Recovering the Clipped OFDM Figure Based on the Conic Function

Linjun Wu   Shihua Zhu   Xingle Feng

Abstract—In Orthogonal Frequency Division Multiplexing (OFDM) systems, the peak to average power ratio (PAR) is much high. The clipping signal scheme is a useful method to reduce PAR. Clipping the OFDM signal, however, increases the overall noise level by introducing clipping noise. It is necessary to recover the figure of the original signal at receiver in order to reduce the clipping noise. Considering the continuity of the signal and the figure of the peak, we obtain a certain conic function curve to replace the clipped signal module within the clipping time. The results of simulation show that the proposed scheme can reduce the systems' BER (bit-error rate) 10 times when signal-to-interference-and noise-ratio (SINR) equals to 12dB. And the BER performance of the proposed scheme is superior to that of kim's scheme, too.

Keywords—Orthogonal Frequency Division Multiplexing (OFDM), peak-to-average power ratio (PAR), clipping time, conic function.

I. INTRODUCTION

ORTHOGONAL frequency division multiplexing (OFDM) technique, known to be well suited for high rate transmission over time-dispersive channel, has been accepted for many applications. However, one of the limitations of using OFDM is the high peak-to-average power ratio (PAR) of the transmitted signal. A large PAR leads to disadvantages such as increased complexity of the analog to digital converter (A/D) and reduced efficiency of the radio frequency (RF) amplifier. If power amplifiers are not operated with large linear-power back-offs, it is impossible to keep the out-of-band power below imposed limits. This leads to very inefficient amplification, expensive transmitters and causing intermodulation among the subcarriers and undesired out-of-band radiation. The PAR reduction techniques are therefore of great importance for OFDM systems.

Partial transmit sequences (PTS)[1], selected mapping (SLM) [2], active constellation extension (ACE) [3], Golay sequences and Reed-Muller codes (GRC) [4], clipping signal scheme [5][6][7][8], selective scrambling (SLS)[9] and windowing signal scheme were recently proposed to reduce PAR. The side

information (SI) must be transmitted to the receiver when using PTS, SLM and SLS methods, and then the channel efficiency drops a little. The complexity of using GRC method is too high to employ at real-time practice, and it is only efficacious when the subcarrier number equals to several special values as well. The ACE scheme wastes the precious power, especially when the mobile's power is very limited. The clipping signal scheme is relatively simpler than others. And the performance of the clipping scheme is superior to that of the windowing signal scheme in OFDM systems, because the windowing technique distorts all signals, but the clipping technique distorts small portion signal where the peak power exceeds the max-permitted power. In this paper we focus on the clipping signal technique.

When using the clipping scheme, the phase information of the signal is completely transmitted, and the amplitude of the signal is clipped if the power of the signal exceeds the max-permitted power. We propose a conic function scheme for replacing the original signal amplitude at receiver, while mitigating the clipping noise. Considering the continuity of signal and the figure of the signal peak, we obtain a certain conic function curve to replace the received signal amplitude within the clipping time. This paper is organized as follow. Starting with an overview of OFDM transmission, then the PAR problem defined in OFDM systems and the clipping scheme are shown in Section 2. Section 3 describes the fundamental of the proposed conic function scheme to recover the original signal. Section 4 gives the performance of the systems. Some simulation results and conclusions are given in section 5 and section 6.

II. ORTHOGONAL FREQUENCY DIVISION MULTIPLEXING

In OFDM systems, an input bit stream is mapped into N symbols, $\{X_n, n = 0, 1, \cdots, N-1\}$, and is modulated to N subcarriers. A continuous-time domain OFDM signal $y(t)$ is the sum of N independent signals that are modulated into subchannels of equal bandwidth with frequency separation of $1/T$ between adjacent subcarriers.

$$y(t) = \frac{w(t)}{\sqrt{N}} \sum_{n=0}^{N-1} X_n \exp(j2\pi \frac{n}{T} t) \tag{1}$$

where $w(t)$ is a rectangular window with amplitude one over the interval $(0, T)$, T is the OFDM symbol duration. A cyclic prefix (CP) is then appended to minimize interblock interference and aid the frequency-domain equalizer at the receiver. The CP does not affect our analysis results, so the CP is not considered in this paper. Digital-to-analog (D/A)

conversion and analog filtering are performed. Signals are fed to a high-powered amplifier (HPA), which drives the antenna load. Because of the limited transmitting power, the PAR must be reduced to a proper value, which is operated easy and satisfy systems requirement.

