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# Modeling and Optimization of Abrasive Waterjet Parameters using Regression Analysis

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***Abstract***—Abrasive waterjet is a novel machining process capable of processing wide range of hard-to-machine materials. This research addresses modeling and optimization of the process parameters for this machining technique. To model the process a set of experimental data has been used to evaluate the effects of various parameter settings in cutting 6063-T6 aluminum alloy. The process variables considered here include nozzle diameter, jet traverse rate, jet pressure and abrasive flow rate. Depth of cut, as one of the most important output characteristics, has been evaluated based on different parameter settings. The Taguchi method and regression modeling are used in order to establish the relationships between input and output parameters. The adequacy of the model is evaluated using analysis of variance (ANOVA) technique. The pairwise effects of process parameters settings on process response outputs are also shown graphically. The proposed model is then embedded into a Simulated Annealing algorithm to optimize the process parameters. The optimization is carried out for any desired values of depth of cut. The objective is to determine proper levels of process parameters in order to obtain a certain level of depth of cut. Computational results demonstrate that the proposed solution procedure is quite effective in solving such multi-variable problems.

***Keywords***— AWJ cutting, Mathematical modeling, Simulated Annealing, Optimization.

## I. INTRODUCTION

MANUFACTURING industry is becoming ever more time conscious with regard to the global economy. The need for rapid prototyping and small production batches is increasing in modern industries. These trends have placed a premium on the use of new and advanced technologies for quickly turning raw materials into usable goods; with no time being required for tooling. Abrasive waterjet (AWJ) machining technology has been found to be one of the most recent developed advanced non-traditional methods used in industry for material processing with the distinct advantages of no thermal distortion, high machining versatility, high flexibility and small cutting forces [1].

There are several distinguished advantages of AWJ technique. It is less sensitive to material properties and hence does not cause chatter, has no thermal effects, imposes minimal stresses on the workpiece, and has high machining versatility and flexibility. However, AWJ has some limitations and drawbacks. It may generate loud noise and a messy working environment. It may also create tapered edges on the kerf, especially when cutting at high traverse rates [2, 3].

As in the case of every machining process, the quality of process in AWJ is significantly affected by the process tuning parameters. There are several process parameters in this technique, among which water pressure, abrasive flow rate, jet traverse rate and diameter of focusing nozzle are of great importance and precisely controllable[4, 5]. The main process quality measures include attainable depth of cut, kerf width and its regularity and surface finish. Therefore, it is of great importance to study the effects of the process parameters on the process response characteristics. In this study, the depth of cut is considered as the performance measure as in many industrial application it is the main constraint on the process applicability.

A few attempts have been made to model and optimize the process parameters in AWJ. The approaches employed in this direction include design of experiments (DOE), regression modeling, analysis of variance (ANOVA), fuzzy logics and artificial neural networks. Some of these studies gave rise to various mathematical equations developed for predicting the output parameters [6]. Hashish [7] was among the first who developed a set of mathematical model to relate the process parameters settings to the process output variables in water jet technique. Later Ramulu and Arola [8] used regression analysis to predict depth of cut due to cutting and deformation wear for graphite/epoxy composite materials.

In recent years, determining an optimal set of process parameters values to achieve a certain output characteristics has been the prime interest by many researchers. Chakravarthy and Babu [9] employed a fuzzy-genetic approach for selection of process parameters in maintaining the desired depth of cut with a fixed size of orifice and focusing nozzle. More recently, the same approach has been employed by Srinivasu and Babu [10] to model and optimize the varying conditions of focusing nozzle in AWJ. Their study aims at selecting suitable process parameters that can control the depth of cut within the desired limits; taking into account nozzle wear.

Although there are few studies in modeling and optimization of process parameters in AWJ, most of them are limited to the particular circumstances and are computationally complex. The present study attempts to make use of available experimental data to relate important process parameters to process output variables, through developing empirical regression models for various target parameters. In

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the next stage, the proposed model is implanted into a simulated annealing (SA) optimization procedure to identify a proper set of process parameters that can produce the desired depth of cut.

