A closed form solution for hydrodynamic pressure of gravity dams reservoir with effect of viscosity under dynamic loading

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***Abstract*—**Hydrodynamic pressures acting on upstream of concrete dams during an earthquake are an important factor in designing and assessing the safety of these structures in Earthquake regions. Due to inherent complexities, assessing exact hydrodynamic pressure is only feasible for problems with simple geometry. In this research, the governing equation of concrete gravity dam reservoirs with effect of fluid viscosity in frequency domain is solved and then compared with that in which viscosity is assumed zero. The results show that viscosity influences the reservoir's natural frequency. In excitation frequencies near the reservoir's natural frequencies, hydrodynamic pressure has a considerable difference in compare to the results of non-viscose fluid.

***Keywords*—**Closed form solution, Concrete dams reservoir, viscosity, Dynamic loads, Hydrodynamic pressure.

I. Introduction

THE issue of determining hydrodynamic pressures applied on concrete dams under the effect of earthquake has been largely studied by researchers. Westergaard[1] was the first who put forward an analytical response for determining hydrodynamic pressure on a rigid dam under harmonic loads with the assumption that the reservoir fluid has no viscosity.

Kotsubo[2] showed that Westergaard's response in valid only when excitation period in larger than the reservoir's natural period. Barthez and Heilborn [3] showed that if the reservoir is of limited length, upstream, with upstream side unmoving, for $L/H > 2$ (L= length of reservoir, H=Depth of reservoir) with elongation of length, pressure rise wont exceed 0.5% (compared to a reservoir with infinite length) and if it's assumed that the upstream of reservoir vibrates along with the earth, for $L/H > 3$ effect of length is inconsiderable. Hoskins and Jacobsen's experimental studies confirm the results above[4].

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Werner and Sundquist[5] also showed that pressure response is independent of the reservoir length.

Bustamante et all. [6] then in 1961 inspected the effect of reservoir length on spans of wider excitation periods than previously studied spans and concluded that for excitation periods larger than the reservoir's natural period, the effect of reservoir's length is inconsiderable, but for shorter periods length has an important role in harmonic loading.

However Kotsubo[7] put forward the hydrodynamic response of a reservoir and cylindrical arch dam for harmonic loading. By applying polar coordinate, he suggested a very simple analysis. Bustamante et all. [6] then studied the effect of reservoir's length on span of wider excitation periods than previously studied spans and concluded that for excitation periods larger than the reservoir's natural period, the effect of reservoir's length is inconsiderable, but for shorter periods the modeled length of reservoir plays an important role in harmonic movement. He also surveyed the effect surface waves for harmonic excitation and calculated the error due to ignoring surface waves.

In further researches Chopra [8] in 1967 calculated the hydrodynamic pressure on dams and in 1968 the dam response under horizontal and vertical acceleration of earthquake of any value considering the effect of compressibility of the fluid conducting further researches. Chopra and Chakrabarti [9] calculated the hydrodynamic pressure in a dam reservoir caused by vertical component of earthquake.

Lee and Tsai [10] developed the enclosed solution to the analysis of the dam-reservoir in time domain.

They studied the response for the dam when the reservoir is both empty and full considering the interaction between fluid and structure and using standard methods. They also worked out the effect of material on responded by numerical values and proved that reservoir fluid and flexibility of dam structure itself has great impact on interactive behavior of both structure and fluid and thus leads to huge responses.

Chwang[11] found the analytical answers for a rigid dam with a sloped upstream face and also a sloped reservoir bottom in case of a reservoir with infinite length. He used the boundary integration method in this research.

Liu [12] put forward semi-analytical solution for rigid dam which actually were development of those of Chwang.

Aviles [13] gave semi-analytical answers for rigid dam, with sloped upstream face and also with viscous and compressible fluid. He also calculated hydrodynamic pressure using boundary integration method and a series of certain functions that were to satisfy boundary conditions and governing equation and stated the answer for different cases of upstream shape of this dam with the aid of numerical problems.

Zingales[14] found the hydrodynamic pressure for dam-reservior system using earthquake stochastic analysis. This was carried out considering a compressible fluid and a vibration model.

