

Fatigue Properties and Strength Degradation of Carbon Fiber Reinforced Composites

Pasquale Verde, and Giuseppe Lamanna

Abstract—A two-parameter fatigue model explicitly accounting for the cyclic as well as the mean stress was used to fit static and fatigue data available in literature concerning carbon fiber reinforced composite laminates subjected tension-tension fatigue. The model confirms the strength-life equal rank assumption and predicts reasonably the probability of failure under cyclic loading. The model parameters were found by best fitting procedures and required a minimum of experimental tests.

Keywords—Fatigue life, strength, composites, Weibull distribution.

I. INTRODUCTION

THE fatigue behavior of composites is very peculiar, resulting in the accumulation of diffuse damage having different origin and location, rather than the propagation of a single crack as in metals. Fiber fracture, matrix cracking, matrix crazing, fiber buckling, fiber-matrix interface failure and delamination are interacting sources of damage accumulation showing different growth rates [1]-[8]. Further the fatigue lifetime of composite materials is influenced primarily from the matrix and fibers type, the geometry of reinforcement (unidirectional, mat, fabric, braiding), the laminate stacking sequence, the environmental conditions (mainly temperature and moisture), the loading history (stress ratio, R , cyclic frequency) and boundary conditions. A subtle phenomenon influencing fatigue of polymer based-composites is the viscoelastic nature of the matrix that accumulates residual stresses depending on the manufacturing process characteristics. Thermal cycling plays an important role during service lifetime even if this aspect has been rarely attacked systematically. Furthermore it was shown that the structural relaxation of the in situ resin (i.e. the resin constrained into the fibers lattice) may influence its nominal glass transition [9]-[15]. Samples conditioned in the vicinity of T_g showed higher characteristic strength, which resulted in a correspondingly higher fatigue life compared to the as manufactured materials. All the above phenomena would be preferably included in a fatigue damage modeling. However, this task is really difficult and expensive. The first reasons are the several scales where damage mechanisms are present. Secondly, it is impossible reproducing perfectly identical specimens for fatigue as well as static characterization. In turn, the different micro-structural specimens features claims for a statistical approach that requires time-consuming testing procedures.

On the other hand many models have been established for laminates with a particular stacking sequence, under given loading conditions. Thus, all the above considerations

explain why the extrapolation to real structures is almost impossible, owing to the fact that the stacking sequence may be a variable and the “in service” loading history is much more complex than that used in a laboratory environment. The complexity of factors influencing fatigue and the usually large scatter in mechanical properties explain why one of the most intricate tasks in designing with composite laminates is the definition of reliable allowable strength. Among the others the residual stresses arising in the polymer matrix that act as non-mechanical loads and influence the strength of the composites [11]-[18]. The difficulty to predict the long-term behavior of carbon fiber reinforced composites comes from the thermodynamic intricacies resulting from exposure to adverse conditions, e.g. aggressive environments, near glass transition, T_g , service temperature or cyclic loadings [19]-[28]. For instance, in the vicinity of the glass transition temperature the structural relaxation alters the viscoelastic behavior of the matrix and affects its long-term behavior [29]-[37].

This complicates the modeling of composites response to fatigue, which is also strongly dependent on external factors such as maximum applied stress, stress ratio, frequency and stress history.

Commonly, the cyclic fatigue data are termed S-N data and are presented as the maximum cyclic stress, σ_{max} , or the normalized maximum stress, σ_{max}/σ_0 , (σ_0 being the static material strength) or stress amplitude, $\sigma_{max} - \sigma_{min}$, as a function of number of cycles to failure, N . The fatigue data are obtained at a fixed value of the maximum stress and the stress ratio, $R = \sigma_{min}/\sigma_{max}$, the latter parameter representing the severity of loading conditions. However, it is a matter of fact that, given σ_{max} , higher stress ratios, R , imply higher cycles to failure, thus the phenomenological analytic models appeared in literature assume R as the main variable [5]. The loading frequency is strictly related to temperature problems arising from hysteretic heating. In fact, for most thermoplastic and thermoset systems the temperature rise on the surface of the specimen should be monitored in exploratory tests and should not exceed a few degrees Celsius, which is usually unavoidable. No standard frequency is universally recognized. However, values between one and ten Hertz are typically selected to ensure against hysteretic heating. Further, in [3], [5], [7], [13] the loading frequency was adjusted as function of maximum stress amplitude to give a quasi-constant loading rate.

