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$(\tau_1, \tau_2)^*$-semi star generalized locally closed sets

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Abstract—The aim of this paper is to continue the study of $(\boldsymbol{\tau_1}, \boldsymbol{\tau_2})^*$-semi star generalized closed sets by introducing the concepts of $(\boldsymbol{\tau_1}, \boldsymbol{\tau_2})^*$-semi star generalized locally closed sets and study their basic properties in bitopological spaces.

Keywords—$(\tau_1, \tau_2)^*$-semi star generalized locally closed sets, $\tau_1\tau_2$-semi star generalized closed sets, $(\tau_1, \tau_2)^*$-semi generalized locally closed sets, $(\tau_1, \tau_2)^*$-generalized locally closed sets, $(\tau_1, \tau_2)^*$-generalized semi locally closed sets.

I. Introduction

THE study of generalization of closed sets has been found to ensure some new separation axioms which have been very useful in the study of certain objects of digital topology. In recent years many generalizations of closed sets have been developed by various authors. K. Chandrasekhara Rao and K. Joseph [3] introduced the concepts of semi star generalized open sets and semi star generalized closed sets in unital topological spaces.

Ganster and Reilly [14] introduced locally closed sets in topological spaces and Stone called locally closed sets as FG sets. H. Maki, P. Sundaram and K. Balachandran [26] introduced the concept of generalized locally closed sets and obtained seven different notions of generalized continuities.

Ganster, Arockiarani and Balachandran [13] introduced regular generalized locally closed sets and RGLC continuous functions and discussed some of their properties. K. Chandrasekhara Rao and K. Kannan [6], [7] introduced the concepts of semi star generalized locally closed sets and s^*g-submaximal spaces in unital topological spaces.

Mean while J.C. Kelly [21] introduced the study of bitopological spaces. M. Jelic [19] introduced locally closed sets and lc-continuity in bitopological settings. K. Chandrasekhara Rao and K. Kannan [4], [5] introduced the concepts of semi star generalized closed sets in bitopological spaces.

They [8] also introduced the concepts of $\tau_1\tau_2$-semi star generalized locally closed sets and pairwise s^*g-submaximal spaces with the help of s^*g-closed sets and studied their basic properties in bitopological spaces.

In this sequel the aim of this paper is to introduce the concepts of $(\tau_1, \tau_2)^*$-semi star generalized locally closed sets, pairwise $^*s^*g$ submaximal spaces and study their basic properties in bitopological spaces. In the next section some prerequisites and abbreviations are established.

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II. Preliminaries

Let (X, τ_1, τ_2) or simply X denote a bitopological space. By $\tau_1\tau_2$-$S^*GO(X, \tau_1, \tau_2)$ {resp. $\tau_1\tau_2$-$S^*GC(X, \tau_1, \tau_2)$}, we shall mean the collection of all $\tau_1\tau_2$-s^*g open sets (resp. $\tau_1\tau_2$-s^*g closed sets) in (X, τ_1, τ_2). For any subset $A \subseteq X$, τ_i-int(A) and τ_i-cl(A) denote the interior and closure of a set A with respect to the topology τ_i respectively. A^C denotes the complement of A in X unless explicitly stated. We shall require the following known definitions.

Definition 2.1: A subset of a bitopological space (X, τ_1, τ_2) is called

(a) $\tau_1\tau_2$-semi open if there exists a τ_1-open set U such that $U \subseteq A \subseteq \tau_2$-cl (U).

(b) $\tau_1\tau_2$-semi closed if $X - A$ is $\tau_1\tau_2$-semi open. Equivalently, a subset A of a bitopological space (X, τ_1, τ_2) is called $\tau_1\tau_2$-semi closed if there exists a τ_1-closed set F such that τ_2-int $(F) \subseteq A \subseteq F$.

(c) $\tau_1\tau_2$-generalized closed ($\tau_1\tau_2$-g closed) if τ_2-cl $(A) \subseteq U$ whenever $A \subseteq U$ and U is τ_1-open in X.

(d) $\tau_1\tau_2$-generalized open ($\tau_1\tau_2$-g open) if $X - A$ is $\tau_1\tau_2$-g closed.

(e) $\tau_1\tau_2$-semi star generalized closed ($\tau_1\tau_2$-s^*g closed) if τ_2-cl $(A) \subseteq U$ whenever $A \subseteq U$ and U is τ_1-semi open in X.

(f) $\tau_1\tau_2$-semi star generalized open ($\tau_1\tau_2$-s^*g open) if $X - A$ is $\tau_1\tau_2$-s^*g closed in X.

(g) $\tau_1\tau_2$-semi generalized closed ($\tau_1\tau_2$-sg closed) if τ_2-scl $(A) \subseteq U$ whenever $A \subseteq U$ and U is τ_1-semi open in X.

(h) $\tau_1\tau_2$-semi generalized open ($\tau_1\tau_2$-sg open) if $X - A$ is $\tau_1\tau_2$-sg closed in X.