The essential cause of the PAR problem is a result of weighted sums of random variables to have a Gaussian-like distribution for a large N. The PAR can reach to $10\log N$ dB for large N. To transmit large peaks, the HPA must support a very large dynamic range, which is either impractical or expensive. The PAR for given OFDM block can be written as

$$PAR = \frac{\max|y(t)|^2}{E[|y(t)|^2]}, \quad 0 \le t \le T \tag{2}$$

where $E[.]$ denotes expectation. One method to reduce the peak power is to clip the signal. We define the clipping ratio γ is the ratio of the max-permitted power value (MPTV) to the average power of signal, which is

$$\gamma = \frac{MPTV}{E[|y(t)|^2]} \tag{3}$$

Let $\rho(t)e^{j\theta(t)}$ denotes the output complex signal $y(t)$ in polar coordinates. If the signal power exceeds to $\gamma \cdot E[|y(t)|^2]$, the transmitted signal module is clipped to a constant λ, and express as follow.

$$y_{tran}(t) = \begin{cases} y(t) & \rho(t)^2 < \gamma \cdot E[|y(t)|^2] \\ \lambda \arg(y(t)) & \rho(t)^2 \ge \gamma \cdot E[|y(t)|^2] \end{cases} \tag{4}$$

where $y_{tran}(t) = \rho_{tran}(t)e^{j\theta(t)}$ (polar coordinates) is the final transmitted signal, constant λ is the threshold value and equals to $\sqrt{\gamma \cdot E[|y(t)|^2]}$. For large N, $y(t)$ is an approximately complex Gaussian random process, so the signal amplitude is Rayleigh distribution. The probability of the peak power is relatively small compared with the whole signal, although the clipping scheme destroys the system's orthogonality. The performance of systems does not decline seriously under a proper clipping ratio. But the frequency set-off and the channel's imperfect also affect the systems' performance. If high quality transmission is required, it is essential to accurately recover the original signal as soon as possible.

III. CONIC FUNCTION SCHEME

In this paper, we adopt a conic function to be close to the original signal module at receiver. Fig.1 and Fig.2 exhibit the original and the clipping peak signal module in enlarged-time domain. The original signal module (see Fig.1 curve ABCDE) is clipped by the threshold value to the clipped signal module (see Fig.2 curve ABFDE). If using ideal power control to counteract the influence of wireless channel in systems and not considering other factors (such as frequency set-off), the received signal amplitude also shows as the same as Fig.2 plus to AWGN.

Because the received signal is clipped within the clipping time segment (see Fig.2 line from point B to point D), there is

not any information to recover the original signal. Fortunately, beside the clipping time segment, the redundancy of signal can
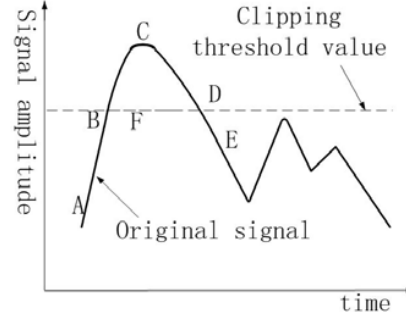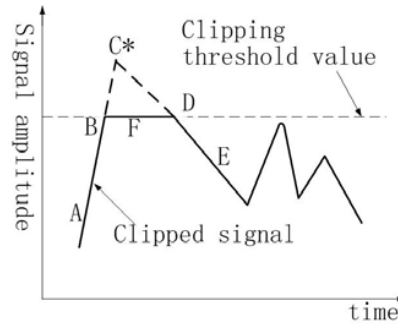