The phenomenological fatigue life models extract information from the S-N curves and possibly propose a fatigue failure criterion. Thus, they do not take into account damage accumulation, but predict the number of cycles at which fatigue failure occurs under fixed loading conditions.

This paper presents a re-elaboration of static and fatigue data available in literature [38], [39] concerning carbon fiber reinforced laminates. The data were used to obtain the parameters of a recent model [3] accounting for both mean

Pasquale Verde, and Giuseppe Lamanna are with the Department of Industrial and Information Engineering, the Second University of Naples, 81031 Aversa, Italy (e-mail: giuseppe.lamanna@unina2.it).

stress and stress amplitude.

Vol:7, No:10, 2013, when (4) is solved for σ_0 one obtains:

II. MODELLING

In [3] a two parameter model was proposed for the prediction of the fatigue lifetime of Random Glass Fiber Reinforced Plastics (RGFRP). The starting point of the model is the hypothesis that the material strength undergoes a continuous decrease with fatigue cycle evolution, according to a power law.

Expressing the cyclic and the mean stress as function of the stress ratio $R = \sigma_{\min}/\sigma_{\max}$, and using the boundary condition $n = 1$ i.e. $\sigma_n = \sigma_0$, where σ_0 is the strength of the virgin material, the following relationship was obtained:

$$\sigma_0 - \sigma_n = \sigma_{\max} \left(\frac{\varphi}{2\beta} \right) (n^\beta - 1) \quad (1)$$

$$\varphi = \alpha(1-R) - (1-\alpha)(1+R)$$

where σ_n is the residual material strength after n cycles, α and β are positive constants and $\Delta\sigma = (\sigma_{\max} - \sigma_{\min})/2$, and $\sigma_{\text{mean}} = (\sigma_{\max} + \sigma_{\min})/2$ are the cyclic and the mean stresses, respectively. α ($0 < \alpha < 1$) is the parameter partitioning the fatigue sensitivity to cyclic and mean stress.

According to (1), the evolution of strength degradation with fatigue cycling can be calculated, provided the constants α and β , only dependent on the material and loading conditions, are known. Assuming that the failure will happen when the residual strength equals the maximum applied stress during fatigue, the critical number of cycles for failure, N , can be calculated putting $\sigma_n = \sigma_{\max}$ in (1). Solving for N , we obtain:

$$N = \left[\left(\frac{\sigma_0}{\sigma_{\max}} - 1 \right) \frac{2\beta}{\varphi} + 1 \right]^{1/\beta} \quad (2)$$

$$\varphi = \alpha(1-R) - (1-\alpha)(1+R)$$

Equation (2) can be rearranged to obtain:

$$(N^\beta - 1) \frac{\varphi}{2\beta} = \left(\frac{\sigma_0}{\sigma_{\max}} - 1 \right) \quad (3)$$

$$\varphi = \alpha(1-R) - (1-\alpha)(1+R)$$

or, equivalently:

$$\frac{\sigma_{\max}}{\sigma_0} = \left[(N^\beta - 1) \frac{\varphi}{2\beta} + 1 \right]^{-1} \quad (4)$$

$$\varphi = \alpha(1-R) - (1-\alpha)(1+R)$$

from which the parameters α and β are obtained by best fitting procedure from fatigue data usually expressed in terms of σ_{\max}/σ_0 vs. number of cycles to failure, N , at given stress ratio, R .

$$\begin{aligned} \sigma_0 &= \sigma_{0N} = \\ &= \sigma_{\max} \left[(N^\beta - 1) \frac{\varphi}{2\beta} + 1 \right] \end{aligned} \quad (5)$$

$$\varphi = \alpha(1-R) - (1-\alpha)(1+R)$$

From a physical viewpoint (5) indicates that the ultimate strength of the virgin material can be calculated from the fatigue life, N . The symbol σ_{0N} has been used in (5) just to distinguish the calculated strength from that, σ_0 directly measured in a static characterization test.

However the reliability of our approach has still to be assessed for carbon fiber laminates. To do that, in this paper, the scheme already adopted in references [3], [9] will be followed. The experimental data are taken from reference [38], where both monotonic and fatigue tests were carried out on T300/934 graphite/epoxy laminates. In Reference [38], tension-tension fatigue test were performed with $R=1/36$ and $R=0$, using different levels for σ_{\max} . The different stress ratios were adopted for laminates having different stacking sequences. In this paper we use the static and the fatigue data obtained at $R=1/36$.