## II. Modeling Development

The important controlling process parameters in AWJ cutting include water pressure (P), jet traverse rate (V), abrasive flow rate ($M_f$) and diameter of focusing nozzle ($d_f$). In this study, depth of cut has been chosen as the main process response characteristics to investigate the influence of the above parameters. We first develop a mathematical model to relate the process control parameters to the process response characteristics. The empirical model for the prediction of depth of cut in terms of the controlling parameters will be established by means of piecewise linear regression analysis. The experimental results were obtained using design of experiment (DOE) technique. For illustrative purposes, the data presented by Srinivasu et al. [13] is used here. Table I shows some of the experiment settings obtained by Taguchi DOE matrix.

TABLE I
DOE Matrix and Results for the AWJ

| No. | $d_f$ (mm) | $P$ (MPa) | $M_f$ (kg/min) | $V$ (mm/min) | $h$ (mm) |
|---|---|---|---|---|---|
| 1 | 0.8 | 206 | 0.035 | 125 | 11.37 |
| 2 | 1.0 | 168 | 0.056 | 119 | 10.32 |
| 3 | 1.1 | 165 | 0.059 | 110 | 10.35 |
| 4 | 1.2 | 165 | 0.071 | 123 | 10.00 |
| 5 | 1.3 | 177 | 0.065 | 114 | 10.45 |
| . | . | . | . | . | . |
| . | . | . | . | . | . |
| . | . | . | . | . | . |
| . | . | . | . | . | . |
| 30 | 1.0 | 234 | 0.081 | 38 | 47.38 |
| 31 | 1.1 | 232 | 0.084 | 37 | 48.05 |
| 32 | 1.2 | 237 | 0.085 | 37 | 48.94 |
| 33 | 1.3 | 232 | 0.091 | 38 | 48.61 |
| 34 | 1.4 | 232 | 0.094 | 37 | 47.78 |
| 35 | 1.5 | 239 | 0.095 | 37 | 46.59 |

As shown a total of 35 experiments were performed to gather the required data. In this table, the first four columns show the process parameters settings given by Taguchi DOE matrix. The last column (h) is the measured process output resulted from different experiments.

The general form of a regression mathematical model is as follows:

$$Y= a_0 + a_1.P + a_2.V + a_3.M_f + a_4.d_f + a_{11}.P^2 + a_{22}.V^2 + a_{33}.M_f^2 + a_{44}.d_f^2 + a_{12}.P.V + a_{13}.P.M_f + a_{14}.P.d_f + a_{23}.V.M_f + a_{24}.V.d_f + a_{34}.M_f.d_f \quad (1)$$

Different regression functions (linear, curvilinear, logarithmic, etc.) are fitted to the above data and the coefficients values ($a_i$) are calculated using regression analysis. The best model is the most fitted function to the experimental data. Such a model can accurately represent the actual AWJ process. Therefore, in this research, the adequacies of various functions have been evaluated using analysis of variance (ANOVA) technique.

The model adequacy checking includes test for significance of the regression model and test for significance on model coefficients [11]. ANOVA results recommend that the quadratic model is statistically the best fit in this case. Statistical analysis show that the associated P-value for the model is lower than 0.05; i.e. $\alpha$=0.05, or 95% confidence. This illustrates that the model is statistically significant. Based on ANOVA, the values of $R^2$ and adjusted $R^2$ are over 99% for h. This means that regression model provides an excellent explanation of the relationship between the independent variables and h response.

Table II shows the values of "T-value"' and "P-Value" for each term on the performances of h. In the case of depth of cut (h) the $d_f$ , $d_f^2$ and P.V can be regarded as significant term due to their "P-value" being less than 0.05. The backward elimination process removes the rest of insignificant terms to adjust the fitted quadratic model. The final proposed curvilinear model is presented below:

$$h = -61.1 + 73.7\, d_f + 0.365\, P - 29.3\, d_f^2 + 3031\, M_f^2 + 0.00173\, V^2 - 0.0781\, d_f . P - 0.00190\, P. V - 2.61\, M_f. V \quad (2)$$

For illustrative purposes, the distributions of real data around regression lines for these models are illustrated in Fig. 1. This figure demonstrates a good conformability of the developed models to the real process and hence is used to represent the actual process.

