortexes increases with increasing Gr number.

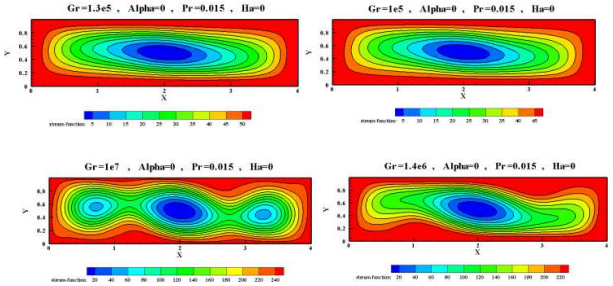


Fig. 3a Effect of Grashof number on streamlines

Fig. 3(b) illustrates the temperature counters in the flow region. The high temperature region remains near the left side of the computational domain and the isothermal lines are nearly linear and parallel to the vertical walls for $Gr=10^5$. These lines become more curved with of growing of Gr. The isothermal lines concentrate near the hot and cold walls for larger values of Gr. With increasing Grashof number the thickness of thermal boundary layer on the hot and cold walls decreases. Therefore, local heat transfer coefficient on the wall will increase.

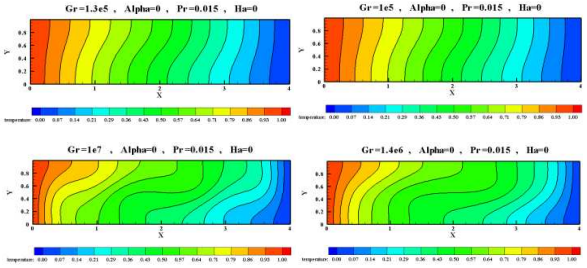


Fig. 3b Effect of Grashof number on temperature counter

The effect of orientation of magnetic field on the flow field is depicted in Fig. 4(a) where $Gr=1.4\times10^6$, $\Pr=0.015$ and $Ha=40$. The streamlines contain a rotating cell at $\alpha=0$. The size of this vortex increases with increasing orientation of magnetic field so that it covers almost the whole of the cavity and this is suitable for better mixing of fluid. Fig. 4(b) illustrates the temperature field in the flow region. The thermal field becomes more compressed at the hot and cold walls of the cavity with decreasing α. So, the high temperature region remains near the hot wall of the computational domain increases and the isothermal lines are become more linear and parallel to the vertical walls with increasing α. It is obvious that with increasing α the Nusselt number on the hot wall decreases because the thermal boundary layer on the wall increases.

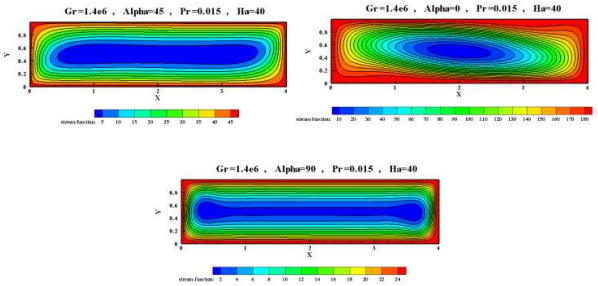


Fig. 4a Effect of orientation of magnetic field on streamlines inside the cavity

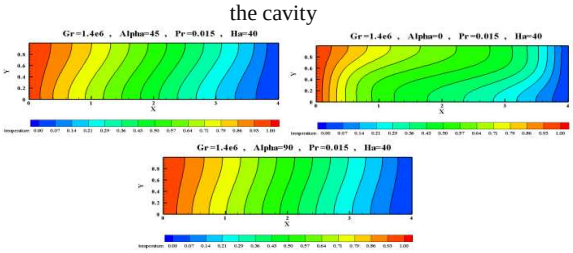


Fig. 4b Effect of orientation of magnetic field on temperature field inside the cavity

Fig. 5 (a,b) shows the effect of Hartmann number Ha (from 0 to 80) on flow field at $Gr=10^7$, $\Pr=0.015$, $\alpha=0$. The non-dimensional number (Ha) shows the power of magnetic force on the flow field and at the absence of magnetic field, the streamlines consist of two recirculation cells including one at the left side of cavity and a secondary eddy at the right side of cavity. As seen, these vortexes loss their strength and finally are disappeared with rising Ha while larger vortex produced at the center of cavity. The corresponding temperature field shows that the concentrated region near the walls becomes more compressed and the isothermal lines are more bend from the right top corner due to the elevating Hartmann number. It means that the magnetic field significantly affects the flow and thermal fields in the cavity.