The static strength data are presented in Fig. 1 (open circle) where the characteristic strength, $\sigma_0=608.6$ MPa, and the shape parameter, $\gamma=21.6$, of the two-parameter Weibull distribution are evaluated by best fitting procedures.

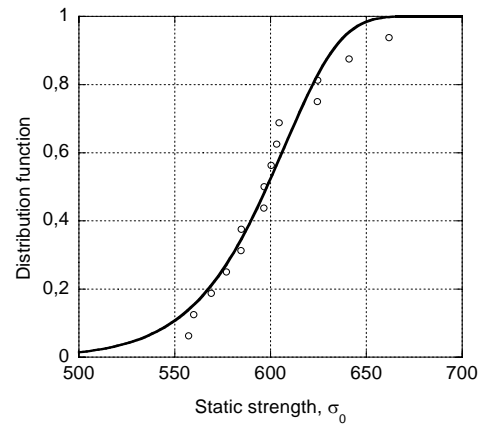


Fig. 1 Statistical distribution of the measured monotonic strength, σ_0 , for T300/934 graphite/epoxy laminates. Experimental data taken from reference [38]

The tension-tension fatigue data obtained with $R=1/36$, are reported in Fig. 2 in a classical σ_{\max}/σ_0 vs. the number of cycles to failure, N , together with the best fit curve based on (4). The fitting process allowed obtaining the model parameters, $\alpha=0.536$, $\beta=0.11$.

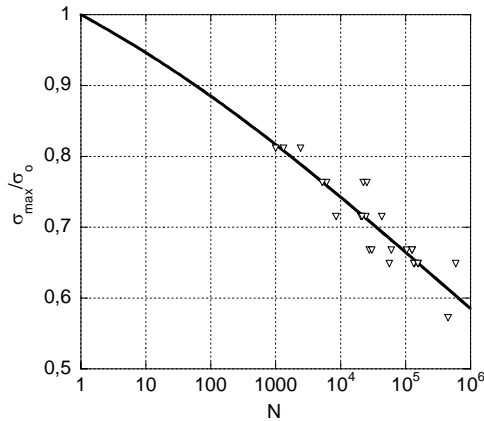


Fig. 2 Re-elaboration of tension-tension fatigue data according to (4) Experimental data taken from reference [38]

A useful way to test the model reliability consists of its potential ability to predict the monotonic strength from fatigue data as described by (5).

It assumed that the scatter in the monotonic material strength, σ_0 , is well represented by a two-parameter Weibull distribution. Therefore the probability of finding a σ_0 value $\leq x$ is given by:

$$F_{\sigma_0}(x) = P(\sigma_{\max} \leq x) = 1 - \exp\left[-\left(\frac{x}{\gamma}\right)^\delta\right] \quad (6)$$

where γ is the scale parameter or the characteristic strength, and δ is the shape parameter. Accordingly, in Fig. 3 the static strength and the theoretical static strength, σ_{0N} , coming out from our analytical model expressed by (6) and resulting from fatigue data, are reported.

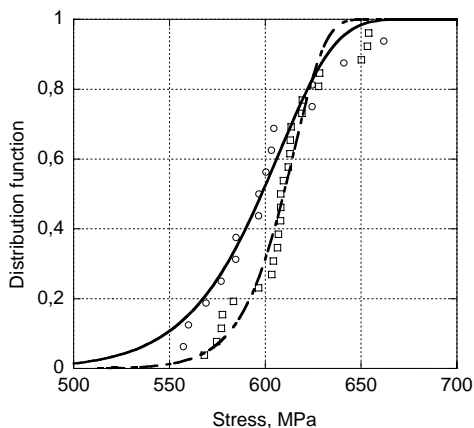


Fig. 3 Statistical distribution of static strength σ_0 (circle symbols) and the calculated, according to (6), monotonic strength, σ_{0n} (square symbols), for T300/934 graphite/epoxy laminates. Broken and continuous lines represent the best fitting with a two parameters Weibull distribution for σ_0 and σ_{0n} , respectively. Experimental data taken from reference [38]

reasonable agreement between the two statistical distributions can be recognized. The observable discrepancies between the Weibull parameters, (namely, the characteristic strength and the shape parameter) are attributed to the different sampling of experimental static strength data, σ_0 , and the calculated monotonic strength, σ_{0N} , spurted from fatigue data according to (5).

It appears that the scatter in fatigue life is substantially due to the variability in monotonic strength according to the strength-life equal rank assumption [10], [11].