Fig.1 the original signal

Fig.2 the clipped signal

be used to help us to recover the original signal module. That is by using the redundancy of curve AB and curve DE (Fig.2) construct a high order curve to be close to the original signal module. In here, the conic function is used to recover the original signal module. The conic function $w(t)$ can be expressed as follow

$$[w(t)]^2 + \beta_1 t^2 + \beta_2 w(t) \cdot t + \beta_3 w(t) + \beta_4 t + \beta_5 = 0. \tag{5}$$

where $\beta_1, \beta_2, \beta_3, \beta_4, \beta_5$ is constant, respectively. $w(t)$ is the signal amplitude. If constants $\beta_1 \sim \beta_5$ are obtained, the shape of carve is fixed.

Transmitted signals are composed of N time-continuous subcarrier signals. At point B and D (fig.2), signal modules are smooth and derivative. So within a relative small domain of point B and point D, the first and second order derivatives of $w(t)$ ought to equal to those of $\rho_{tran}(t)$.

The first and second order derivatives of $w(t)$ are expressed as follow

$$\frac{d}{dt}w(t) = -\frac{2\beta_1 t + \beta_2 w(t) + \beta_4}{2w(t) + \beta_2 t + \beta_3} \tag{6}$$

$$\frac{d^2}{dt^2}w(t) = -2\frac{\beta_1 + [\frac{2\beta_1 t + \beta_2 w(t) + \beta_4}{2w(t) + \beta_2 t + \beta_3}]^2 - \beta_2 \frac{2\beta_1 t + \beta_2 w(t) + \beta_4}{2w(t) + \beta_2 t + \beta_3}}{2w(t) + \beta_2 t + \beta_3} \tag{7}$$

Let the received clipping signal's first and second order left derivatives of point B be α_1 and α_2, these of right derivatives of point D be α_3 and α_4. Because of the signals' continuity,

the derivatives at point B and D are exclusive. So we obtain follow equations.

$$-\frac{2\beta_1 t + \beta_2 w(t) + \beta_4}{2w(t) + \beta_2 t + \beta_3}\bigg|_B = \alpha_1 \qquad (8)$$

$$-2\frac{\beta_1 + [\frac{2\beta_1 t + \beta_2 w(t) + \beta_4}{2w(t) + \beta_2 t + \beta_3}]^2 - \beta_2 \frac{2\beta_1 t + \beta_2 w(t) + \beta_4}{2w(t) + \beta_2 t + \beta_3}}{2w(t) + \beta_2 t + \beta_3}\bigg|_B = \alpha_2 \qquad (9)$$

$$-\frac{2\beta_1 t + \beta_2 w(t) + \beta_4}{2w(t) + \beta_2 t + \beta_3}\bigg|_D = \alpha_3 \qquad (10)$$

$$-2\frac{\beta_1 + [\frac{2\beta_1 t + \beta_2 w(t) + \beta_4}{2w(t) + \beta_2 t + \beta_3}]^2 - \beta_2 \frac{2\beta_1 t + \beta_2 w(t) + \beta_4}{2w(t) + \beta_2 t + \beta_3}}{2w(t) + \beta_2 t + \beta_3}\bigg|_D = \alpha_4 \qquad (11)$$

The duration of every peak is very small, the figure of every peak likes a "splinter", so we extend line l_1, which passes point B and the slope is α_1, and line l_2, which passes point D and the slope is α_3. These two lines intersect to point c^*, which is an approximate estimation of point c. Let the conic function $w(t)$ pass point c^*.

The functions of line l_1 and l_2 are

$$l_1 : y(t) = B_y + \alpha_1(t - B_t) \qquad (12)$$

$$l_2 : y(t) = D_y + \alpha_2(t - D_t) \qquad (13)$$

where B_y, B_t is the signal module value and time of the point B, $B_y = \lambda$; D_y, D_t is the signal module value and time of the point D, $D_y = \lambda$.