(i) $\tau_1\tau_2$-generalized semi closed ($\tau_1\tau_2$-gs closed) if τ_2-scl $(A) \subseteq U$ whenever $A \subseteq U$ and U is τ_1-open in X.

(j) $\tau_1\tau_2$-generalized semi open ($\tau_1\tau_2$-gs open) if $X - A$ is $\tau_1\tau_2$-gs closed in X.

(k) $(\tau_1, \tau_2)^*$-generalized closed $\{(\tau_1, \tau_2)^*$-g closed $\}$ [23] if $\tau_1\tau_2$-$cl(A) \subseteq U$ whenever $A \subseteq U$ and U is $\tau_1\tau_2$-open in X.

(l) $(\tau_1, \tau_2)^*$-generalized open $\{(\tau_1, \tau_2)^*$-g open $\}$ [23] if $X - A$ is $(\tau_1, \tau_2)^*$-g closed.

(m) $(\tau_1, \tau_2)^*$-semi generalized closed $\{(\tau_1, \tau_2)^*$-sg closed $\}$ [23] if $(\tau_1, \tau_2)^*$-$scl(A) \subseteq U$ whenever $A \subseteq U$ and U is $(\tau_1, \tau_2)^*$-semi open in X.

(n) $(\tau_1, \tau_2)^*$-semi generalized open $\{(\tau_1, \tau_2)^*$-sg open $\}$ [23] if $X - A$ is $(\tau_1, \tau_2)^*$-sg closed.

(o) $(\tau_1, \tau_2)^*$-generalized semi closed $\{(\tau_1, \tau_2)^*$-gs closed $\}$ if $(\tau_1, \tau_2)^*$-$scl(A) \subseteq U$ whenever $A \subseteq U$ and U is $\tau_1\tau_2$-open in X.

(p) $(\tau_1,\tau_2)^*$-generalized semi open $\{(\tau_1,\tau_2)^*$-gs open$\}$ if $X - A$ is $(\tau_1,\tau_2)^*$-gs closed.
(q) $(\tau_1,\tau_2)^*$-semi star generalized closed $\{(\tau_1,\tau_2)^*$-s^*g closed $\}$ if $\tau_1\tau_2$-$cl(A) \subseteq U$ whenever $A \subseteq U$ and U is $\tau_1\tau_2$-semi open in X.
(r) $(\tau_1,\tau_2)^*$-semi star generalized open $\{(\tau_1,\tau_2)^*$-s^*g open$\}$ if $X - A$ is $(\tau_1,\tau_2)^*$-s^*g closed.

Definition 2.2: A subset A of a bitopological space (X,τ_1,τ_2) is said to be a

(a) $\tau_1\tau_2$-locally semi closed set if $A = G \cap F$ where G is τ_1-open and F is τ_2-semi closed in X.
(b) $\tau_1\tau_2$-semi locally closed set if $A = G \cap F$ where G is τ_1-semi open and F is τ_2-semi closed in X.
(c) $\tau_1\tau_2$-g locally closed set if $A = G \cap F$ where G is τ_1-g open and F is τ_2-g closed in X.
(d) $\tau_1\tau_2$-sg locally closed set if $A = G \cap F$ where G is τ_1-sg open and F is τ_2-sg closed in X.
(e) $\tau_1\tau_2$-sg locally closed* set if $A = G \cap F$ where G is τ_1-sg open and F is τ_2-closed in X.
(f) $\tau_1\tau_2$-sg locally closed** set if $A = G \cap F$ where G is τ_1-open and F is τ_2-sg closed in X.
(g) $\tau_1\tau_2$-gs locally closed set if $A = G \cap F$ where G is τ_2-gs open and F is τ_2-gs closed in X.
(h) $\tau_1\tau_2$-s^*g locally closed if $A = G \cap F$ where G is a τ_1-s^*g open set and F is a τ_2-s^*g closed set in X,
(i) $(\tau_1,\tau_2)^*$-g locally closed set if $A = G \cap F$ where G is $\tau_1\tau_2$-g open set and F is $\tau_1\tau_2$-g closed set in X.
(j) $(\tau_1,\tau_2)^*$-sg locally closed set if $A = G \cap F$ where G is $\tau_1\tau_2$-sg open set and F is $\tau_1\tau_2$-sg closed set in X.
(k) $(\tau_1,\tau_2)^*$-gs locally closed set if $A = G \cap F$ where G is $\tau_1\tau_2$-gs open set and F is $\tau_1\tau_2$-gs closed set in X.

III. $(\tau_1,\tau_2)^*$-Semi Star Generalized Locally Closed Sets

Definition 3.1: A subset A of a bitopological space (X,τ_1,τ_2) is said to be

(a) $(\tau_1,\tau_2)^*$-s^*g locally closed set if $A = G \cap F$ where G is $\tau_1\tau_2$-s^*g open set and F is $\tau_1\tau_2$-s^*g closed set in X.
(b) $(\tau_1,\tau_2)^*$-s^*g locally closed* if $A = G \cap F$ where G is $\tau_1\tau_2$-s^*g open set and F is τ_2-closed in X.
(c) $(\tau_1,\tau_2)^*$-s^*g locally closed** if $A = G \cap F$ where G is τ_1-open and F is $\tau_1\tau_2$-s^*g closed in X.

