

# Method of Finding Aerodynamic Characteristic Equations of Missile for Trajectory Simulation

Attapon Charoenpon, Ekkarach Pankeaw

**Abstract**—This paper present a new way to find the aerodynamic characteristic equation of missile for the numerical trajectories prediction more accurate. The goal is to obtain the polynomial equation based on two missile characteristic parameters, angle of attack ( $\alpha$ ) and flight speed ( $v$ ). First, the understudied missile is modeled and used for flow computational model to compute aerodynamic force and moment. Assume that performance range of understudied missile where range  $-10 < \alpha < 10$  and  $0 < v < 200$ . After completely obtained results of all cases, the data are fit by polynomial interpolation to create equation of each case and then combine all equations to form aerodynamic characteristic equation, which will be used for trajectories simulation.

**Keywords**—Aerodynamic, Characteristic Equation, Angle of Attack, Polynomial interpolation, Trajectories

## I. INTRODUCTION

In recently, modern missile design and analysis are performed by estimating data such as aerodynamic coefficient from historical database. Firstly, the shape is roughly modeled and all of coefficient are interpolating obtained. Then rocket stability and trajectory are calculated. The result obtained still insufficient accuracy due to historical based calculation. To increase accuracy of results we will use the exactly coefficient of missile which obtained from CFD or experimental data and form them to be the aerodynamic equation that describe actual performance of missile called aerodynamic characteristic equation. Then these equations will be used to calculate its trajectory. The concept of this paper is shown in Fig. 1

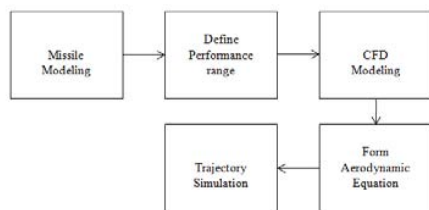


Fig. 1 Concept of Work Procedure

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## II. CONCEPT OF POLYNOMIAL AERODYNAMIC EQUATION

Considering the aerodynamic force acting on the missile as shown in Fig. 2

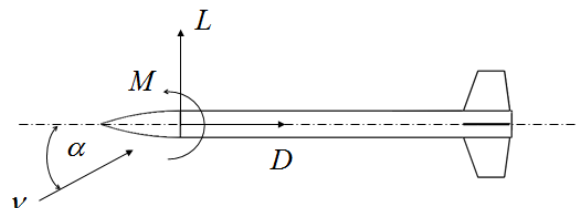


Fig. 2 Aerodynamic force and moment acting on missile

The missile has flight speed  $v$  and angle of attack  $\alpha$  cause the resultant force and moment are generated on the center of pressure of missile. There is slightly complicate to calculate the center of pressure so we will locate new position for resultant force and moment to junction between nosecone and body tube and namely to Transverse force ( $L$ ), Axial force ( $D$ ) and Moment ( $M$ ). The assumptions are the rocket has no axial spinning, symmetry both up-down and left-right side and no surface roughness. So we can derive the relation between force flight speed and angle of attack so that the equation will be [1]

$$L = 0.5 \rho v^2 S C_l \alpha \quad (1)$$

$$D = 0.5 \rho v^2 S C_d \alpha \quad (2)$$

$$M = 0.5 \rho v^2 S l C_m \alpha \quad (3)$$

Where  $S$  = Reference Area  
 $l$  = Length Area

Rearrange the equation (1) though (3) (Shown only equation (1))

$$L = 0.5 \rho v^2 S a_l \alpha$$

$$L = [P \alpha] v^2 \quad (4)$$

And the other equations will be

$$D = [R \alpha] v^2 \quad (5)$$

$$M = [S \alpha] v^2 \quad (6)$$

Where  $P, R, S$  are coefficients

We see that equations are parabolic equation which is one kind of polynomial equation scheme. So we extend them more term of polynomial, we get

$$L = \left[ A_1 \alpha^4 + B_1 \alpha^3 + C_1 \alpha^2 + D_1 \alpha + E_1 \right] v^3 + \left[ F_1 \alpha^4 + G_1 \alpha^3 + H_1 \alpha^2 + I_1 \alpha + J_1 \right] v^2 \quad (7)$$

$$D = \left[ A_2 \alpha^4 + B_2 \alpha^3 + C_2 \alpha^2 + D_2 \alpha + E_2 \right] v^3 + \left[ F_2 \alpha^4 + G_2 \alpha^3 + H_2 \alpha^2 + I_2 \alpha + J_2 \right] v^2 \quad (8)$$

$$M = \left[ A_3 \alpha^4 + B_3 \alpha^3 + C_3 \alpha^2 + D_3 \alpha + E_3 \right] v^3 + \left[ F_3 \alpha^4 + G_3 \alpha^3 + H_3 \alpha^2 + I_3 \alpha + J_3 \right] v^2 + \left[ K_3 \alpha^4 + L_3 \alpha^3 + M_3 \alpha^2 + N_3 \alpha + O_3 \right] v \quad (9)$$

Firstly we assume that the highest order of variable is 3 to ensure that the dominant term are order 2 by classical aerodynamic theory as indicated in equation (1)-(3) and the order of variable is 3 initially.