If this is true, then the modeling approach could be further implemented statistically. In fact, according to (5) and (6), it can be easily recognized that the relationship allowing for the calculation of the probability $F(N^*)$ to find an N lower than N^* can be written as follows:

$$F_N(N^*) = 1 - \exp\left\{\frac{\sigma_{\max}^\delta [(N^* - 1)\varphi + 1]^\delta}{\gamma^\delta}\right\} \quad (7)$$

$$\varphi = \frac{\alpha(1-R) - (1-\alpha)(1+R)}{2\beta}$$

Where the parameters α, β, γ and δ have been already defined.

This approach was verified by one of us [40] for laminates with different stacking sequence and stress ratio (namely, $R=0$). Some small discrepancies between the experimental data and the model prediction were observed [40].

The present approach yielded similar response compared to the data elaborated in references [9], [38], with negligible discrepancies. To this end, (5) still remains an empirical expression. However it has the potential to describe both the fatigue life and the scatter in tension-tension fatigue of composite laminates.

III. CONCLUSION

A fatigue model explicitly accounting for the cyclic as well as the mean stress was tested on the basis of static and fatigue data for a graphite/epoxy laminate. The model confirms the strength-life equal rank assumption and predicts reasonably the probability of failure under cyclic loading. The model has the potential of being statistically implemented assuming a distribution of static strength according to a two-parameter Weibull distribution. However the phenomenological nature of the model limits its applicability when the parameters governing the fatigue life are modified. Of course, the laminate stacking sequence dictate also the evolution of fatigue damage and the strength degradation kinetics, given the loading history, so that generalizing the model to different stacking sequences appears rather difficult if not impossible. We restrict our future work trying to extend the phenomenological approach for uniaxial constant amplitude loading to more general loading condition, such as block type and spectrum loading and to take into account the effect of cyclic frequency and multiaxial loads.