Remark 3.2: (a) The class of all $(\tau_1,\tau_2)^*$-s^*g locally closed sets in (X,τ_1,τ_2) is denoted by $(\tau_1,\tau_2)^*$-$S^*GLC(X,\tau_1,\tau_2)$.
(b) The class of all $(\tau_1,\tau_2)^*$-s^*g locally closed* sets in (X,τ_1,τ_2) is denoted by $(\tau_1,\tau_2)^*$-$S^*GLC^*(X,\tau_1,\tau_2)$.
(c) The class of all $(\tau_1,\tau_2)^*$-s^*g locally closed** sets in (X,τ_1,τ_2) is denoted by $(\tau_1,\tau_2)^*$-$S^*GLC^{**}(X,\tau_1,\tau_2)$.

Example 3.3: Let $X = \{a,b,c\}, \tau_1 = \{\phi, X, \{a\}\}$, $\tau_2 = \{\phi, X, \{b,c\}\}$. Then $\tau_1\tau_2$-s^*g open sets in (X,τ_1,τ_2) are $\phi, X, \{a\}, \{b,c\}$ and $\tau_1\tau_2$-s^*g closed sets in (X,τ_1,τ_2) are $X, \phi, \{a\}, \{b,c\}$. Then

(a) $(\tau_1,\tau_2)^*$-s^*g locally closed sets in (X,τ_1,τ_2) are $\phi, X, \{a\}, \{b,c\}$.
(b) $(\tau_1,\tau_2)^*$-s^*g locally closed* sets in (X,τ_1,τ_2) are $\phi, X, \{a\}, \{b,c\}$.
(c) $(\tau_1,\tau_2)^*$-s^*g locally closed** sets in (X,τ_1,τ_2) are $\phi, X, \{a\}, \{b,c\}$.

Theorem 3.4: In any bitopological space (X,τ_1,τ_2),

(i) $A \in (\tau_1,\tau_2)^*$-$S^*GLC^*(X,\tau_1,\tau_2) \Rightarrow A \in (\tau_1,\tau_2)^*$-$S^*GLC(X,\tau_1,\tau_2)$.
(ii) $A \in (\tau_1,\tau_2)^*$-$S^*GLC^{**}(X,\tau_1,\tau_2) \Rightarrow A \in (\tau_1,\tau_2)^*$-$S^*GLC(X,\tau_1,\tau_2)$.
(iii) $A \in \tau_1\tau_2$-$S^*GC(X,\tau_1,\tau_2) \Rightarrow A \in (\tau_1,\tau_2)^*$-$S^*GLC(X,\tau_1,\tau_2)$.
(iv) $A \in \tau_1\tau_2$-$S^*GO(X,\tau_1,\tau_2) \Rightarrow A \in (\tau_1,\tau_2)^*$-$S^*GLC(X,\tau_1,\tau_2)$.

Proof: (i) Since A is $(\tau_1,\tau_2)^*$-s^*g locally closed* subset in (X,τ_1,τ_2), we have $A = G \cap F$ where G is $\tau_1\tau_2$-s^*g open set and F is τ_2-closed in X. Since every τ_2-closed set is $\tau_1\tau_2$-s^*g closed in $(X,\tau_1,\tau_2), A = G \cap F$ where G is $\tau_1\tau_2$-s^*g open and F is $\tau_1\tau_2$-s^*g closed in (X,τ_1,τ_2). Therefore, $A \in (\tau_1,\tau_2)^*$-$S^*GLC(X,\tau_1,\tau_2)$.
(ii) Since A is $(\tau_1,\tau_2)^*$-s^*g locally closed** subset in (X,τ_1,τ_2), we have $A = G \cap F$ where G is τ_1-open and F is $\tau_1\tau_2$-s^*g closed in (X,τ_1,τ_2). Since every τ_1-open set is $\tau_1\tau_2$-s^*g open in $(X,\tau_1,\tau_2), A = G \cap F$ where G is $\tau_1\tau_2$-s^*g open and F is $\tau_1\tau_2$-s^*g closed in (X,τ_1,τ_2). Therefore, $A \in \tau_1\tau_2$-$S^*GLC(X,\tau_1,\tau_2)$.
(iii) Since $A = A \cap X$ and A is $\tau_1\tau_2$-s^*g closed and X is $\tau_1\tau_2$-s^*g open in (X,τ_1,τ_2), we have $A \in (\tau_1,\tau_2)^*$-$S^*GLC(X,\tau_1,\tau_2)$.
(iv) Since $A = A \cap X$ and A is $\tau_1\tau_2$-s^*g open and X is $\tau_1\tau_2$-s^*g closed in (X,τ_1,τ_2), we have $A \in (\tau_1,\tau_2)^*$-$S^*GLC(X,\tau_1,\tau_2)$. ■

Remark 3.5: The converses of (i), (ii), (iii) and (iv) of the above theorem are not true in general as can be seen from the following examples.