Variable  $A, B, C, D, E, F, G, H, I, J, K, M, N$  and  $O$  are "Specified Characteristic coefficient" of missile which described the missile performance and trajectory of specific missile.

### III. MISSILE AND FLOW SIMULATION MODELING

The understudied missile had been modeled with tangent ogive nose cone, four symmetrical fins. Its length is 1240mm and center of pressure 893mm from nose tip [2].

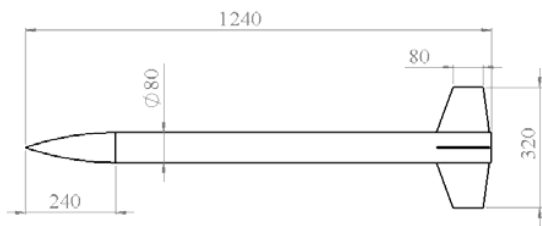


Fig. 3 Schematic of missile model

Using commercial CFD software to solve flow simulation with tetrahedron element 136644 elements (total 280508 elements). The boundary conditions are specified by two variables, velocity which range 0 to 200 m/s and angle of attack -10 to 10deg (required performance).

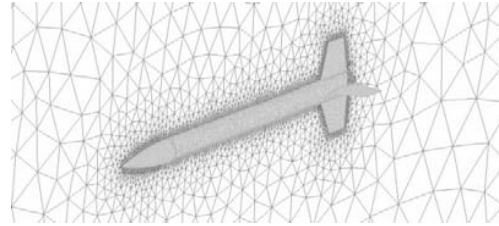


Fig. 4 Meshing with Tetrahedron Element

### IV. FLOW SIMULATION RESULT

The results were plotted between aerodynamic force/moment and velocity to find the relationship between them. Obviously, axial force is slightly constant independent to angle of attack. This is due to when angle of attack was changed, the velocity along axial was changed a little bit so the axial force is almost the same at difference angle of attack.

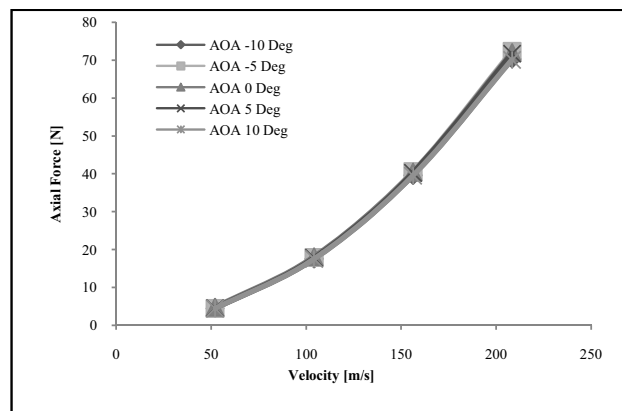


Fig. 5 Schematic of Axial force versus Velocity

For transverse force and moment, the results are the same patterning which similar to parabolic curve and symmetry with horizontal axis.

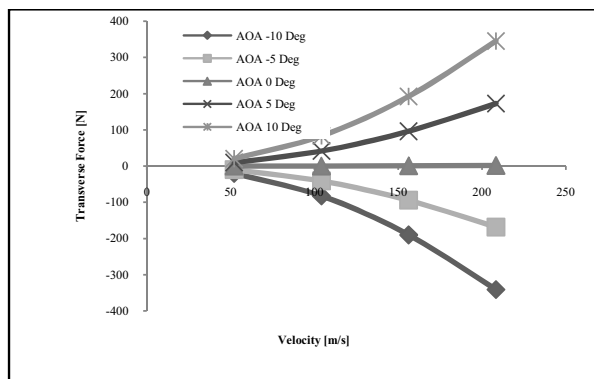


Fig.6 Schematic of Transverse Force versus Angle of Attack

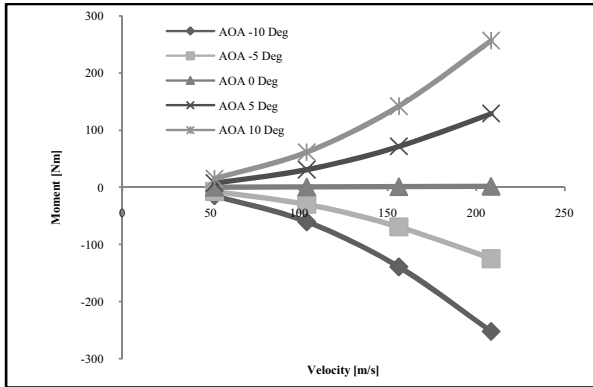


Fig. 7 Schematic of Moment versus Velocity

#### V. FORMING AERODYNAMIC CHARACTERISTIC EQUATION

To form the aerodynamic characteristic equation, we made a fitted curve from flow simulation result. Firstly we formed the equation that relate between aerodynamic force/moment and velocity. The suitable order of velocity which conformed to result data is 3 that the same as we estimated previously.