Attarnejad and Farsad [15] studied the closed form solution for dam- reservoir system in time domain for a variable-thickness dam. By considering the interaction between the structure and fluid showed the answers for a dam with both empty and full reservoir for several earthquakes using numerical examples.

Bouaanani et all. [16] considering the effect of bottom absorption. And horizontal earthquake acceleration found the earthquake semi-closed form answer using approximate method.

Pressure wave absorption by sediment at the bottom of the reservoir is also an important factor in assessing the hydrodynamic pressure which was fist studied by Fenves and Chopra[17] and then showed that sediment at the bottom of a reservoir plays and important role in assessing the true Hydrodynamic pressure applied on the dam.

Kucukarslan et all. [18] investigated the effect of pressure wares absorption by reservoir bottom sediment and its impact on the value of hydrodynamic pressure applied on the dam by modeling it with finite elements and two-layer boundary elements.

The aim of this research is solve governing differential equation of reservoir containing viscous fluid under harmonic loading. Thus horizontal acceleration in the dam's reservoir due to dam body vibration excluding structure-fluid interaction is investigates.

II. Governing Equation

Using Navier-Stock and continuity relations equation governing circumference of concrete dam's reservoir can be written as:

$$\nabla^2 P + \frac{\mu}{K}\frac{\partial}{\partial t}(\nabla^2 P) = \frac{1}{C^2}\frac{\partial^2 P}{\partial t^2} \tag{1}$$

where P and μ are hydrodynamic pressure and viscosity respectively, K and C represents Bulk modulus and elastic waves velocity respectively.

Following assumptions have been made in obtaining the above equation: acceleration of transfer" terms in Navier-Stock relations are minor, fluid is isotropic, homogeneous with linear behavior. Amplitude of movement of reservoir water is low and irrigational.

A. Boundary conditions and initial value

Four boundary conditions with S1, S2 , S3 and S4, as boundaries are shown in fig 1 and assuming horizontal component of earth's movement is perpendicular to dam's axis, dam is so wide that it can be surveyed as in 2 and dam reservoir is infinite upstream, two initial conditions can be stated follows:

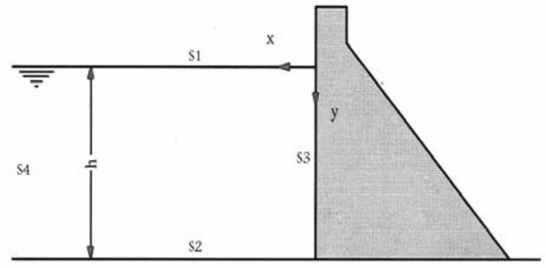

Fig. 1 Dam-Reservoir System and boundary conditions

Boundary condition in reservoirs free surface (S1)

With neglecting of surface waves, boundary condition in reservoir's free surface can be taken P=0 with a good approximation.

Boundary condition at reservoir's bottom (S2)

If the reservoir's bottom is assumed horizontal and rigid, boundary condition can be obtained as $\frac{\partial P}{\partial y} = 0$

Based on this assumption Reservoirs bottom bears no deformation and dynamic load directly reaches to dam's reservoir during the application of earthquake load.

Boundary condition between dam and reservoir (S3)

Normal velocity in the interface of dam and fluid must be equal so that neither vacuum nor opening is made between dam and reservoir. In other words, fluid is in constant contact with solid boundary to satisfy so, velocity component any where in solid boundary perpendicular to surface must be equal to velocity in that very direction in fluid boundary. Assuming that fluid velocity is negligible and consequently shear stress where fluid and dam body meets, boundary condition in boundary between dam and reservoir could be writhen as: $\frac{\partial P}{\partial n} = -\rho\, a_n$

Where p is hydrodynamic pressure, an is the exerted acceleration on a point in boundary and n is the vector perpendicular to the intended point upstream.

Boundary condition in upstream of reservoir (S4) or propagation condition

Considering that in analytical solution it is assumed that the dam spreads infinitely. Hydrodynamic pressure is gradually reduced till it converges to zero at last.