TABLE II
DOE Matrix and Results for the AWJ

| symbol | Degree of freedom | h | |
|---|---|---|---|
| | | T-valu | P-valu |
| P | 1 | 0.80 | 0.434 |
| V | 1 | -0.01 | 0.992 |
| $M_f$ | 1 | -0.73 | 0.476 |
| $d_f$ | 1 | 3.78 | 0.001* |
| P*P | 1 | -0.11 | 0.910 |
| V*V | 1 | 2.10 | 0.048 |
| $M_f$*$M_f$ | 1 | 1.95 | 0.065 |
| $d_f$*$d_f$ | 1 | -4.89 | 0.000* |
| P*V | 1 | -3.49 | 0.002* |
| P*$M_f$ | 1 | 1.03 | 0.315 |
| P*$d_f$ | 1 | -1.62 | 0.120 |
| V*$M_f$ | 1 | -0.97 | 0.345 |
| V*$d_f$ | 1 | 0.03 | 0.978 |
| $M_f$*$d_f$ | 1 | 0.65 | 0.522 |
| Residual | 20 | | |
| Total | 34 | | |

*significant

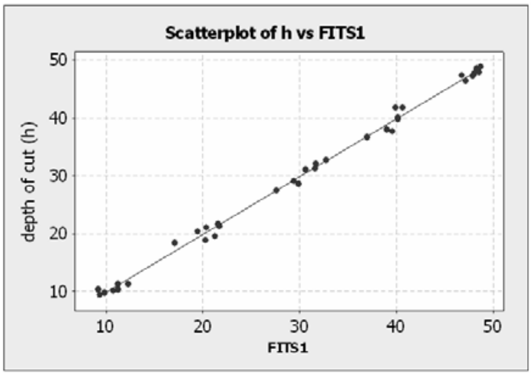


Fig. 1 Predicted *h* versus actual values

## III. Depth of cut response surface

The response surfaces for depth of cut (h) were obtained for the interaction terms in the reduced quadratic model.

The response surface for h in terms of water pressure and focusing nozzle diameter is shown in Figure 2(a). From this figure it can be observed that h increases with an increase in water pressure. However, the increasing the diameter of focusing nozzle would increase the depth of cut until 1.2 and then has a reverse effect on h since behind this diameter the cutting energy would decrease.

Response surface of h versus jet traverse rate and abrasive flow rate is shown in Fig. 2(b). From the figure it can be seen that a high jet traverse rate and abrasive flow rate combination leads to high h. Similarly, from Fig. 2(c) it can be observed that high h is obtained at high water pressure and high jet traverse rate combination. High values of water pressure and jet traverse rate lead to an efficient cutting energy which improves the h. Depth of cut increases by increasing any of the three factors (P, V, $M_f$), but it can be seen that V has the highest effect on h. The effect of jet pressure depends on the jet traverse level.