Example 3.6: Let $X = \{a,b,c,d\}, \tau_1 = \{\phi, X, \{a\}, \{a,b\}\}, \tau_2 = \{\phi, X, \{b\}, \{a,c\}\}$. Then $\{a,b\}$ is $(\tau_1,\tau_2)^*$-s^*g locally closed in (X,τ_1,τ_2), but not $(\tau_1,\tau_2)^*$-s^*g locally closed* in (X,τ_1,τ_2).

Example 3.7: In Example 3.3, $\{b\}$ is $(\tau_1,\tau_2)^*$-s^*g locally closed in (X,τ_1,τ_2), but not $(\tau_1,\tau_2)^*$-s^*g locally closed** in (X,τ_1,τ_2).

Example 3.8: In Example 3.6, $\{a\}$ is $(\tau_1,\tau_2)^*$-s^*g locally closed in (X,τ_1,τ_2), but not $\tau_1\tau_2$-s^*g open in (X,τ_1,τ_2) and $\{a\}$ is $(\tau_1,\tau_2)^*$-s^*g locally closed in (X,τ_1,τ_2), but not $\tau_1\tau_2$-s^*g closed in (X,τ_1,τ_2).

Theorem 3.9: If (X,τ_1,τ_2) is pairwise door space, then every subset of X is both $(\tau_1,\tau_2)^*$-s^*g locally closed and $(\tau_2,\tau_1)^*$-s^*g locally closed.

Proof: Since (X,τ_1,τ_2) is pairwise door space, every subset of (X,τ_1,τ_2) is either τ_1-open or τ_2-closed and τ_2-open or τ_1-closed. Since every τ_1-open (resp. τ_2-closed) subset of (X,τ_1,τ_2) is $\tau_1\tau_2$-s^*g open (resp. $\tau_1\tau_2$-s^*g closed), we have every subset of (X,τ_1,τ_2) is either $\tau_1\tau_2$-s^*g open or $\tau_1\tau_2$-s^*g closed. Since every $\tau_1\tau_2$-s^*g open and $\tau_1\tau_2$-s^*g closed subset of (X,τ_1,τ_2) is $(\tau_1,\tau_2)^*$-s^*g locally closed, we have every subset of X is $(\tau_1,\tau_2)^*$-s^*g locally closed. Similarly we can prove that every subset of X is $(\tau_2,\tau_1)^*$-s^*g locally closed. ■

Theorem 3.10: For a subset A of a bitopological space (X,τ_1,τ_2), the following are equivalent.

(a) $A \in (\tau_1, \tau_2)^*\text{-}S^*GLC^*(X, \tau_1, \tau_2)$.
(b) $A = G \cap [\tau_2\text{-cl }(A)]$ for some $\tau_1\tau_2\text{-}s^*g$ open set G.
(c) $A \cup \{X - [\tau_2\text{-cl }(A)]\}$ is $\tau_1\tau_2\text{-}s^*g$ open.
(d) $[\tau_2\text{-cl }(A)] - A$ is $\tau_1\tau_2\text{-}s^*g$ closed.