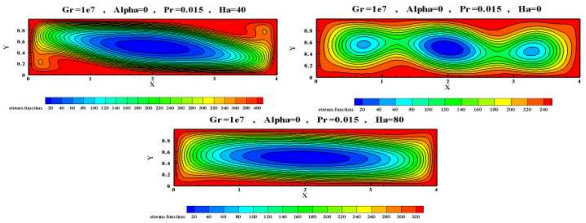


Fig. 5a Effect of Hartmann number on streamlines inside the cavity

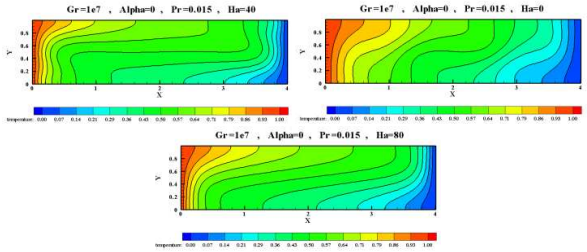


Fig. 5b Effect of Hartmann number on temperature field inside the cavity

Fig. 6 (a,b) show the effect of Prandtl number (Pr) on flow field at $Gr = 1.4\times10^{6}$, $Ha = 40$, $\alpha = 0$. As seen at both Prantdl numbers, the streamlines consist of a recirculation cell at the center of cavity but with rising Pr, the vortex become stronger and finally is covered the most of computational domain. The corresponding temperature field shows the concentrated region near the walls becomes more compressed and the isothermal lines at the whole of domain except near vertical walls are become more linear and parallel to horizontal walls due to the elevating Prandtl number. It is clear that with increasing Prandtl number the thermal boundary layer on the walls decreases and the heat transfer coefficient on the walls increase.

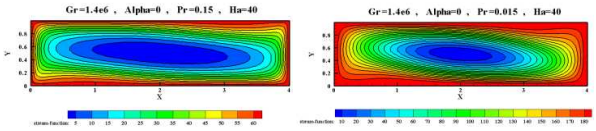


Fig. 6a Effect of Prandtl number on streamlines inside the cavity

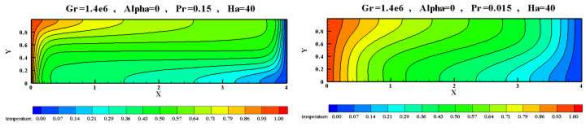
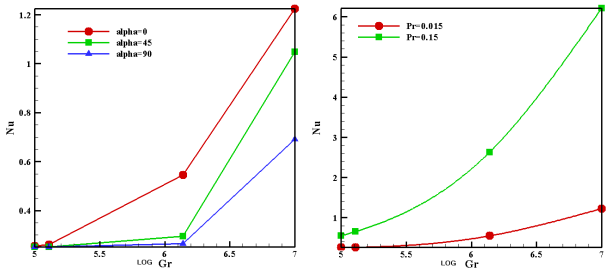


Fig. 6b Effect of Prandtl number on temperature field inside the cavity

In order to show how the presence of Prandtl number, magnetic field and its orientation affects the heat transfer rate along the heated surface, the average Nusselt number is plotted as a function of Grashof number as shown in Fig 7. It is observed that Nu rises with increasing Grashof and Prandtl numbers and decreasing Hartmann and orientation of magnetic field. The maximum heat transfer rate is obtained for the lowest Ha and the highest Gr, because the magnetic field tends to concentrate motion.The flow at the center of cavity. It is worth mentioning that the influence of mentioned parameters on Nusselt number is not very sensitive at lower Grashof numbers.



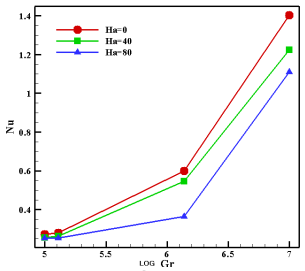


Fig. 7 Effect of $\alpha, \mathrm{Pr}, Gr, Ha$ on Nusselt number

V. Conclusion

In the present investigation, we studied the effect of magnetic field on natural convection flow in a rectangular enclosure filled with an electrically conducting fluid. The governing equations along the appropriate boundary conditions for the present problem are first transformed into a non-dimensional form and the resulting non linear system of partial differential equations are then solved numerically using finite volume method. The influence of Grashof number, Prantdl number of fluid, Hartmann number and orientation of magnetic field on the flow and heat transfer characteristic such as average Nusselt number, streamlines and isotherms is performed. It is observed that Nu rises with increasing Grashof and Prandtl numbers and decreasing Hartmann and orientation of magnetic field.

Nomenclature

C_p	Heat capacity of fluid at constant pressure ($J/kg.K$)
H	Height of cavity (m)
k	Thermal conductivity ($W/m.K$)
L	Length of cavity (m)
p	Pressure (pa)
$\hat{p}$	Non-dimensional pressure
Pr	Prandtl number
T	Temperature (K)
Nu	Nusselt number
Gr	Grashof number
Ha	Hartmann number
B	Magnetic field
g	Acceleration due to gravity
u, v	Velocity components in x and y directions (m/s)
$\hat{u}, \hat{v}$	Non-dimensional velocity components in X and Y directions
N_x, N_y	Number of grid in x and y directions
x, y	Cartesian coordinates(m)
$\hat{x}, \hat{y}$	non-dimensional Cartesian coordinates

Greek Symbols

α	Orientation angle of magnetic field
θ	non-dimensional temperature

μ dynamic viscosity(kg/ms^2)

ρ Density (kg/m^3)

β Thermal expansion coefficient

σ Electrical conductivity of fluid

SUBSCRIPTS

h Hot

c Cold

$\circ$ Previous iteration

x in x direction

y in y direction

z in z direction

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