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Phenomenological and Theoretical Analysis of Relativistic Temperature Transformation and Relativistic Entropy

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***Abstract*—**There are three possible effects of Special Theory of Relativity (STR) on a thermodynamic system. Planck and Einstein looked upon this process as isobaric; on the other hand Ott saw it as an adiabatic process. However plenty of logical reasons show that the process is isotherm. Our phenomenological consideration demonstrates that the temperature is invariant with Lorenz transformation. In that case process is isotherm, so volume and pressure are Lorentz covariant. If the process is isotherm the Boyles law is Lorentz invariant. Also equilibrium constant and Gibbs energy, activation energy, enthalpy entropy and extent of the reaction became Lorentz invariant.

***Keywords*—**STR, relativistic temperature transformation, Boyle's law, equilibrium constant, Gibbs energy.

I. Introduction

SPECIAL Theory of Relativity (STR) was published in 1905. In 1907 Planck has demonstrated that pressure is invariant with Lorentz transformations.

$$Po=P'$$

Hereafter, all variables with subscript (o) refer to the inertial frame at rest regarding observer, while variables with (‘) refer to inertial frame K’ moving at relative speed w. Planck[1] concludes that the temperature is Lorenz covariant. So

$$T'=To\sqrt{1-\frac{v^2}{c^2}} \tag{1}$$

In 1963, Ott[2] deduced exactly the opposite transformation law

$$T'=To\,/\sqrt{1-\frac{v^2}{c^2}} \tag{2}$$

After that, many papers dealing with thermodynamics have shown with “a simple experiment is described, using a constant-volume gas thermometer at rest with a body to show that the ideal-gas scale is Lorentz invariant. The statement that thermodynamic temperature is Lorentz invariant is then equivalent to the requirement that the thermodynamic temperature scale and the ideal-gas scale should be identical in all frames of reference.”[3] Some papers are explicit: “since any valid Lorentz transformation of temperature must be able to deal with black-body radiation, it is concluded that a universal and continuous temperature transformation does not exist.”[4] Also “The non-existence of a relativistic temperature transformation is due to the fact that an observer moving in a heat reservoir cannot detect a blackbody spectrum.” [15] The conclusion that: “all thermodynamic relations become Lorentz-invariant” have been made by some authors.[5] At the end:” one has to conclude that the temperature is invariant with Lorentz transformations.”[6] So

$$T'=To \tag{3}$$

“There is no universal relativistic temperature transformation“claims E. Bormashenko 2007.[7]

In the Avramov paper we can find another conclusion: ”If temperature is invariant with speed, then entropy with respect to the Boltzmann constant is not. This put serious problems on the statistical physics” . So

$$S' = So\sqrt{1-\frac{v^2}{c^2}} \tag{4}$$

The set of relativity transformation laws for the volume V, temperature T and pressure P

$$V' = Vo\sqrt{1-\frac{v^2}{c^2}} \tag{5}$$

$$P'=Po$$

$$T'=To$$

is made by Avramov [6] So Boyle’s law must be Lorentz covariant. Some authors have different opinion: “the obvious relativistic transformation $p = p_o$ are not needed”[8]

On the other hand, process can not be in same time both isothermal and isobaric.

The aim of this paper is to show that some thermodynamic relations become Lorentz-invariant. Also, considerations should demonstrate that temperature and pressure can’t be Lorenz invariant in the same time because process can’t be in the same time both isothermal and isobaric. Using equation of the state of ideal gas it will be shown that relativistic transformation of temperature does not exist.

II. Phenomenological Consideration

There is some logical indication that temperature is invariant with the speed of inertial frame.

“For instance, if the system consists of three equilibrium phases (triple point), the temperature is known. The data of any thermometer should be calibrated according to this

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temperature. Say the triple point of water is by definition 273,16 K and all observers from all frames, no matter fast moving or not, inertial or not, will see the three phases in equilibrium. This means that they will decide the system is at the same temperature T'=To."[6]

"The ligt spectrum of the moving system depends on the product of Planck constant h, of Boltzmann constant b, and of the temperature T. If the Planck transformation is valid, one must conclude that fast moving galaxies should be cold and invisible. On the other hand, with the Ott's transformation these stars should be infinitively bright. As soon as either of these happens, one has to expect that temperature is invariant with the speed."[6]

Note that there is an important difference between the frequency shift of light caused by the Doppler effect and a shift caused by temperature change. According to Planck's formula, the intensity of light *I* depends on frequency n and temperature T as follows:

$$I = \frac{c^3}{8\pi h} \frac{\nu^3}{exp\left(\frac{h\nu}{k_b T}\right) - 1}$$

It is seen that it is possible to distinguish the intensity change caused by the Doppler effect from the shift caused by the Lorenz transformation of the $h/k_b T$ product. Moreover the Doppler effect depends on whether the object is moving towards or away from the observer.