Initial value

Since fluid is stationary in (t=0), hydrodynamic pressure inside the reservoir is zero. So initial conditions are described with following values at t=0:

$$P(x,y,z,t)=0\ ,\qquad \frac{\partial P}{\partial t}(x,y,z,t)=0$$

III. Solution to the Governing Equation

To solve the governing equation (1) boundary conditions with respect to fig (1) are introduced as:

$$\begin{aligned}&1)\ p(x,0,t)=0\\ &2)\ \frac{\partial p}{\partial y}(x,h,t)=0\\ &3)\ \frac{\partial p}{\partial x}(0,y,t)=-\rho\,\alpha\,g\,e^{iwt}\\ &4)\ p(\infty,y,t)=0\end{aligned}\tag{2}$$

where α and h are earthquake factor and reservoir fluid head respectively.

If the excitation of real part is e^{iwt}, the answer could be written in real form as: $\overline{P}(x,y,w).e^{iwt}$, In other words assuming that the harmonic response is $P(x,y,t)=\overline{P}(x,y,w)e^{iwt}$ equation (1) in frequency domain is transformed as:

$$\nabla^2\overline{p}+s^2\overline{p}=0\tag{3}$$

where

$$s^2=\frac{w^2}{C^2+\dfrac{\mu\,\mathrm{i}\,w}{\rho}}\tag{4}$$

By applying separation method and inserting it in equation (3) and then simplifying the relation above and also applying boundary conditions (S1) and (S4) in the relation:

$$\overline{p}(x,y,w)=Ge^{-\sqrt{\lambda^2-s^2}\,x}.\sin\lambda y\tag{5}$$

where G in a constant coefficient which will be obtained later. By applying boundary Condition (S2) in relation above it gives:

$$\begin{aligned}&\frac{\partial\overline{p}}{\partial y}(x,h,w)=G\lambda\,e^{-\sqrt{\lambda_n^2-s^2}\,x}.\cos\lambda h=0\rightarrow\\ &\cos\lambda h=0\rightarrow\lambda=(2m-1)\frac{\pi}{2h}\end{aligned}\tag{6}$$

λ is special value for Sturm-Liouville problem (in this research Sturm-Liouville problem is a governing equation with its boundary conditions) and can be described as a sequence.

$$\overline{p}(x,y,w)=\sum_{n=1,3,\ldots}^{\infty}G_n e^{-\sqrt{\lambda_n^2-s^2}\,x}.\sin\lambda_n y\tag{7}$$

by applying boundary condition (S3), in relation above coefficient G would be obtained:

$$G_n=\frac{2\rho\alpha g}{h\lambda_n}\times\frac{1}{\sqrt{\lambda_n^2-s^2}}\tag{8}$$

By inserting G_n in relation (7), the final response can be described as:

$$P(x,y,t)=-\frac{2\alpha\rho\,\mathrm{g}}{\mathrm{h}}e^{i\omega t}\sum_{n=1,3,..}^{\infty}\frac{1}{\lambda_n\sqrt{\lambda_n^2-s^2}}e^{-x\sqrt{\lambda_n^2-s^2}}\sin\lambda_n y\tag{9}$$

Statement above is the answer to equation (1) assuming that acceleration is horizontal. By simplifying we would get:

$$P(x,y,t)=-\frac{8\alpha\rho\,\mathrm{g}\,h}{\pi^2}e^{i\omega t}\sum_{n=1,3,..}^{\infty}\frac{1}{n^2\sqrt{1-\dfrac{s^2}{\lambda_n^2}}}e^{-\frac{n\pi}{2h}\sqrt{1-\frac{s^2}{\lambda_n^2}}\,x}\sin\lambda_n y\tag{10}$$