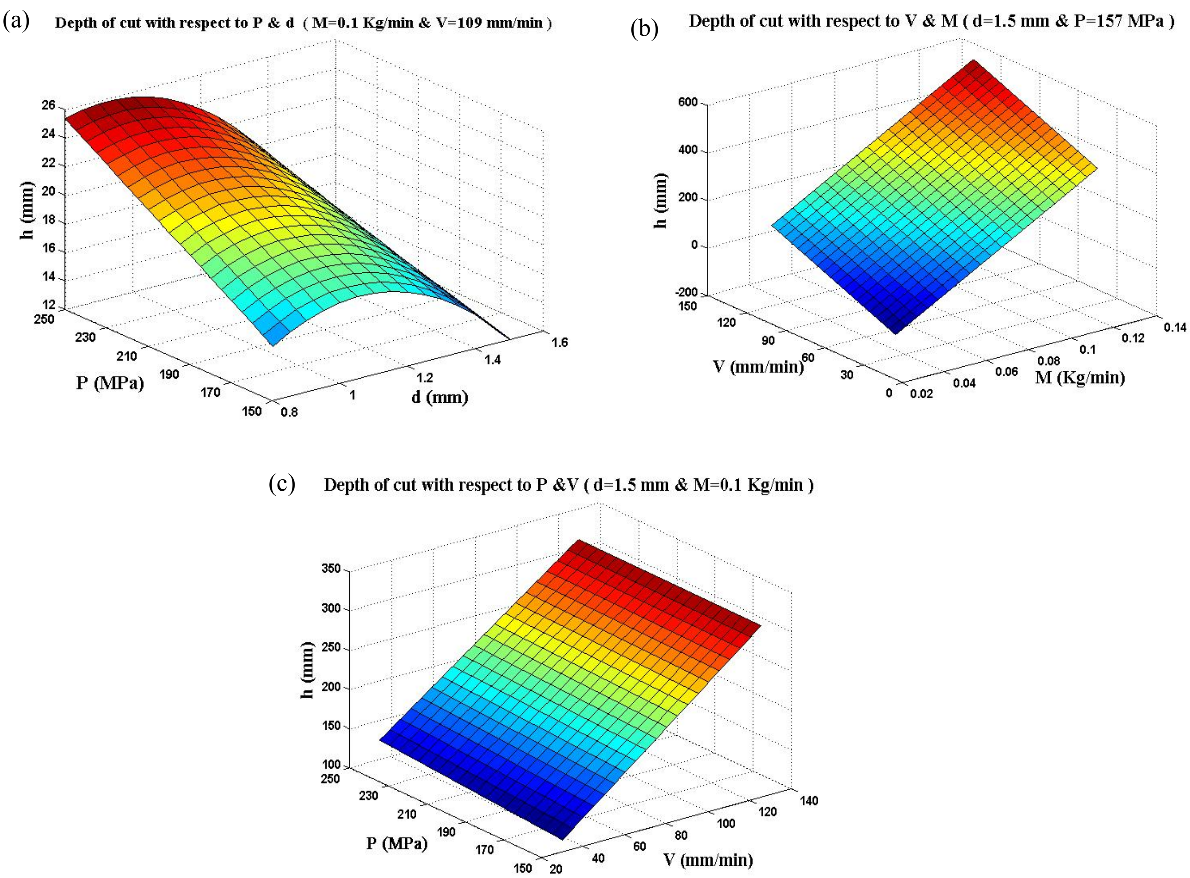


Fig 2. Response surface of depth of cut versus: (a) water pressure and diameter of focusing nozzle, (b) jet traverse rate and abrasive flow rate (c) water pressure and jet traverse rate.

## IV. The optimization procedure

The mathematical models furnished above provide one to one relationships between process parameters and AWJ cutting response characteristic, depth of cut. They can be used in two ways:

1) Predicting AWJ cutting response characteristic (h) for any given set of input parameters.

2) Determining a set of process parameters values for a desired AWJ characteristic specification (h).

In many practical situations, one needs to set the process parameters in such a way that a desired output is obtained (in this case depth of cut, h).Since finding the optimal set of input parameters for a given h is the problem of combination explosion, evolutionary algorithms can be employed as the optimizing procedure. These techniques would make the combination converge to solutions that are globally optimal or nearly so. Evolutionary algorithms are powerful optimization techniques widely used for solving combinatorial problems. As a new and promising approach, one of these algorithms, called SA, is implemented for optimization purposes in this research.

Simulated annealing (SA) is one of the novel algorithms which was initially presented by Kirkpatrick et.al [12]. Simulated Annealing solution methodology resembles the cooling process of molten metal through annealing. At high temperature, the atoms in the molten metal can move freely with respect to each other, but as the temperature is reduced, the movement of the atoms gets restricted. The atoms start to get arranged and finally form crystals having the minimum possible energy which depends on the cooling rate. If the temperature is reduced at a very fast rate, the crystalline state may not be achieved at all and instead, the system may end up in a polycrystalline state, which may have a higher energy state than the crystalline state. Therefore, in order to achieve the absolute minimum energy state, the temperature needs to be reduced at a slow rate. The process of slow cooling is known as annealing process.