Proof: $(a) \Rightarrow (b)$:
Since A is $(\tau_1, \tau_2)^*\text{-}s^*g$ locally closed* set in (X, τ_1, τ_2), we have $A = G \cap F$ where G is $\tau_1\tau_2\text{-}s^*g$ open set and F is τ_2-closed in X. Since $A \subseteq \tau_2\text{-cl }(A)$ and $A \subseteq G$, we have $A \subseteq G \cap [\tau_2\text{-cl }(A)]$ ....(1)
Since $A \subseteq F$ and F is τ_2-closed in X, we have $\tau_2\text{-cl }(A) \subseteq F$. Therefore $G \cap [\tau_2\text{-cl }(A)] \subseteq G \cap F = A$. Hence $G \cap [\tau_2\text{-cl }(A)] \subseteq A$ ....(2)
From(1) and (2), we have $A = G \cap [\tau_2\text{-cl }(A)]$ for some $\tau_1\tau_2\text{-}s^*g$ open set G in (X, τ_1, τ_2).
$(b) \Rightarrow (a)$:
Suppose that $A = G \cap [\tau_2\text{-cl }(A)]$ for some $\tau_1\tau_2\text{-}s^*g$ open set G in (X, τ_1, τ_2). Since $\tau_2\text{-cl }(A)$ is τ_2-closed in (X, τ_1, τ_2) and G is $\tau_1\tau_2\text{-}s^*g$ closed in (X, τ_1, τ_2), we have $A \in (\tau_1, \tau_2)^*\text{-}S^*GLC^*(X, \tau_1, \tau_2)$
$(b) \Rightarrow (c)$:
Since $A = G \cap [\tau_2\text{-cl }(A)]$ for some $\tau_1\tau_2\text{-}s^*g$ open set G in (X, τ_1, τ_2), we have $A \cup \{X - [\tau_2\text{-cl }(A)]\} = \{G \cap [\tau_2\text{-cl }(A)]\} \cup \{X - [\tau_2\text{-cl }(A)]\} = G$. Therefore, $A \cup \{X - [\tau_2\text{-cl }(A)]\}$ is $\tau_1\tau_2\text{-}s^*g$ open.
$(c) \Rightarrow (b)$:
Suppose that $A \cup \{X - [\tau_2\text{-cl }(A)]\}$ is $\tau_1\tau_2\text{-}s^*g$ open in (X, τ_1, τ_2). Let $G = A \cup \{X - [\tau_2\text{-cl }(A)]\}$. Then G is $\tau_1\tau_2\text{-}s^*g$ open in (X, τ_1, τ_2). Now, $G \cap [\tau_2\text{-}cl(A)] = [A \cup \{X - [\tau_2\text{-}cl(A)]\}] \cap [\tau_2\text{-}cl(A)] = \{[A \cup [\tau_2\text{-}cl(A)]^C\} \cap = \{A \cap [\tau_2\text{-}cl(A)]\} \cup \{[\tau_2\text{-}cl(A)]^C \cap [\tau_2\text{-}cl(A)]\} = A \cup \phi = A$. Therefore, $A = G \cap [\tau_2\text{-cl }(A)]$ for some $\tau_1\tau_2\text{-}s^*g$ open set G in (X, τ_1, τ_2).
$(c) \Rightarrow (d)$:
Suppose that $A \cup \{X - [\tau_2\text{-cl }(A)]\}$ is $\tau_1\tau_2\text{-}s^*g$ open in (X, τ_1, τ_2). Let $G = A \cup \{X - [\tau_2\text{-cl }(A)]\}$. Since G is $\tau_1\tau_2\text{-}s^*g$ open in (X, τ_1, τ_2), we have $X - G$ is $\tau_1\tau_2\text{-}s^*g$ closed in (X, τ_1, τ_2). Now, $X - G = X - [A \cup \{X - [\tau_2\text{-}cl(A)]\}] = (X - A) \cap \{X - [\tau_2\text{-}cl(A)]\} = (X - A) \cap [\tau_2\text{-}cl(A)] = \tau_2\text{-}cl(A) - A$. Therefore, $\tau_2\text{-cl }(A) - A$ is $\tau_1\tau_2\text{-}s^*g$ closed in (X, τ_1, τ_2).
$(d) \Rightarrow (c)$:
Suppose that $\tau_2\text{-cl }(A) - A$ is $\tau_1\tau_2\text{-}s^*g$ closed in (X, τ_1, τ_2). Let $F = \tau_2\text{-cl }(A) - A$. Then F is $\tau_1\tau_2\text{-}s^*g$ closed in (X, τ_1, τ_2) implies that $X - F$ is $\tau_1\tau_2\text{-}s^*g$ open in (X, τ_1, τ_2). Now, $X - F = X - \{[\tau_2\text{-}cl(A)] - A\} = X \cap \{[\tau_2\text{-}cl(A)] - A\}^C = X \cap \{[\tau_2\text{-}cl(A)] \cap A^C\}^C = X \cap \{[\tau_2\text{-}cl(A)]^C \cup (A^C)^C\} = X \cap \{[\tau_2\text{-}cl(A)]^C \cup A\} = \{X \cap [\tau_2\text{-}cl(A)]^C\} \cup \{X \cap A\} = [\tau_2\text{-}cl(A)]^C \cup A = \{X - [\tau_2\text{-}cl(A)]\} \cup A$. Hence $A \cup \{X - [\tau_2\text{-cl }(A)]\}$ is $\tau_1\tau_2\text{-}s^*g$ open in (X, τ_1, τ_2). ■

Theorem 3.11: In a bitopological space (X, τ_1, τ_2), the following are equivalent.

(a) $A - [\tau_1\text{-int }(A)]$ is $\tau_2\text{-}s^*g$open in (X, τ_1, τ_2).
(b) $[\tau_1\text{- int }(A)] \cup [X - A]$ is $\tau_1\tau_2\text{-}s^*g$ closed in (X, τ_1, τ_2).
(c) $G \cup [\tau_1\text{-int }(A)] = A$ for some $\tau_1\tau_2\text{-}s^*g$ open set G in (X, τ_1, τ_2).

