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Mixed Convection Boundary Layer Flow from a Vertical Cone in a Porous Medium Filled with a Nanofluid

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Abstract—The steady mixed convection boundary layer flow from a vertical cone in a porous medium filled with a nanofluid is numerically investigated using different types of nanoparticles as Cu (copper), Al_2O_3 (alumina) and TiO_2 (titania). The boundary value problem is solved by using the shooting technique by reducing it into an ordinary differential equation. Results of interest for the local Nusselt number with various values of the constant mixed convection parameter and nanoparticle volume fraction parameter are evaluated. It is found that dual solutions exist for a certain range of mixed convection parameter.

Keywords—boundary layer, mixed convection, nanofluid, porous medium, vertical cone.

I. Introduction

MANY engineering applications are concerned with the combination of heat transfer in fluid saturated porous media. The interest in this subject has been stimulated by the fact that thermally driven flows in porous media have considerable applications in mechanical, chemical and civil engineering. Applications include fibrous insulation, food processing and storage, thermal insulation of buildings, geophysical systems, electro chemistry, etc. One of the fundamental problems concerning heat transfer in porous media is the mixed convection boundary layer flow from a vertical cone in a porous medium filled with a nanofluid. A wide application of porous media in many practical applications can be found in the well known books by Nield and Bejan [1], Ingham and Pop [2], Pop and Ingham [3], Bejan *et al.* [4], and Vadasz [5].

The term nanofluid has been first introduced by Choi [6] to define the dilution of nanometer-sized particles (smaller than 100nm) in a fluid [7], such as water, ethylene glycol and oil [8]. These nanoparticles, which normally contain metals, oxides, carbides, or carbon nanotubes, have unique chemical and physical properties [9], and due to the nanometer-sized, nanoparticles can easily flow smoothly through the microchannels [10], and as such, nanofluid has better thermal conductivity and convective heat transfer coefficient compared to the base fluid only [11]. Many studies related to nanofluids characteristics are very well described in the book by Das *et al.* [7] and in the review papers by Buongiorno [12], Wang and Mujumdar [13], Kakaç and Pramuanjaroenkij [14], Lee *et al.* [15], Eagen *et al.* [16], and Fan and Wang [17]. By using nanofluids, the heat transfer can be reduced or enhanced and they can be utilized where straight heat transfer enhancement is very important as in many industrial applications, nuclear reactors, transportation as well as electronics and biomedicine.

The principal aim of this paper is to examine the mixed convection boundary layer flow from a vertical cone in a porous medium filled with a nanofluid. Based on the literature review, Ahmad and Pop [18] had carried out a research entitled mixed convection boundary layer flow from a vertical flat plate embedded in a porous medium filled with nanofluids by using different types of nanoparticles as Cu (cuprum), Al_2O_3 (alumina) and TiO_2 (titania). Nazar *et al.* [19] had studied similar problem but with horizontal circular cylinder. They studied in both cases which are heated and cooled cylinder. The solutions for the flow and heat transfer characteristics are evaluated numerically for various values of the governing parameters, namely the nanoparticle volume fraction φ and the mixed convection parameter λ. They also considered the same nanoparticles as Ahmad and Pop [18]. Besides, there are also papers based on the truncated cone. Cheng *et al.* [20] examined the natural convection of a Darcian fluid about a cone followed by the various papers. Ching-Yang Cheng [21] studied a problem of nonsimilar boundary layer analysis of double-diffusive convection from a vertical truncated cone in a porous medium with variable viscosity. The mentioned literature survey indicates that there is still no study regarding this paper.

II. Basic equation

Consider the steady mixed boundary layer flow near a vertical cone (with half angle σ) embedded in a saturated porous medium filled with a nanofluid as shown in Fig. 1. The origin O of the coordinate system is placed at the vertex of the cone, where x is the coordinate measured from the origin O along the surface of the cone and y is the coordinate normal to the surface of the cone. The surface of the cone is held at the contant heat flux q_w, while the temperature of the ambient nanofluid is T_∞. Under the assumption of using the model proposed by Tiwari and Das [22] along with the assumptions of the Darcy-Boussinesq and boundary layer approximations, the basic equations governing the steady free

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or mixed convection flow near a vertical cone embedded in a porous medium filled with a nanofluid are, see also Nield and Bejan [1].