$\sqrt{1-\dfrac{s^2}{\lambda_n^2}}$ can be simplified as follows:

$$\sqrt{1-\frac{s^2}{\lambda_n^2}}=\sqrt{1-\frac{\dfrac{w^2}{C^2+\dfrac{\mu\,\mathrm{i}\,w}{\rho}}}{\lambda_n^2}}=\frac{\sqrt{\mathrm{C}^4+\dfrac{\mu^2w^2}{\rho^2}-\dfrac{w^2C^2}{\lambda_n^2}+i(\dfrac{\mu w^3}{\rho\lambda_n^2})}}{C^2+\dfrac{\mu\,w}{\rho}}\rightarrow\tag{11}$$

$$\rightarrow\frac{1}{2}\frac{1}{C^2+\dfrac{\mu\,w}{\rho}}\left[\sqrt{2\sqrt{R^2+M^2}+2R}+i\,\mathrm{sgn}(M-iR)\sqrt{2\sqrt{R^2+M^2}+2R}\right]$$

where M and R are introduced as:

$$R=\mathrm{C}^4+\frac{\mu^2w^2}{\rho^2}-\frac{w^2C^2}{\lambda_n^2}\ ,\qquad M=\frac{\mu w^3}{\rho\lambda_n^2}\tag{12}$$

so the in the response of governing equation can be written in a complex form To obtain resonance frequencies, we first calculate the roots of and we'll have:

$$\sqrt{1-\frac{s^2}{\lambda_n^2}}=0\ \rightarrow\ \mathrm{T_{Res}}=\frac{-\mu\,\pi}{\rho\,C^2}i\pm\frac{\sqrt{-\mu^2\pi^2+16\rho^2C^2h^2}}{\rho\,C^2}\tag{13}$$

Resonance period order for Chopra's response is then discovered

$$\sqrt{\lambda_n^2-\frac{w^2}{C^2}}=0\ \rightarrow\mathrm{T_{Res}}=\frac{4\,h}{n\,C}\tag{14}$$

By placing real values for the fluid with viscosity 10 times greater than water's we would have:

$$\mathrm{T_{Res}}=0.13888889\quad\text{(Chopra's resonance Period)}\tag{15}$$

$\mathrm{T_{Res}}=-0.00001515\ i\ \pm 0.13888889$ (viscose fluid resonance period).

Thus it is concluded that viscosity in fluid causes the formation of complex in the fluid. It is also concluded from relations (15) that the real purl in resonance period of the fluid is almost like resonance period of in viscid fluid and on the other hand imaginary part in resonance period of the fluid is very small. So it can be observed that existence of viscosity in fluid change the resonance period of reservoir. It should be noted that considering relation (13) if viscosity is very high, resonance period will become totally imaginary.

In fig. 2-10 presented effect of excitation frequency (or period) on response of viscous fluid. In mentions figures viscous and inviscid fluid responses for different viscosity values for $\mu_1 = \mu_{wat}$, $\mu_2 = 10\mu_{wat}$, $\mu_3 = 100\mu_{wat}$ that are respectively shown as vis1, vis2, vis3, for different excitation periods and specified time and location are compared to those of Chopra's.

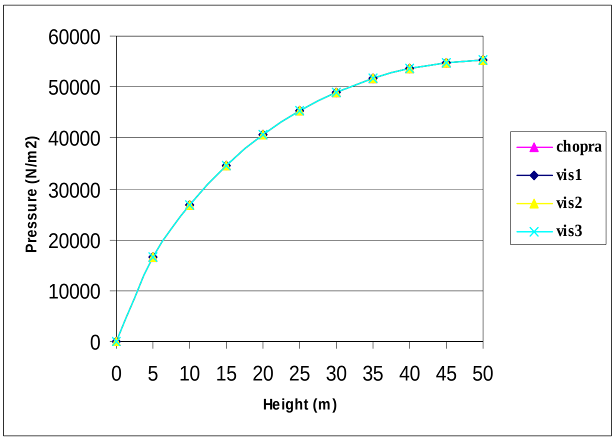

Fig. 2 Result comparison for viscous and inviscid fluid in x=0, t=0 and Ts=1 sec.

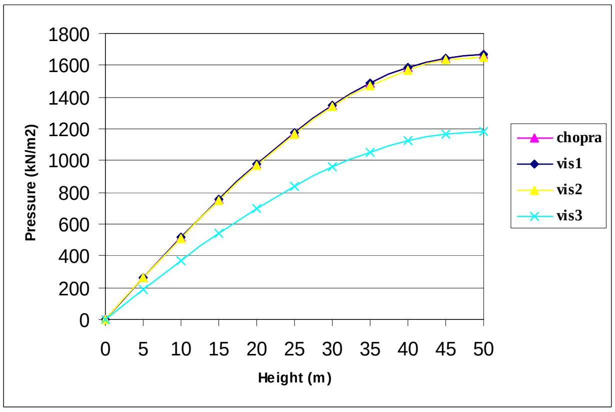

Fig. 3 Result comparison for viscous and inviscid fluid in x=0 , t=0 and Ts=0.1388 sec.