By (12) and (13), we can get the coordinate of point $c^*[t_{\hat{c}}, y_{\hat{c}}]$.

$$t_{\hat{c}} = \frac{\alpha_1 B_t - \alpha_2 D_t}{\alpha_1 - \alpha_2} \qquad (14)$$

$$y_{\hat{c}} = \lambda - \frac{\alpha_1 \alpha_2}{\alpha_1 - \alpha_2}(B_t + D_t) \qquad (15)$$

The coordinate of point $\hat{c}[t_{\hat{c}}, y_{\hat{c}}]$ satisfies the conic function (5). So we get

$$y_{\hat{c}}^2 + \beta_1 t_{\hat{c}}^2 + \beta_2 y_{\hat{c}} \cdot t_{\hat{c}} + \beta_3 y_{\hat{c}} + \beta_4 t_{\hat{c}} + \beta_5 = 0 \qquad (16)$$

Finally, these coefficients of the conic function $w(t)$ are obtained by equations (5), (8), (9), (10), (11) and (16). Suppose there are J domains δ_j $j = 1, \cdots J$, where $|y(t)|^2 \geq \gamma \cdot E[|y(t)|^2]$ $t \in \delta_j$ is tenable. So the recovered signal can be written as follow

$$y_{re}(t) = \begin{cases} y(t) & t \notin \delta_j \\ w(t) \angle y(t) & t \in \delta_j \end{cases} \qquad j = 1,2,\cdots J \qquad (17)$$

We also adopt high order curve function to replace the original signal within the clipping time, but the systems performance is almost as same as that of using conic function.

IV. THE PERFORMANCE OF SYSTEMS

In evidence, the clipping process reduces the output power. For large N, $y(t)$ amplitude is Rayleigh distributed. For clipping ratio γ and input power P_{in}, the output power P_{out} can be obtained as follow

$$P_{out} = (1 - e^{-\gamma^2})P_{in}. \qquad (18)$$

Suppose error item is $e(t)$, the relationship of the original signal and the recovered signal can be expressed

$$y(t) = y_{re}(t) + e(t). \qquad (19)$$

Considering equation (19) at the sampling period $\Delta t = T/N$, the discrete-time signal sampled at time instant $t = k\Delta t$ is then expressed as

$$y(k) = y_{re}(k) + e(k) \qquad (20)$$

By using the N-point discrete Fourier transform (DFT) of the sequence of length N, the transform of equation (20) is

$$X_n = DFT(N, y(k))_n = DFT(N, y_{re}(k))_n + DFT(N, e(k))_n \qquad (21)$$
$$= \widetilde{X}_n + E_n$$

Where $X_n, \widetilde{X}_n, E_n$ is the DFT of the original signal, the retrieved signal and the error. The signal-to-interference-and-noise ratio (SINR) of the kth subcarrier is

$$SINR_k = \frac{E[|\widetilde{X}_k|^2]}{E[|E_k|^2] + \eta_k} = \frac{P_{out,k}}{P_{err,k} + \eta_k} = \frac{(1 - e^{-\gamma^2})P_{in,k}}{e^{-\gamma^2}P_{in,k} + \eta_k} = \frac{(1 - e^{-\gamma^2})}{e^{-\gamma^2} + \eta_k/P_{in,k}} \qquad (22)$$

Where η_k is the noise power of the kth subcarrier. If power is uniformly distributed and noise power is equal to others, equation (22) can be rewritten as follow.

$$SINR_k = \frac{(1 - e^{-\gamma^2})}{e^{-\gamma^2} + N\eta_k/P_{in}} = \frac{1 - e^{-\gamma^2}}{e^{-\gamma^2} + N\eta/P_{in}} \qquad (23)$$

The average channel capacity of OFDM is

$$C = W \log_2(1 + \frac{1 - e^{-\gamma^2}}{e^{-\gamma^2} + N\eta/P_{in}}) \qquad (24)$$

Where W is the bandwidth of subcarrier. If the noise power can be ignored compared with the interference power, equation (24) is rewrited as

$$C = W \cdot \gamma^2 \log_2 e \qquad (25)$$

From equation (25), the channel capacity is proportional to γ^2. So in order to guarantee the channel capacity, the value of γ is as high as possible. But in order to reduce the PAR, we always expect that γ is very small. So how to optimize γ is a trade-off problem.