Proof: $(a) \Rightarrow (b)$:
Now, $X - \{A - [\tau_1\text{-}int(A)]\} = X \cap \{A - [\tau_1\text{-}int(A)]\}^C = X \cap [A \cap \{\tau_1\text{-}int(A)\}^C]^C = X \cap \{A^C \cup [\{\tau_1\text{-}int(A)\}^C]^C\} = X \cap \{A^C \cup [\tau_1\text{-}int(A)]\} = \{A^C \cup [\tau_1\text{-}int(A)]\} = [\tau_1\text{-}int(A)] \cup [X - A]$. Since $A - [\tau_1\text{-int }(A)]$ is $\tau_1\tau_2\text{-}s^*g$ open, we have $X - \{A - [\tau_1\text{-int }(A)]\} = [\tau_1 - int(A)] \cup [X - A]$ is $\tau_1\tau_2\text{-}s^*g$ closed in (X, τ_1, τ_2).
$(b) \Rightarrow (a)$:
Suppose that $[\tau_1\text{-int }(A)] \cup [X - A]$ is $\tau_1\tau_2\text{-}s^*g$ closed in (X, τ_1, τ_2). Since $[\tau_1\text{-int }(A)] \cup [X - A]$ is $\tau_1\tau_2\text{-}s^*g$ closed, we have $X - \{[\tau_1\text{-int }(A)] \cup [X - A]\}$ is $\tau_1\tau_2\text{-}s^*g$ open. Now, $X - \{[\tau_1\text{-}int(A)] \cup [X - A]\} = X \cap \{[\tau_1\text{-}int(A)] \cup [X - A]\}^C = X \cap \{[\tau_1\text{-}int(A)] \cup A^C\}^C = X \cap \{[\tau_1\text{-}int(A)]^C \cap (A^C)^C\} = X \cap \{[\tau_1\text{-}int(A)]^C \cap A\} = A \cap [\tau_1\text{-}int(A)]^C = A - [\tau_1\text{-}int(A)]$. Therefore, $A - [\tau_1\text{-int }(A)]$ is $\tau_1\tau_2\text{-}s^*g$ open in (X, τ_1, τ_2).
$(b) \Rightarrow (c)$:
Suppose that $[\tau_1\text{-int }(A)] \cup [X - A]$ is $\tau_1\tau_2\text{-}s^*g$ closed. Let $U = [\tau_1\text{-int }(A)] \cup [X - A]$. Then U is $\tau_1\tau_2\text{-}s^*g$ closed. Then U^C is $\tau_1\tau_2\text{-}s^*g$ open. Now, $U^C \cup [\tau_1\text{-}int(A)] = \{[\tau_1\text{-}int(A)] \cup [X - A]\}^C \cup [\tau_1\text{-}int(A)] = \{[\tau_1\text{-}int(A)]^C \cap (A^C)^C\} \cup [\tau_1\text{-}int(A)] = \{[\tau_1\text{-}int(A)]^C \cap A\} \cup [\tau_1\text{-}int(A)] = \{[\tau_1\text{-}int(A)]^C \cup [\tau_1\text{-}int(A)]\} \cap \{A \cup [\tau_1\text{-}int(A)]\} = X \cap A = A$. Take $G = U^C$. Then $A = G \cup [\tau_1\text{-int }(A)] = A$ for some $\tau_1\tau_2\text{-}s^*g$ open set in (X, τ_1, τ_2).
$(c) \Rightarrow (b)$:
Suppose that $A = G \cup [\tau_1\text{-int }(A)] = A$ for some $\tau_1\tau_2\text{-}s^*g$ open set G in (X, τ_1, τ_2). Now, $[\tau_1\text{-}int(A)] \cup [X - A] = \tau_1\text{-}int(A) \cup A^C = [\tau_1\text{-}int(A)] \cup \{G \cup [\tau_1\text{-}int(A)]\}^C = [\tau_1\text{-}int(A)] \cup \{G^C \cap [\tau_1\text{-}int(A)]^C\} = \{[\tau_1\text{-}int(A)] \cup G^C\} \cap \{[\tau_1\text{-}int(A)] \cup [\tau_1\text{-}int(A)]^C\} = \{[\tau_1\text{-}int(A)] \cup G^C\} \cap X\} = \{[\tau_1\text{-}int(A)] \cup G^C\} = X - G$. Since G is $\tau_1\tau_2\text{-}s^*g$ open in (X, τ_1, τ_2), we have $X - G$ is $\tau_1\tau_2\text{-}s^*g$ closed in (X, τ_1, τ_2). Therefore $[\tau_1\text{-int }(A)] \cup [X - A]$ is $\tau_1\tau_2\text{-}s^*g$ closed in (X, τ_1, τ_2). ■

Remark 3.12: The union of two $(\tau_1, \tau_2)^*\text{-}s^*g$ locally closed sets in (X, τ_1, τ_2) is not $(\tau_1, \tau_2)^*\text{-}s^*g$ locally closed in general as can be seen from the following example.

Example 3.13: In Example 3.6, $A = \{b\}, B = \{c\}$ are $(\tau_1, \tau_2)^*\text{-}s^*g$ locally closed sets in (X, τ_1, τ_2), but $A \cup B = \{b, c\}$ is not $(\tau_1, \tau_2)^*\text{-}s^*g$ locally closed set in (X, τ_1, τ_2).

Remark 3.14: Even A and B are not $(\tau_1, \tau_2)^*\text{-}s^*g$ locally closed sets in (X, τ_1, τ_2), $A \cup B$ is $(\tau_1, \tau_2)^*\text{-}s^*g$ locally closed in general as can be seen from the following example.