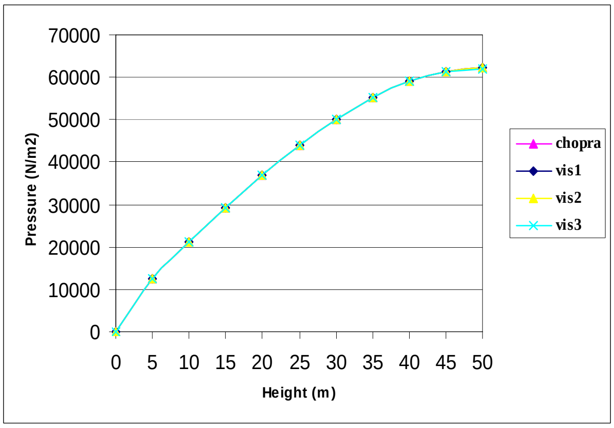

Fig. 4 Result comparison for viscous and inviscid fluid in x=0 , t=0 and Ts=0.1 sec.

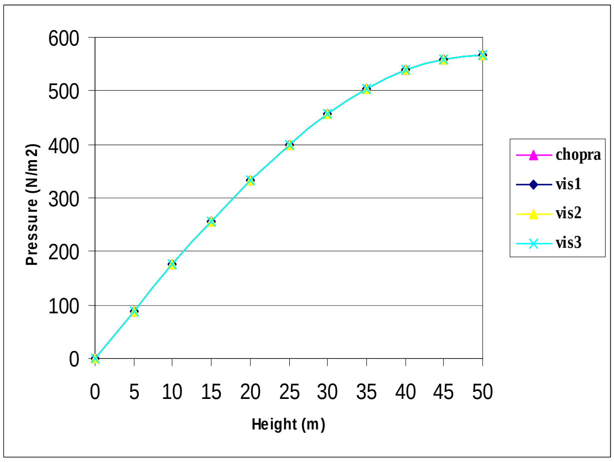

Fig. 5 Result comparison for viscous and inviscid fluid in x=3h , t=0 and Ts=1 sec.

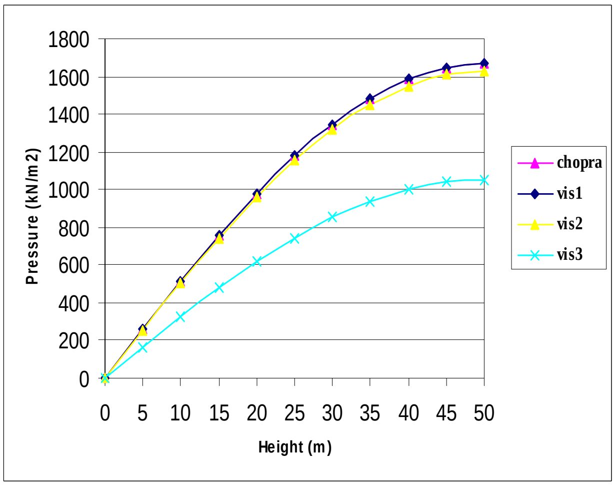

Fig. 6 Result comparison for viscous and inviscid fluid in x=3h, t=0 and Ts=0.1388 sec.

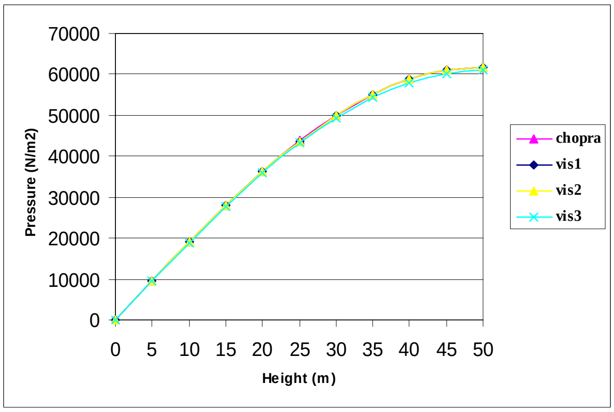

Fig. 7 Result comparison for viscous and inviscid fluid in x=3h, t=0 and Ts=0.1 sec.