The SA parameters are as follows:

$T_0$: Initial temperature,

$C$ : Rate of the current temperature decreases (cooling schedule),

E : Percent of error

$L$ : Number of accepted solution at each temperature,

$S$ : Counter for the number of accepted solution at each temperature,

$X$ : A feasible solution,

$C(X)$ : The value of objective function for $X$,

SA uses a stochastic approach to direct the search. It allows the search to proceed to neighboring state even if the move causes the value of the objective function become worse. This important feature, can allow it to prevent falling in the local optimum trap. SA guides the original local search method in the following way. The algorithm starts with an initial solution for the problem. In the inner cycle of the SA, repeated while $S < L$, a neighboring solution $X_n$ of the current solution $X$ is generated. If this move decreases the objective function, or leaves it unchanged, then the move is always accepted. Moves, which increase the objective function value, are accepted with a probability $e^{-\Delta c/T_0}$ to allow the search to escape a local optimum. The value of the temperature decreases in each iteration of the outer cycle of the algorithm. Obviously the probability of accepting worst solution decreases as the temperature decreases in each outer cycle. Two important issues that need to be defined when adopting this general algorithm to a specific problem are the procedures to generate both initial solution and neighboring solutions. A number of iterations are performed to simulate the thermal equilibrium at a particular temperature, before reducing the temperature further. The algorithm is terminated when a sufficiently small temperature is obtained and a small enough change in function value is found between two consecutive iterations. The initial temperature, cooling rate and number of iterations performed at a particular temperature are the three important parameters which governs the successful working of the Simulated Annealing procedure. The Steps of SA algorithm are shown in Figure 3.

For optimization process, we first define the prediction function as follow:

$$Error = \frac{T\arg et(h) - \Pr edicted(h_d)}{\Pr edicted(h_d)} \tag{3}$$

This function is used as the fitness function in the optimization process. In the above function, $h$ is depth of cut, and $h_d$ is the desired output value for the cutting operation. The objective is to set the process parameters at such levels that these values are achieved. In other words, we want to minimize the difference between the desired output and the output given by the SA algorithm. This is done by minimizing the error function given by equation (3). By doing so, the process parameters are calculated in such way that the AWJ cutting parameters approach their desired values.

## V. An illustrative example

In this section a numerical example is presented to illustrate the performance of the proposed model and the solution procedure. The error function given in (3), along with AWJ cutting model (2) is embedded into SA algorithm. The objective is to determine the values of control parameters (P, V, $M_f$, $d_f$) in such a way that the process output response (h) converges towards its target value.

The algorithm was coded in MATLAB 7.0 software and executed on a Pentium 4 computer. The best set of search parameters, found through several trial runs, is as follow: initial temperature ($T_0$) = 250; cooling rate ($\alpha$) =0.98; and termination criteria = 500 iterations or error less than 0.01.

The comparison between predicted and desired values of process responses is shown in Table III As shown, all the parameters deviate from their desired values by less than 0.5%. These results illustrate that the proposed procedure can be efficiently used to determine optimal process parameters for any desired output values of AWJ cutting process.

$Xbest = \varnothing$
Select an initial solution, $X_0$
$Xbest = X_0$ , $X = X_0$
While ( $E < 0.1$ ) Do
While ( $S < L$ ) Do
Generate solution $X_n$ in the neighborhood of $X$ ,
$\Delta C = C(X_n) - C(X)$
If $\Delta C \leq 0$ then
$X = X_n$
$S = S + 1$
If $C(X_n) < C(X\,best)$ then
$X\,best = X_n$
End If
Else
Generate $y \rightarrow U(0,1)$ Randomly
Set Z= $e^{-\Delta c/T_0}$
If $y < z$ then
$X = X_n$
$S = S + 1$
End If
End If
End While
$T_0 = C \times T_0$
End While

Fig. 3 The logic of SA algorithm

TABLE III
COMPARISON BETWEEN TARGET AND CALCULATED VALUES

| **No.** | **Target** | **Predicted values for process parameters and output** | | | | | **Error** |
|---|---|---|---|---|---|---|---|
| | **$h_d$** | **$d_f$** | **P** | **$M_f$** | **V** | **h** | **(%)** |
| **1** | 11.37 | 0.8 | 100 | 0.098 | 98 | 11.3981 | - 0.24 |
| **2** | 11.44 | 1.5 | 168 | 0.04 | 57 | 11.4851 | - 0.39 |
| **3** | 18.54 | 0.8 | 166 | 0.092 | 92 | 18.5162 | 0.13 |
| **4** | 21.15 | 1.5 | 208 | 0.052 | 56 | 21.0526 | 0.46 |
| **5** | 28.69 | 0.8 | 206 | 0.091 | 76 | 28.7221 | - 0.11 |
| **6** | 32.83 | 1.5 | 205 | 0.092 | 50 | 32.8329 | - 0.01 |
| **7** | 36.83 | 0.8 | 185 | 0.095 | 50 | 36.7814 | 0.13 |
| **8** | 38.15 | 1.5 | 197 | 0.090 | 43 | 38.0513 | 0.25 |
| **9** | 46.59 | 1.5 | 239 | 0.094 | 36 | 46.6053 | - 0.03 |