- [1] J. F. Mandell, "Fatigue behavior of short fiber composite materials" in K. L. Reifsnider Ed., *Fatigue of Composite Materials*, vol. 4, in *Composite Materials series*, ch. 7, Ed. Amsterdam, NL: Elsevier, 1990, pp. 231-237.
- [2] K. Y. Hour, H. Sehitoglu, "Proof testing of composite materials" in *Journal of Composite Materials*, vol. 27(8), Ed. Thousand Oaks CA, USA: Sage, 1993, pp. 782-805.
- [3] A. D'Amore, G. Caprino, P. Stupak, J. Zhou, L. Nicolais, "Effect of stress ratio on the flexural fatigue behaviour of continuous strand mat reinforced plastics", in *Science and Engineering of Composite Materials*, vol. 5, Ed. Boston MA, USA: De Gruyter, 1996, pp. 1-8.
- [4] A. D'Amore, L. Grassia, P. Verde, "Modeling the Flexural Fatigue Behavior of Glass Fiber Reinforced Thermoplastic Matrices" in *Mechanics of Time-Dependent Materials*, Ed. Rotterdam, NL: Springer, 2012, doi: 10.1007/s11043-012-9192-y.
- [5] A. D'Amore, L. Grassia, P. Verde, "Modeling the Fatigue Behavior of Glass Fiber Reinforced Thermoplastic and Thermosetting Matrices", in *AIP American Institute of Physics conference proceedings*, vol. 1459, Ed. Melville New York, USA: American Institute of Physics, 2012, pp. 372-374.
- [6] Z. Fawaz, F. Ellyin, "Fatigue Failure Model for Fibre-Reinforced Materials under General Loading Conditions", in *Journal of Composite Materials*, vol. 28(15), Ed. Thousand Oaks CA, USA: Sage, 1994, pp. 1432-1451.
- [7] G. Caprino, A. D'Amore, "Flexural fatigue behaviour of randomcontinuous-fibre-reinforced thermoplastic composites", in *Composites Science and Technology*, vol. 58(6), Ed. Amsterdam, NL: Elsevier, 1998, pp. 957-965.
- [8] G. Caprino, "Predicting fatigue life of composite laminates subjected to tension-tension fatigue", in *Journal of Composite Materials*, vol. 34(16), Ed. Thousand Oaks CA, USA: Sage, 2000, pp. 1334-1355.
- [9] H.T. Hahn, R.Y. Kim, "Proof testing of composite materials", in *Journal of Composite Materials*, vol. 9(3), Ed. Thousand Oaks CA, USA: Sage, 1975, pp. 297-311.
- [10] P. C. Chou, R. Croman, "Residual Strength in Fatigue Based on Strength-life equal rank assumption", in *Journal of Composite Materials*, vol. 12(2), Ed. Thousand Oaks CA, USA: Sage, 1978, pp. 177-194.
- [11] A. D'Amore, F. Caputo, L. Grassia, M. Zarrelli, "Numerical Evaluation of Structural Relaxation-Induced Stresses in Amorphous Polymers", in *Composites Part A: Applied Science and Manufacturing*, vol. 37(4), Ed. Amsterdam, NL: Elsevier, 2006, pp. 556-564.
- [12] L. Grassia, A. D'Amore, "Residual stresses in amorphous polymers", in *Macromolecular Symposia*, vol. 228, Ed. Hoboken NJ, USA: Wiley, 2005, pp. 1-15.
- [13] M. Zarrelli, I. K. Partridge, A. D'Amore, "Warping induced in bi-material specimens: Coefficient of thermal expansion, chemical shrinkage and viscoelastic modulus evolution during cure", in *Composites Part A: Applied Science and Manufacturing*, vol. 37(4), Ed. Amsterdam, NL: Elsevier, 2006, pp. 565-573-564.
- [14] L. Grassia, A. D'Amore, "Calculation of the Shrinkage Induced Residual Stress in a Viscoelastic Dental Restorative Material", in *Mechanics of Time-Dependent Materials*, Ed. Rotterdam, NL: Springer, 2012, doi: 10.1007/s11043-012-9190-0.
- [15] L. Grassia, A. D'Amore, "Modeling the residual stresses in reactive resins-based materials: A case study of photo-sensitive composites for dental applications", in *AIP American Institute of Physics conference proceedings*, vol. 1255, Ed. Melville New York, USA: American Institute of Physics, 2010, pp. 408-410.
- [16] G. Lamanna, F. Caputo, A. Soprano, "Numerical Modelling and Simulation of a Bolted Hybrid Joint", in *Structural durability and Health Monitoring*, vol. 7(4), Ed. Duluth GA, USA: Tech Science Press, 2011, pp. 283-296.
- [17] G. Lamanna, F. Caputo, A. Soprano, "Geometrical parameters influencing a hybrid mechanical coupling", in *Key Engineering Materials*, vols. 525-526, Ed. Stafa/Zurich, CH: Trans Tech Publications Ltd, 2012, pp. 161-164.
- [18] L. Grassia, M. G. Pastore Carbone, A. D'Amore, "Modeling of the isobaric and isothermal glass transitions of polystyrene", in *Journal of Applied Polymer Science*, vol. 122, Ed. Hoboken NJ, USA: Wiley, 2011, pp. 3752-3757.
- [19] A. D'Amore, L. Grassia, D. Acierno, "Modelling the yield stress and the Poisson's ratio of glassy polymers", in *E-Polymers*, art. no. 052, 2009, pp. 1-9.
- [20] L. Grassia, A. D'Amore, P. Verde, "On The Inter-Conversion Between Viscoelastic Material Functions of Polycarbonate", in *AIP American Institute of Physics conference proceedings*, vol. 1459, Ed. New York, USA: American Institute of Physics, 2012, pp. 375-377.