"Let us consider, for instance, a system consisting of a charged battery connected to a heater with link that is conducting at some temperature interval and isolating outside this interval (wire of a high temperature superconductor being an isolator at room temperature). If the observer at K' is at room temperature he will see no electrical current and the battery will remain charged. At the same time, the observer at Ko should see (if temperature is not invariant) that the battery is discharging, heating the surrounding area. As this process is irreversible, the observer will never see the opposite process with the surroundings cooling spontaneously and recharging the battery even if it accelerates. Therefore, an absurd situation will appear if the second observer arrives to the first one at Ko. At the same space-time point, the first observer should see the battery charged, while the second one should see it uncharged."[6]

We strongly support Avramov's point of view on the question of relativistic temperature transformation.

Historia magistra vitae est. We will use one experiment from history of chemistry to show that temperature is Lorentz invariant. Let us take for example Lavoisier experiment.

Thermodynamic system consist retort with Hg and cylinder with air. Lavoisier doesn't heat the retort, but the whole system in which he is experimenting is moving at relativistic speed. If we use Ott's transformations, the observer in Ko would notice that the temperature in K' is rising, and red oxide of mercury would form on the surface of the mercury in the retort. When no more red powder was formed, observer in Ko would notice that about one-fifth of the air had been used up and that the remaining gas did not support life or burning. The reaction he would notice is

$$2Hg + O_2 \longrightarrow 2HgO$$

For Lavoisier moving in K' the temperature would remain the same, so he wouldn't notice any red oxide of mercury would form on the surface of the mercury in the retort, nor any change of the gas volume in the cylinder. An absurd situation would appear when the observer in K' arrives to Ko. At the same space-time point, one observer would see red oxide of mercury, while the other one would see none of it. One observer would see one gas volume, while the other one would see a different volume.

Life can exist in relatively narrow temperature range. If the observer in K' see his colleague alive, then observer in Ko will both them see alive and has to conclude that the temperature is in the same narrow region.

Let us consider, system made of one ideal crystal. The temperature is known. By the definition ideal crystal is at finite temperature of - 273 C. All observers from all frames, no matter fast moving or not, inertial or not, will see the ideal crystal and therefore conclude that temperature is absolute zero.

In moving system observer measures the body temperature. Temperature is scalar, and therefore it cannot depend on the direction. If observer lay dawn and measures body temperature, then thermometer is normal to the motion axes. Both observer in K' and Ko will see the same temperature, for example 37 ºC. If observer stands then the bulb of thermo-meter is coaxial with the direction of motion. Observer in K' will see no change and see 37, while the one in Ko will see the shorter one. However, this does not mean that temperature is lower. This means that the thermometer has to be recalibrated, because for every point there is only one temperature.

Let us consider a system which includes an autoclave with oleic acid and hydrogen gas. The reaction is hydrogenation of unsaturated oleic acid.

$$CH_3(CH_2)_7CH{=}CH(CH_2)_7COOH + H_2 \rightarrow CH_3(CH_2)_{16}COOH$$

This reaction happens at 175 °C to 200 °C. Catalyst is nickel (also possible with platinum or palladium). The observer within the moving system does not heat the autoclave. If we use Planck's transformations, the observer at Ko will see temperatures high enough for this reaction to happen, and because of that he will see stearic acid forming as sediment. While the observer at K' will see no temperature change, so he won't see any stearic acid forming. Since this reaction is irreversible, an absurd situation will appear if the second observer arrives to the first one at Ko. In the same space-time observer at Ko would claim that there is stearic acid in the autoclave, while the observer at K' would tell him to wear his glasses more often because he doesn't see any solid acid at all.

System which we accelerate contains an observer, and an candle. The candle is unlit. In the process of acceleration the temperature remains the same for the observer situated in the system K'., and the candle will not change its shape. The observer in Ko according to Planck's relativistic temperature should see the candle crack, and fall to pieces because of the cold, if we use Ott's relativistic temperature the observer in Ko should see the candle melt (even though it is unlit). Again, an absurd situation will appear if the observer situated in K' arrives to the observer in Ko, in the same space/time three observers (one in K', Planck and Ott) will see three different shapes of candle.

Let us consider, for instance, a system consisting of a system for electrolysis of water connected to a source of electricity via link that is conducting at some temperature interval and isolating outside this interval (wire of a high temperature superconductor being an isolator at room temperature). If the observer at K' is at room temperature he will see no electrolysis process, and the water level will stay unchanged. At the same time, the observer at Ko should see (if temperature is not invariant) that the electrolysis process is going on, and he will see the water level changing because H2 and O2 are evaporating to the atmosphere. As this process is irreversible, the observer will never see the opposite process. Therefore, an absurd situation will appear if the second observer arrives to the first one at Ko. At the same space-time point, the first observer should see unchanged water level, while the second one should see a different water level.