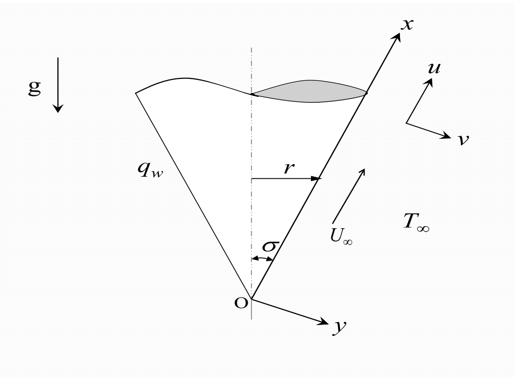

Fig. 1. Physical model and coordinate system

$$\frac{\partial}{\partial x}(ru) + \frac{\partial}{\partial y}(rv) = 0, \tag{1}$$

$$\frac{\mu_{nf}}{\mu_f} u = \frac{\mu_{nf}}{\mu_f} u_e(x) + \frac{gK[\varphi\rho_s\beta_s + (1-\varphi)\rho_f\beta_f]}{\mu_f}(T - T_\infty), \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha_{nf}\frac{\partial^2 T}{\partial y^2}, \tag{3}$$

subject to the boundary conditions

$$v = 0, \quad -k_{nf}\frac{\partial T}{\partial y} = q_w \quad \text{at} \quad y = 0,$$
$$u = u_e(x), \quad T = T_\infty \quad \text{as} \quad y \to \infty. \tag{4}$$

Here u and v are the velocity components along x and y axes, respectively, T is the temperature of the nanofluid, g is the acceleration due to gravity, $r = x\sin\sigma$, φ is the nanoparticle volume fraction, β_f and β_s are the coefficients of the thermal expansion of the fluid and of the solid, respectively, μ_{nf} is the viscosity of the nanofluid, α_{nf} is the thermal diffusivity of the nanofluid and k_{nf} is the thermal conductivity of the nanofluid, which are given by

$$\mu_{nf} = \frac{\mu_f}{(1-\varphi)^{2.5}}, \quad \alpha_{nf} = \frac{k_{nf}}{(\rho C_p)_{nf}},$$

$$(\rho C_p)_{nf} = (1-\varphi)(\rho C_p)_f + \varphi(\rho C_p)_s,$$

$$\frac{k_{nf}}{k_f} = \frac{(k_s + 2k_f) - 2\varphi(k_f - k_s)}{(k_s + 2k_f) + \varphi(k_f - k_s)}, \tag{5}$$

where μ_f is the dynamic viscosity of the base fluid and its expression has been proposed by Brinkman [23], k_f and k_s are the thermal conductivities of the base fluid and of the solid, respectively, and $(\rho C_p)_{nf}$ is the heat capacitance of the nanofluid. It is worth mentioning that the expressions (5) are restricted to spherical nanoparticles where it does not account for other shapes of nanoparticles. The thermophysical properties of fluid and nanoparticles are given in Table 1 (Oztop and Abu-Nada, [24]).

Now, we look for a solution of Eqs. (2) and (3) of the form

$$\psi = \alpha_f r Pe_x^{1/2} F(\eta), \quad \eta = Pe_x^{1/2}\frac{y}{x},$$

$$T - T_\infty = \frac{x q_w}{k_{nf}} Pe_x^{-1/2} G(\eta), \tag{6}$$

where $Pe_x = u_e(x)x/\alpha_f$ is the local Péclet number. In order that Eqs. (2) and (3) admit similarity solutions, we assume that $u_e(x) = U_\infty x^{1/2}$, where U_∞ is a characteristic velocity. Thus, we have

$$u = U_\infty x^{1/2} F'(\eta), \quad v = -\alpha_f (Pe_x^{1/2}/4x)[7F(\eta) - \varsigma F'(\eta)], \tag{7}$$

where prime now denotes differentiation with respect to η . Substituting (7) into Eqs. (2) and (3), we get