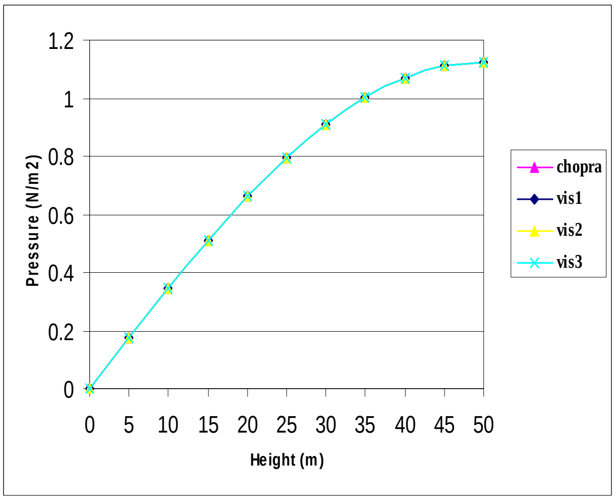

Fig. 8 Result comparison for viscous and inviscid fluid in x=7h, t=0 and Ts=1 sec.

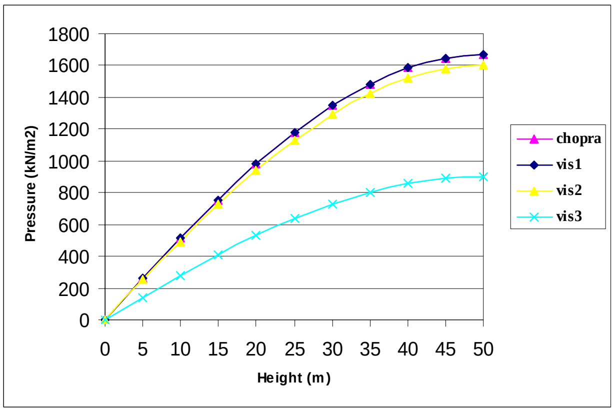

Fig. 9 Result comparison for viscous and inviscid fluid in x=7, t=0 and Ts=0.1388 sec.

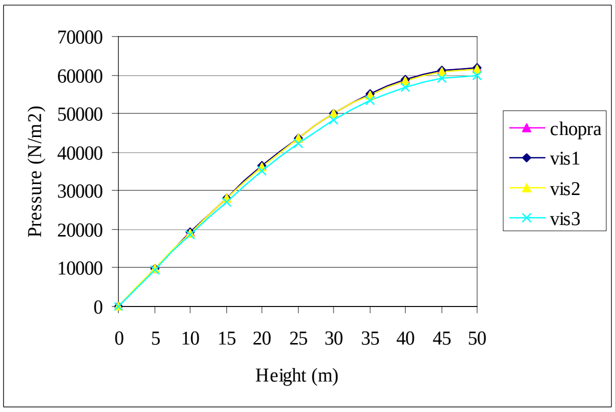

Fig. 10 Result comparison for viscous and inviscid fluid in x=7h, t=0 and Ts=0.1 sec.

III. CONCLUSION

As it is clear from above result, in excitation periods near to natural period of the reservoir the response difference between viscous and inviscid fluid rises in comparison to other periods which confirms that the reservoir contains viscose fluid has complex resonance period. Away upstream face of the dam, pressure value wont change in Chopra's solution in shorter period than natural periods of reservoir and this proves the dynamic state of a non-viscose system while away upstream face of the dam for inviscid fluid pressure reduction is observable which proves that energy damping occurs in form of viscosity in fluid. It should be stated that this energy damping is more boilable in higher viscosity. In greater excitation periods than fluid reservoir's natural period (first resonance frequency) viscosity has less effect on responses.

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