Example 3.15: In Example 3.3, $A = \{b\}, B = \{c\}$ are not $(\tau_1, \tau_2)^*\text{-}s^*g$ locally closed sets in (X, τ_1, τ_2), but $A \cup B = \{b, c\}$ is $(\tau_1, \tau_2)^*\text{-}s^*g$ locally closed set in (X, τ_1, τ_2).

Remark 3.16: Since every $(\tau_1, \tau_2)^*\text{-}s^*g$ locally closed set is the intersection of a $\tau_1\tau_2\text{-}s^*g$ open set and $\tau_1\tau_2\text{-}s^*g$ closed set, we can conclude the following.

Theorem 3.17: A subset A of (X, τ_1, τ_2) is $(\tau_1, \tau_2)^*\text{-}s^*g$ locally closed if and only if A^C is the union of a $\tau_1\tau_2\text{-}s^*g$ open set and $\tau_1\tau_2\text{-}s^*g$ closed set.

Remark 3.18: Every τ_1-open set {resp. τ_2-closed set} is $\tau_1\tau_2\text{-}s^*g$ open {resp. $\tau_1\tau_2\text{-}s^*g$ closed}. Accordingly, we conclude the following.

Theorem 3.19: (a) Every τ_1-open set is $(\tau_1, \tau_2)^*\text{-}s^*g$ locally closed and every τ_2-closed set is $(\tau_1, \tau_2)^*\text{-}s^*g$ locally closed.

(b) Every $\tau_1\tau_2$-locally closed set is $(\tau_1,\tau_2)^*$-s^*g locally closed, $(\tau_1,\tau_2)^*$-s^*g locally closed* and $(\tau_1,\tau_2)^*$-s^*g locally closed**.

Remark 3.20: But the converses of the assertions of above theorem are not true in general as can be seen in the following examples.

Example 3.21: (a) In Example 3.3, $\{b,c\}$ is $(\tau_1,\tau_2)^*$-s^*g locally closed set in (X,τ_1,τ_2), but $\{b,c\}$ is not τ_2-closed in (X,τ_1,τ_2).

(b) In Example 3.3, $\{b,c\}$ is $(\tau_1,\tau_2)^*$-s^*g locally closed set in (X,τ_1,τ_2), but $\{b,c\}$ is not τ_1 - open in (X,τ_1,τ_2).

(c) Let $X = \{a,b,c,d\}$, $\tau_1 = \{\phi,X,\{a,b\}\}$, $\tau_2 = \{\phi,X,\{b\},\{c\},\{b,c\}\}$. Then $\{b\}$ is a $(\tau_1,\tau_2)^*$-s^*g locally closed set, $(\tau_1,\tau_2)^*$-s^*g locally closed* set and $(\tau_1,\tau_2)^*$-s^*g locally closed** set but not $\tau_1\tau_2$-locally closed in (X,τ_1,τ_2).

Remark 3.22: Since every $\tau_1\tau_2$-s^*g closed set is $\tau_1\tau_2$-g closed, $\tau_1\tau_2$-sg closed and $\tau_1\tau_2$-gs closed, we conclude the following.

Theorem 3.23: (a) Every $(\tau_1,\tau_2)^*$-s^*g locally closed is $(\tau_1,\tau_2)^*$-g locally closed.

(b) Every $(\tau_1,\tau_2)^*$-s^*g locally closed is $(\tau_1,\tau_2)^*$-sg locally closed.

(c) Every $(\tau_1,\tau_2)^*$-s^*g locally closed is $(\tau_1,\tau_2)^*$-gs locally closed.

Remark 3.24: But none of the assertions of the above theorem are reversible in general as can be seen in the following example.

Example 3.25: Let $X = \{a,b,c\}$, $\tau_1 = \{\phi,X,\{a\}\}$, $\tau_2 = \{\phi,X,\{a\},\{a,c\}\}$. Then $\{a,b\}$ is a $(\tau_1,\tau_2)^*$-g locally closed set, $(\tau_1,\tau_2)^*$-sg locally closed set and $(\tau_1,\tau_2)^*$-gs locally closed set, but not $(\tau_1,\tau_2)^*$-s^*g locally closed in (X,τ_1,τ_2). From the above results we conclude the following.