- [21] L. Grassia, A. D'Amore, "Thermal residual stresses in amorphous thermoplastic polymers", in *AIP American Institute of Physics conference proceedings*, vol. 1255, Ed. Melville New York, USA: American Institute of Physics, 2010, pp. 414-416.
- [22] L. Grassia, A. D'Amore, "Finite element calculation of residual stress in dental restorative material", in *AIP American Institute of Physics conference proceedings*, vol. 1459, Ed. Melville New York, USA: American Institute of Physics, 2012, pp. 312-315.
- [23] A. Martone, L. Grassia, M. Zarrelli, M. Giordano, A. D'Amore, "Enthalpy relaxation of an epoxy matrix/carbon nanotubes", in *AIP American Institute of Physics conference proceedings*, vol. 1459, Ed. Melville New York, USA: American Institute of Physics, 2012, pp. 347-349.
- [24] R. Alfani, P. Colombet, A. D'Amore, N. Rizzo, L. Nicolais, "Effect of temperature on thermo-mechanical properties of Macro-Defect-Free cement-polymer composite", in *Journal of Materials Science*, vol. 34, Ed. Rotterdam, NL: Springer, 1999, pp. 5683-5687.
- [25] A. D'Amore, G. Caprino, L. Nicolais, G. Marino, "Long-term behaviour of PEI and PEI-based composites subjected to physical aging", in *Composites Science and Technology*, vol. 59, Ed. Amsterdam, NL: Elsevier, 1999, pp.1993-2003.
- [26] G. Caprino, A. D'Amore, F. Facciolo, "Fatigue sensitivity of random glass fibre reinforced plastics", in *Journal of Composite Materials*, vol. 32(12), Ed. Thousand Oaks CA, USA: Sage, 1998, pp. 1203-1220.
- [27] P. Netti, A. D'Amore, D. Ronca, L. Ambrosio, L. Nicolais, "Structure-mechanical properties relationship of natural tendons and ligaments", in *Journal of Materials Science: Materials in Medicine*, vol. 7, Ed. Rotterdam, NL: Springer, 1996, pp. 525-530.
- [28] Y. Miyano, M.K. MacMurray, J. Enyama, M. Nakada, "Loading rate and temperature dependence on flexural fatigue behavior of a satin woven CFRP laminate", in *Journal of Composite Materials*, vol. 28(13), Ed. Thousand Oaks CA, USA: Sage, 1994, pp. 1250-1260.
- [29] L. Grassia, A. D'Amore, S. L. Simon, "On the viscoelastic Poisson's ratio in amorphous polymers", in *Journal of Rheology*, vol. 54, Ed. Pasadena CA, USA: Jor, 2010, pp. 1009-1022.
- [30] L. Grassia, A. D'Amore, "Constitutive law describing the phenomenology of subyield mechanically stimulated glasses", in *Physical Review E - Statistical, Nonlinear, and Soft Matter Physics*, vol. 74(2), Ed. College Park MD, USA: APS, 2006, art. no. 021504 (13 pages).
- [31] Y. Miyano, M. Nakada, H. Kudok, R. Muki, "Prediction of tensile fatigue of for unidirectional CRFP", in *Journal of Composite Materials*, vol. 34(7), Ed. Thousand Oaks CA, USA: Sage, 2000, pp. 538-550.
- [32] G. McCrum, C. P. Bickley, C. B. Bucknall in *Principles of Polymer Engineering*, Oxford University Press, 1989.
- [33] I. Chung, C. T. Sun, I. Y. Chang, "Modeling Creep in Thermoplastic Composites", in *Journal of Composite Materials*, vol. 27(10), Ed. Thousand Oaks CA, USA: Sage, 1993, pp. 1009-1029.
- [34] L. Grassia, A. D'Amore, "Isobaric PVT behavior of Poly(Carbonate) (PC)", in *AIP American Institute of Physics conference proceedings*, vol. 1255, Ed. Melville New York, USA: American Institute of Physics, 2010, pp. 417-419.
- [35] L. Grassia, A. D'Amore, "Isobaric and isothermal glass transition of PMMA: Pressure-volume- temperature experiments and modelling predictions", in *Journal of Non-Crystalline Solids*, vol. 357, Ed. Amsterdam, NL: Elsevier, 2011, pp. 414-418.
- [36] L. Grassia, M. G. Pastore Carbone, G. Mensitieri, A. D'Amore, "Modeling of density evolution of PLA under ultra-high pressure/temperature histories", in *Polymer*, vol. 52, Ed. Amsterdam, NL: Elsevier, 2011, pp. 4011-4020.
- [37] Y. Yang, A. D'Amore, Y. Di, L. Nicolais, B. Li, "Effect of physical aging on phenolphthalein polyethersulfone/poly(phenylene sulfide) blend. I. Mechanical properties", in *Journal of Applied Polymer Science*, vol. 59, Ed. Hoboken NJ, USA: Wiley, 1996, pp. 1159-1166.
- [38] J.N. Yang, M. D. Liu, "Residual strength degradation model and theory of periodic proof tests for graphite/epoxy laminates", in *Journal of Composite Materials*, vol. 11, Ed. Thousand Oaks CA, USA: Sage, 1977, pp. 176-203.
- [39] J.N. Yang, "Effect of high load on statistical fatigue of unnotched graphite/epoxy laminates", in *Journal of Composite Materials*, vol. 14, Ed. Thousand Oaks CA, USA: Sage, 1980, pp. 82-94.
- [40] P. Verde, "Modeling the Strength Degradation and Fatigue of Carbon Fiber Reinforced Composites", in *The Open Materials Science Journal*, vol. 6, Ed. Bussum, NL: Bentham Open Publishers, 2012, pp. 77-82.