Thermodynamically system consisted of Zn and HCl. The reaction is going as:

$$Zn + 2\,HCl \rightarrow Zn(Cl)_2 + H_2\uparrow$$

The hydrogen molecules evaporate from the system. If the observer at K' is at room temperature he will see certain reaction rate and a certain quantity of hydrogen evaporating. At the same time, the observer at Ko should see (if temperature is not invariant) a different reaction rate according to Arrhenius equitation, and a different quantity of hydrogen released from the system. As this process is irreversible, the observer will never see the opposite process. Therefore, an absurd situation will appear if the second observer arrives to the first one at Ko. At the same space-time point, two observers would see different quantities of Zn, HCl, $Zn(Cl(_2$ and H_2 .

III. Theoretical Analysis

Let us consider. If the chemical system consists of two gases A and B then the state of the system is given as:

$$PV_{A+B} = (n_A + n_B)RT. \quad (6)$$

As the definition of the molarity is $[A] = n_A / V_{A+B}$ by changing

$[A] = (n_A /(n_A + n_B))(P/RT)$ where $(n_A /(n_A + n_B)$ is the moll fraction of the substance, χ_A. Therefore

$$[A] = \chi_A\, P/RT \quad (7)$$

presents the relations between the two ways of showing the concentration. Therefore

$$\chi_A = [A]RT/P \quad (8)$$

or

$$\chi_A = n_A RT/PV_{A+B} \quad (9)$$

Because $[A] = n_A / V$

If the system moves the relation has the shape

$$\chi' = [A]'RT'/P \quad (10)$$

because of the relativistic effects.

As the mol fraction is not affected to the effects of the relativity $\chi = \chi'$ it comes that

$$[A]RT/P = [A]'\,RT'/P',$$

where $[A]' = [A]/(1-\beta^2)1/2$, $P' = P/(1-\beta^2)1/2$

R Is a constant, then it comes that To = T''. Temperature is Lorenz invariant.

All observers will have to determine the same temperature regardless of the relative speed of their frames.

Or if

$$\chi = n_A\, R\, T/ P\, V \quad (11)$$

This formula is valid for the system that stands still, and, if the system moves the relation has the shape

$$\chi' = n_A\, R\, T'/ P'\, V' \quad (12)$$

As the mol fraction is not affected to the effects of the relativity $\chi = \chi'$ it comes that

$$n_A\, R\, T/ P\, V = n_A'\, RT'/ P'V'$$

n=n', R is constant Boyle's law is PV = P'V' then, To=T'. Temperature is Lorenz invariant. "We conclude that there is no universally general temperature transformation "say Newburgh [10]. Landsberg and Johns say $T=T_o$[11].

All observers will have to determine the same temperature regardless of the relative speed of their frames.

Equitation of State of Ideal Gas

If

$$PV = nRT \quad (13)$$

Then

$$P'V' = n'R'T' \quad (14)$$

Where P' is relativistic pressure, V' is relativistic volume, T' relativistic temperature. Since T=T', R=R', n=n'

$$V' = V\sqrt{1-\frac{v^2}{c^2}} \quad (15)$$

Volume is Lorenz covariant. If there is no relativistic temperature transformation ($T'=T_0$), relativistic concentration[9] is given $[A]' = [A]/\sqrt{1-\frac{v^2}{c^2}}$ we can write

$$PV = nRT$$

$$P = \frac{n}{V}RT$$

$$PV = [A]RT$$

in relativistic conditions

$$P'V' = [A]'RT'$$

so

$$P' = \frac{[A]}{\sqrt{1-\frac{v^2}{c^2}}}RT$$

so

$$P' = \frac{P}{\sqrt{1-\frac{v^2}{c^2}}} \quad (16)$$

Pressure is Lorenz covariant. Some authors have same opinion: "the obvious relativistic transformation $p = p_0$ are not needed".[8] The set of relativity transformation laws, as given by Avramov,[6] claims opposite.

$$V' = Vo\sqrt{1-\frac{v^2}{c^2}}$$

$$P' = Po$$

$$T' = To$$

Pauli [13] have the same opinion $p' = p_o$, but Pauli claim temperature is Lorentz covariant.

Here we must say that temperature and pressure can't be Lorenz invariant in the same time because process can't be in the same time both isothermal and isobaric.