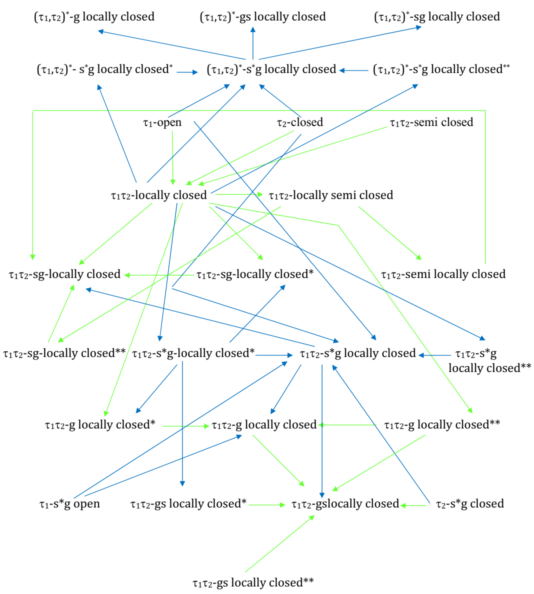

Fig 1. Relationship between several locally closed sets

Since the finite intersection of τ_1-open sets is τ_1-open and the intersection of two $\tau_1\tau_2$-s^*g closed sets is $\tau_1\tau_2$-s^*g closed, we immediately get

Theorem 3.26: In any bitopological space (X,τ_1,τ_2), intersection of two $(\tau_1,\tau_2)^*$-s^*g locally closed** sets is $(\tau_1,\tau_2)^*$-s^*g locally closed**.

In this sequel our next result exhibits the intersection of a $(\tau_1,\tau_2)^*$-s^*g locally closed set and a τ_2-closed set in a bitopological space.

Theorem 3.27: If $A \in (\tau_1,\tau_2)^*$-$S^*GLC(X,\tau_1,\tau_2)$ and B is τ_2-closed in X, then $A \cap B \in (\tau_1,\tau_2)^*$-$S^*GLC(X,\tau_1,\tau_2)$.

Proof: It is obvious since every τ_2-closed set is $\tau_1\tau_2$-s^*g closed and the intersection of two $\tau_1\tau_2$-s^*g closed sets is $\tau_1\tau_2$-s^*g closed. ■

Our next result is an immediate consequence of the above theorem.

Theorem 3.28: If $A \in (\tau_1,\tau_2)^*$-$S^*GLC(X,\tau_1,\tau_2)$ and B is $\tau_1\tau_2$-s^*g closed in X, then $A \cap B \in (\tau_1,\tau_2)^*$-$S^*GLC(X,\tau_1,\tau_2)$.

Remark 3.29: The complement of a $(\tau_1,\tau_2)^*$-s^*g locally closed set in (X,τ_1,τ_2) is not $(\tau_1,\tau_2)^*$-s^*g locally closed in general and hence the finite union of $(\tau_1,\tau_2)^*$-s^*g locally closed sets need not be $(\tau_1,\tau_2)^*$-s^*g locally closed in (X,τ_1,τ_2). The next examples show the claim.

Example 3.30: In Example 3.6, $\{a\}$ is a $(\tau_1,\tau_2)^*$-s^*g locally closed set,but its complement $\{b,c\}$ is not $(\tau_1,\tau_2)^*$-s^*g locally closed in (X,τ_1,τ_2).

Example 3.31: In Example 3.6, $A = \{b\}$, $B = \{c\}$ are $(\tau_1,\tau_2)^*$-s^*g locally closed sets, but $A \cup B = \{b,c\}$ is not $(\tau_1,\tau_2)^*$-s^*g locally closed in (X,τ_1,τ_2).

Theorem 3.32: In a bitopological space (X,τ_1,τ_2), the following are equivalent.

(a) A is $(\tau_1,\tau_2)^*$-s^*g locally closed if and only if A^C is $(\tau_1,\tau_2)^*$-s^*g locally closed.

(b) $(\tau_1,\tau_2)^*$-s^*g locally closed sets are closed under finite union.

Proof: $(a) \Rightarrow (b)$: Suppose that A is $(\tau_1,\tau_2)^*$-s^*g locally closed if and only if A^C is $(\tau_1,\tau_2)^*$-s^*g locally closed. Let A, B be $(\tau_1,\tau_2)^*$-s^*g locally closed. Then by our assumption, A^C, B^C are $(\tau_1,\tau_2)^*$-s^*g locally closed. Consequently, $(A \cup B)^C = A^C \cap B^C$ is $(\tau_1,\tau_2)^*$-s^*g locally closed. Therefore, $A \cup B$ is $(\tau_1,\tau_2)^*$-s^*g locally closed.

$(b) \Rightarrow (a)$: Suppose that $(\tau_1,\tau_2)^*$-s^*g locally closed sets are closed under finite union. Let A be $(\tau_1,\tau_2)^*$-s^*g locally closed. Then $A = G \cap F$ where G is $\tau_1\tau_2$-s^*g open and F is $\tau_1\tau_2$-s^*g closed in X. Since G^C is $\tau_1\tau_2$-s^*g closed and F^C is $\tau_1\tau_2$-s^*g open in X and every $\tau_1\tau_2$-s^*g open is $(\tau_1,\tau_2)^*$-s^*g locally closed and $\tau_1\tau_2$-s^*g closed set is $(\tau_1,\tau_2)^*$-s^*g locally closed, we have A^C is $(\tau_1,\tau_2)^*$-s^*g locally closed by our assumption. Similarly, we can prove if A^C is $(\tau_1,\tau_2)^*$-s^*g locally closed then A is $(\tau_1,\tau_2)^*$-s^*g locally closed. ■

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