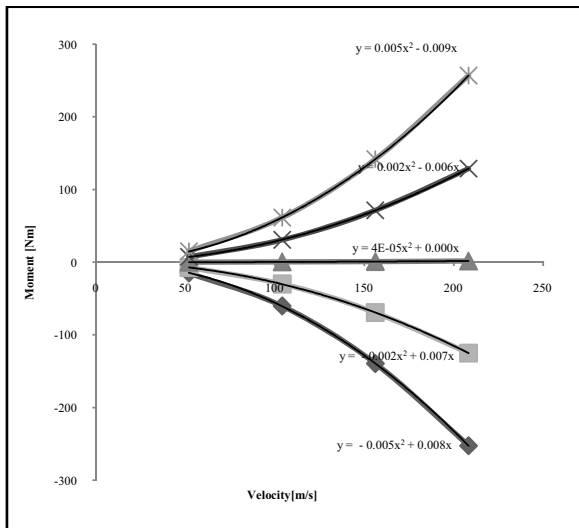


Fig. 8 Schematic of Curve fitting to form equation: illustration of Moment Result

The coefficients of each term we obtained in this stage is still depend on angle of attack, so to extract angle of attack parameter from these coefficients we would plot coefficient versus angle of attack and made a fitted curve again. In this stage we obtained the equation that relate between coefficient value and angle of attack. The highest order of angle of attack is 4 (Previously, we assumed 3).

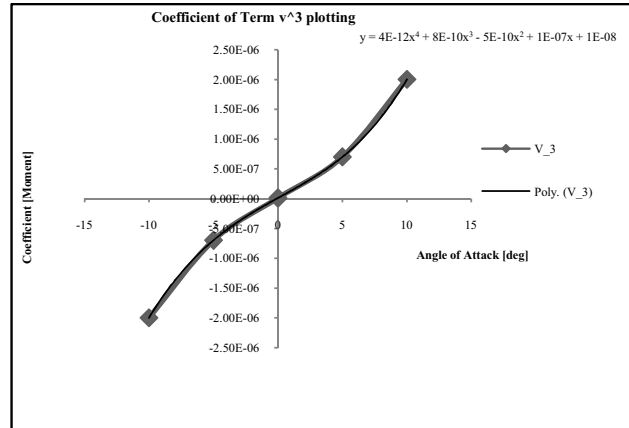


Fig. 9 Schematic of Curve fitting of coefficient versus angle of attack: illustration of moment result

Then we combined all of equations together and got completely equations that describe how aerodynamic force and moment change with missile flight velocity and angle of attack as defined below.

$$L = \left[ -2 \times 10^{-11} \alpha^4 + 9 \times 10^{-10} \alpha^3 + 2 \times 10^{-9} \alpha^2 + 7 \times 10^{-9} \alpha + 3 \times 10^{-8} \right] v^3 + \left[ -4 \times 10^{-9} \alpha^4 - 7 \times 10^{-8} \alpha^3 + 5 \times 10^{-7} \alpha^2 + 0.0008 \alpha + 4 \times 10^{-5} \right] v^2 + \left[ -4 \times 10^{-7} \alpha^4 + 9 \times 10^{-6} \alpha^3 + 2 \times 10^{-5} \alpha^2 - 0.0035 \alpha + 0.0004 \right] v \quad (10)$$

$$D = \left[ 4 \times 10^{-11} \alpha^4 - 2 \times 10^{-10} \alpha^3 - 7 \times 10^{-9} \alpha^2 + 1 \times 10^{-8} \alpha + 3 \times 10^{-7} \right] v^3 + \left[ -7 \times 10^{-8} \alpha^4 + 1 \times 10^{-7} \alpha^3 + 8 \times 10^{-6} \alpha^2 - 1 \times 10^{-5} \alpha + 0.0015 \right] v^2 + \left[ 4 \times 10^{-6} \alpha^4 - 3 \times 10^{-6} \alpha^3 - 0.0005 \alpha^2 - 0.0003 \alpha + 0.01 \right] v \quad (11)$$

$$M = \left[ 4 \times 10^{-12} \alpha^4 + 8 \times 10^{-10} \alpha^3 - 5 \times 10^{-10} \alpha^2 + 1 \times 10^{-7} \alpha + 1 \times 10^{-8} \right] v^3 + \left[ -4 \times 10^{-9} \alpha^4 - 2 \times 10^{-7} \alpha^3 + 5 \times 10^{-7} \alpha^2 + 0.0006 \alpha + 4 \times 10^{-5} \right] v^2 + \left[ -3 \times 10^{-6} \alpha^4 + 7 \times 10^{-6} \alpha^3 + 2 \times 10^{-5} \alpha^2 - 0.0016 \alpha + 0.0002 \right] v \quad (12)$$