V. SIMULATIONS

To justify the proposed algorithm, supposing under the ideal power control, the wireless channel fading does not affect system's performance, no other factors (such as frequency set-off) contribute to the interference. In order to obtain $\alpha_1, \alpha_2, \alpha_3, \alpha_4$ exactly, it is necessary to oversample the received signal for mitigating the effect of AWGN. α_1, α_2 is the average value of the first and second order derivatives of the small domain around point B, respectively. So to α_3, α_4. Selecting the clipping ratio γ is 1.5, 2.0, 3.0, 4.0, 5.0, ∞. The BER (bit-error-ratio) performance of different γ values and the proposed conic function scheme are discussed in AWGN

channel. In order to exactly obtain $\alpha_1, \alpha_2, \alpha_3, \alpha_4$, the received signal is oversampled by a factor of 8.

Fig.3 gives the BER performance to SINR (signal-to-interference-noise-ratio) of subcarriers when adopting QPSK modulation and N= 64. The BER of ideal systems ($\gamma \to \infty$) is smallest because there is only channel AWGN. The bigger γ is, the smaller the systems' BER. The BER of the proposed scheme with γ=1.5 is smaller than that of no using proposed scheme with γ=5.0. When SINR=12, the BER of the proposed scheme is nearly 10^{-4}, the BER of no using the proposed scheme is nearly 10^{-3}. The BER descends 10 times.

Fig.4 compares the BER performance of the proposed scheme with that of Kim's [5] scheme with γ=3.0 and N=64. When SINR becomes bigger, the BER of proposed scheme is much better than that of Kim's.

From the above, we can get that the proposed scheme is useful to recover the clipping signal and to reduce the clipping noise.
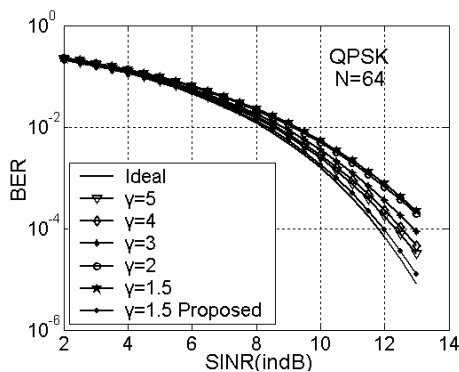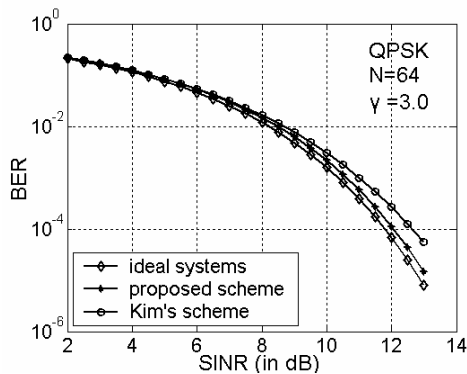

Fig.3　BER to SNR for N=64 and QPSK

Fig.4 BER to SNR for γ=3.0 (QPSK N=64)

VI. Conclusions

The conic function scheme is proposed to reduce the clipping noise. At receiver, let the curve of conic function in the clipping time domain be almost close to the continuous-time original signal module, and let this curve replace the received signal module within the clipping time. The BER of systems can be reduced 10 times by using the proposed conic scheme. The performance of proposed scheme is superior to the kim's scheme.

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