Equilibrium Constant

For the reaction

$$A + B \rightleftarrows C + D \tag{17}$$

Equilibrium constant is given as

$$K = \frac{k_1}{k_2} = \frac{[C][D]}{[A][B]} \tag{18}$$

Then relativistic equilibrium constant is given as

$$K' = \frac{k'_1}{k'_2} = \frac{[C]'[D]'}{[A]'[B]'} \tag{19}$$

Having in mind $[A]' = \frac{[A]}{\sqrt{1-\frac{v^2}{c^2}}}$, $[B]' = \frac{[B]}{\sqrt{1-\frac{v^2}{c^2}}}$, $[C]' = \frac{[C]}{\sqrt{1-\frac{v^2}{c^2}}}$, $[D]' = \frac{[D]}{\sqrt{1-\frac{v^2}{c^2}}}$ it leads

$$K = K' \tag{20}$$

Equilibrium constant is Lorenz invariant.

Equilibrium constant is given

$$K = \frac{[C][D]}{[A][B]}$$

For ideal gas we can write

$$K = \frac{p_C p_D}{p_A p_B}$$

Or using moll fraction

$$K = \frac{XCXD}{XAXB}$$

then

$$K' = \frac{X'CX'D}{X'AX'B}$$

As the mol fraction is not affected to the effects of the relativity $\chi = \chi'$ it comes that

$$K = K'$$

K is Lorenz invariant.

Gibbs Energy

If

$$\Delta G = -RT\log K \tag{21}$$

Then relativistic transformation is given

$$\Delta G' = -R'T'\log K' \tag{22}$$

Having in mind , $K = K'$, $R = R'$, $T = T'$ so

$$\Delta G = \Delta G' \tag{23}$$

Gibbs energy is Lorenz invariant.

Activation Energy

If

$$\ln\frac{K}{\Delta T} = \frac{E_a}{RT^2} \tag{24}$$

$$\ln\frac{K'}{\Delta T'} = \frac{E_a'}{R'T'^2} \tag{25}$$

Since $\Delta T = \Delta T'$, $K = K'$, $T = T$

$$E_a = E_a' \tag{26}$$

Activation energy is Lorenz invariant.

Enthalpy

If

$$\Delta H = E_a - RT \tag{27}$$

$$\Delta H' = E_a' - R'T' \tag{28}$$

Since $E_a = E_a'$, $R = R'$, $T = T'$

$$\Delta H = \Delta H' \tag{29}$$

Entropy

If

$$\Delta S = \frac{\Delta H}{\Delta T} \tag{30}$$

$$\Delta S' = \frac{\Delta H'}{\Delta T'} \tag{31}$$

Since $\Delta H = \Delta H'$, $\Delta T = \Delta T'$

$$\Delta S = \Delta S' \tag{32}$$

Entropy change is Lorenz invariant. I. Avramov[6] claims opposite. If entropy changes when a n moll of gas drops from volume V_1 to volume V_2 without the performance of external work is according to Tolman [12]

$$\Delta S = nR \ln V_2/V_1$$

also

$$\Delta S = nR \ln P_1/P_2$$

so

$$nR \ln V_2/V_1 = nR \ln P_1/P_2$$

ΔS Is Lorentz invariant, so in relativistic condition $n'R' \ln V_2'/V_1' = n'R' \ln P_1'/P_2'$

if $V' = Vo\sqrt{1-\frac{v^2}{c^2}}$ then $P' = Po\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$

Pressure and volume are Lorentz covariant, and then Boyle's law is Lorentz invariant.

Quantity of Substance

If

$$n = \frac{N}{N_a} \tag{33}$$

N is the number of particles, Na is Avogadro's number. Since N and Na is constant

$$n = n'$$

Also

$$n = \frac{m}{M_r} \tag{34}$$

Relativistic mass transformation is given

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$$m' = \frac{m}{\sqrt{1-\frac{v^2}{c^2}}} \tag{35}$$

v is the velocity of the object under study, c is the velocity of light.*m*' is relativistic mass, while *m* is the mass in resting system. So

$$M'_r = \frac{M_r}{\sqrt{1-\frac{v^2}{c^2}}} \tag{36}$$

So,

$$n = n' \tag{37}$$

n is Lorenz invariant

The Extent of Reaction

The extent of reaction is defined

$$\xi = \frac{\Delta n}{\nu_A} \tag{38}$$

Where, $\nu_A$is stoichiometry coefficient.
Relativistic transformation of the extent is given

$$\xi' = \frac{\Delta n'}{\nu_A{}'}$$

Since , $\Delta N = \Delta n'$

$$\xi = \xi \tag{39}$$

The extent of the reaction is Lorenz invariant.

IV. Conclusion

1. The temperature is invariant with Lorentz transformations.
2. Relation between molarity and moll fraction is given as $\chi = n_A R T / P V$ or $\chi_A = [A]RT/P$.
3. The set of relativity transformation laws for the volume, temperature and pressure

$$V' = Vo\,(1-v^2/c^2)^{1/2}$$
$$P' = Po/(1-v^2/c^2)^{1/2}$$
$$T = To$$

Equilibrium constant, Gibbs energy, Boyle's law, activation energy, enthalpy, entropy and extent of the reaction are Lorentz invariant.

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