To estimate the error from curve fitting process, using equation (13) to calculate error of each aerodynamic force and moment

$$er_i = 200 \times \left[ \frac{Actual - CurveFit}{Actual + CurveFit} \right]_i \% \quad (13)$$

And the total error of all fitting process

$$er_{total} = \sqrt{\sum er_i^2} \quad (14)$$

We can calculate the average total error which is 4.687%. That will affect the trajectory within  $\pm 2.34\%$ .

#### VI. APPLY EQUATION TO SIMULATE MISSILE TRAJECTORY

Considering motion of missile in the inertia reference frame  $R_E(X_E, Y_E, Z_E)$  and let body reference frame  $R_B(X_B, Y_B, Z_B)$  be on missile body as shown in Fig. 10.

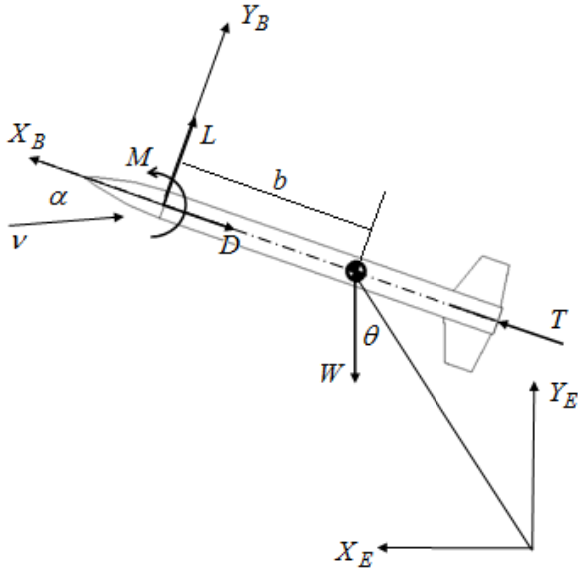


Fig. 10 Definition of Equation of motion

We can write the equation of motion respect to body reference frame that

$$\ddot{X}_B = \frac{(T - D - W \sin \theta)g}{W} \quad (15)$$

$$\ddot{Y}_B = \frac{(L - W \cos \theta)g}{W} \quad (16)$$

$$\ddot{\alpha}_B = \frac{M - Lb}{I_Z} \quad (17)$$

And the relation between body and inertia reference frame is

$$\begin{Bmatrix} X_E \\ Y_E \end{Bmatrix} = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix} \begin{Bmatrix} X_B \\ Y_B \end{Bmatrix} \quad (18)$$

We can solve these equations of motions easily with numerical result to find the trajectory. In this paper using the Rungutta-Merson method to solve the equation. With RATTI90 (174.1N-max, 6.43s burned time) propellant and weight of missile at 1.2kg-constant we could calculated the trajectory as shown in Fig. 11

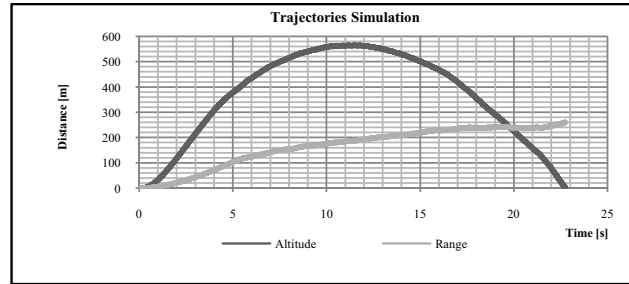


Fig. 11 Trajectory simulation result

#### VII. CONCLUSION

The goal of this research was to develop Aerodynamic Characteristic for using in trajectories simulation. The equations are formed by curve fitting data obtained from flow simulation.

The equations were constructed in form of perpendicular force, axial force and moment which depended on two variable parameters, velocity and angle of attack. The obtained equation has power 3 of velocity term as we expected and corresponded with the classical aerodynamic theory. And power of angle of attack was 4.

The contained error both from flow simulation and curve fitting process are calculated first from the curve fitting process the error is 2.34% and second from the flow simulation which the wind tunnel experimental data is under studying to compare the result. To improve that error an actual data from experiment is preferable and more polynomial orders are required.

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