## VI. CONCLUSION

In this study, the effects of process parameters settings on Abrasive Waterjet machining of 6063-T6 aluminum alloy have been investigated. Statistical regression analysis have been employed to develop mathematical models relating such process parameters as water pressure, jet traverse rate, abrasive flow rate and diameter of focusing nozzle to the depth of cut. A set of experimental data, based on Taguchi method, has been used for model development. Various functions were fitted on the data among which the second order model was found to be the best one to represent relationship between input process parameters and depth of cut. The adequacy of the proposed model was then investigated using ANOVA technique. The results of ANOVA indicate that the proposed model has very good conformability to the real process.

The purpose of developing the mathematical model is to facilitate the optimization of AWJ cutting parameters. Therefore, a Simulated Annealing based procedure has been developed to predict the best process parameters values for any desired cutting characteristic. Computational results have proven that the proposed SA method can efficiently and accurately determine cutting parameters so as a desired depth of cut is obtained. From the above analyses, we conclude that the SA algorithm provides effective and efficient solutions for the specific parameter estimation problem we have defined in this study.

## REFERENCES


[1] U. Çaydaş, A. Hasçalik, "A study on surface roughness in abrasive waterjet machining process using artificial neural networks and regression analysis method", *J. Mater. Process. Techno*., vol. 2 0 2, pp. 574 –582, 2008.

[2] M.A. Azmir, A.K. Ahsan, "Investigation on glass/epoxy composite surfaces machined by abrasive water jet machining*", J. Mater. Process. Techno.*, vol.198, pp.122–128, 2008.

[3] C. Ma, R.T. Deam, "A correlation for predicting the kerf profile from abrasive water jet cutting", *Experimental Thermal and Fluid Science*, vol.30, pp.337–343, 2006.

[4] J. John Rozario Jegaraj, N. Ramesh Babu, "A soft computing approach for controlling the quality of cut with abrasive waterjet cutting system experiencing orifice and focusing tube wear*", J. Mater. Process. Techno.*, vol.185, no.1–3, pp.217–227, 2007.

[5] M. Hashish, "Optimization factors in abrasive waterjet machining", *ASME J. Eng. Ind.,* vol.113, pp.29–37, 1991.

[6] R. Kovacevic, M. Fang, "Modeling of the influence of the abrasive waterjet cutting parameters on the depth of cut based on fuzzy rules", *Int. J. Mach. Tools Manuf.,* vol. 34, no.1, pp.55–72, 1994.

[7] M. Hashish, "A modeling study of metal cutting with abrasive water jets", Transactions of ASME *J. Eng. Mater. Technol.,* vol.106, no.1, pp.88–100, 1984.

[8] M. Ramulu, D. Arola, "The influence of abrasive waterjet cutting conditions on the surface quality of graphite/epoxy laminates", *Int. J. Mach. Tools Manuf.,* vol.34, no.3, pp.295–313, 1994.

[9] P.S. Chakravarthy, N. Ramesh Babu, "A hybrid approach for selection of optimal process parameters in abrasive water jet cutting", in, Proceedings of the Institution of Mechanical Engineers, Part B: *J. Eng. Manuf.*, vol.214, pp.781–791, 2000.

[10] D.S. Srinivasu, N. Ramesh Babu, "A neuro-genetic approach for selection of process parameters in abrasive water jet cutting considering variation in diameter of focusing nozzle", *Appl. Soft Comp*., Vol.8, pp.809–819, 2008.

[11] D.C. Montgomery, E.A. Peck, G.G. Vining, *Introduction to Linear Regression Analysis*. third ed., Wiley, New York, 2003.

[12] Kirkpatrick, S., Gelatt, C. & Vecchi, M. "Optimization by simulated annealing". *Science*, vol.220, pp.671-680, 1983.


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