$$\frac{1}{(1-\varphi)^{2.5}}F' = \frac{1}{(1-\varphi)^{2.5}} + \left[1 - \varphi + \varphi(\frac{\rho_s}{\rho_f})(\frac{\beta_s}{\beta_f})\right]\lambda G, \tag{8}$$

$$\frac{k_{nf}/k_f}{(1-\varphi) + \varphi(\rho C_p)_s/(\rho C_p)_f} G'' + \frac{7}{4}FG' - \frac{1}{4}F'G = 0, \tag{9}$$

along with the boundary conditions

$$F(0) = 0, \quad \frac{k_{nf}}{k_f}G'(0) = -1, \quad G(\infty) = 0, \tag{10}$$

where λ is the constant mixed convection parameter, which is defined as

$$\lambda = \frac{Ra_x}{Pe_x^{3/2}}. \tag{11}$$

It should be mentioned that $\lambda > 0$ for an assisting flow ($q_w > 0$), $\lambda < 0$ for an opposing flows ($q_w < 0$) and $\lambda = 0$ for a forced convection flow, respectively.

The local Nusselt number is given in this case by

$$Pe_x^{-1/2} Nu_x = \frac{k_{nf}}{k_f}\frac{1}{G(0)}. \tag{12}$$

TABLE I
THERMOPHYSICAL PROPERTIES OF FLUID AND NANOPARTICLES [24]

Physical properties	Fluid phase (water)	Cu	Al_2O_3	TiO_2
$C_p(J/kgK)$	4179	385	765	686.2
$\rho(kg/m^3)$	997.1	8933	3970	4250
$k(W/mK)$	0.613	400	40	8.9538
$\beta \times 10^{-5}(1/K)$	21	1.67	0.85	0.9

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III. Results and discussion

Numerical solutions to the governing ordinary differential equation with the boundary conditions are obtained using the shooting method. The effects of the solid volume fraction of nanofluid φ and the mixed convection parameter λ are analyzed for a porous medium filled with a regular fluid ($\varphi = 0$) and a porous medium filled with three different nanofluids as Cu (copper), Al_2O_3 (alumina) and TiO_2 (titania) as working fluids. The values of φ that considered are φ = 0, 0.1 and 0.2 while for λ we considered both, the positive values with corresponds to assisting flow and negative values with corresponds to the opposing flows. The results of the local Nusselt number $Pe_x^{-1/2} Nu_x$ have been shown in Figs. 2 and 3 with the variation of λ. It is seen that for positive values of λ there is only one solution. For negative values of λ, there is a critical value λ_c for which the upper branch solution meets the lower branch solution. Values of λ_c for each φ are stated in the Figs. 2 and 3. The boundary layer separates from the surface at $\lambda = \lambda_c$, thus we are unable to get the solution for $\lambda < \lambda_c$ by using the boundary layer approximations. It is found from Figs. 2 and 3 that the lower branch solutions for various φ and various nanoparticles; respectively intersect at $\lambda_s = -0.001$.

It is noticed from Fig. 2 that for the forced convection flow ($\lambda = 0$) and the assisting flow ($\lambda > 0$), the value of local Nusselt number increases as the values of φ increase. These results prove that nanofluid is a better heat transfer fluid compared to the regular (pure) fluid. Further, Fig. 3 shows the variation of the local Nusselt number for Cu, Al_2O_3 and TiO_2 with mixed convection parameter λ when $\varphi = 0.1$. It is shown from Fig. 3 that for forced and assisting flows, Cu-water nanofluid has higher local Nusselt number followed by Al_2O_3-water and TiO_2-water nanofluids. Physically it is because the thermal conductivity *k* of copper is the highest among alumina and titania, as given in Table 1.

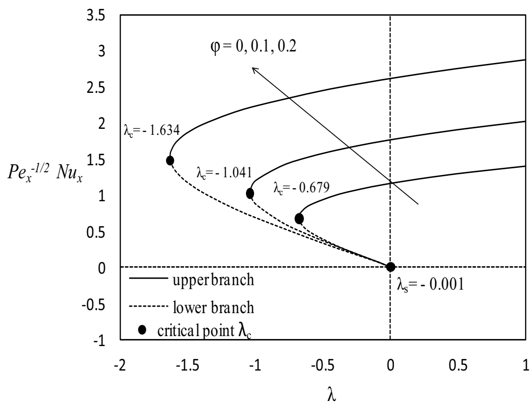

Fig. 2. Variation of $Pe_x^{-1/2} Nu_x$ for Cu with parameter λ for various values of φ

Figs. 4 and 5 illustrated the temperature profiles $G(\eta)$ for regular fluid ($\varphi = 0$) and Cu-water nanofluid with $\varphi = 0.1$, respectively. These two figures show the exists of upper and lower branches. It is also shown from Figs. 4 and 5 that the

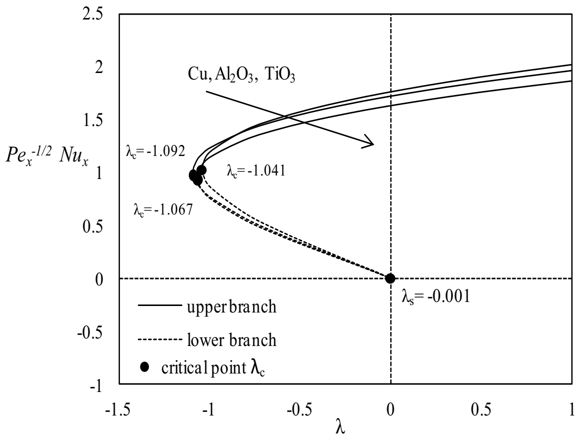

Fig. 3. Variation of $Pe_x^{-1/2} Nu_x$ for Cu, Al_2O_3 and TiO_2 with parameter λ when $\varphi = 0.1$

upper and lower branch solutions satisfy the boundary condition asymptotically, which support the obtained numerical results.

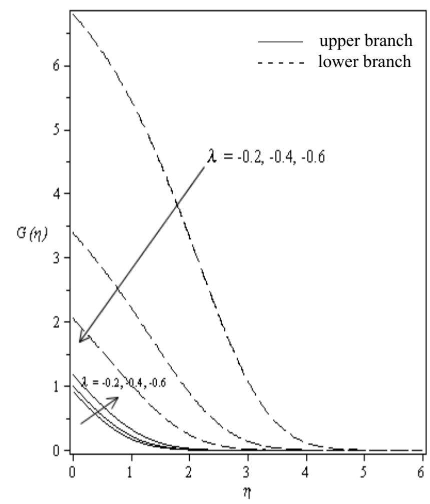

Fig. 4. Temperature profiles of the upper and lower branch solution for regular fluid ($\varphi = 0$) under different values of λ

IV. Conclusion

We have theoretically studied the similarity solutions of the mixed convection boundary layer flow from a vertical cone in a porous medium filled with a nanofluid. We discussed the effects of the nanoparticle volume fraction parameter and constant mixed convection parameter on the heat transfer characteristics. It is seen that there are regions of unique solution when $\lambda > 0$, dual solutions when $\lambda_c < \lambda \leq 0$ and no solution when $\lambda < \lambda_c < 0$. So, the solutions only can be reached up to the critical value $\lambda = \lambda_c < 0$, that's when the boundary layer separates from the surface and the

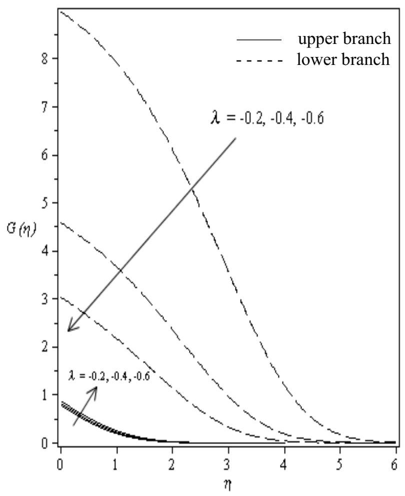

Fig. 5. Temperature profiles of the upper and lower branch solution for Cu when $\varphi = 0.1$ under different values of λ

solution based upon the boundary layer approximations are not possible.

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