

MPSO based Model Order Formulation Technique for SISO Continuous Systems

S. N. Deepa, G. Sugumaran

Abstract—This paper proposes a new version of the Particle Swarm Optimization (PSO) namely, Modified PSO (MPSO) for model order formulation of Single Input Single Output (SISO) linear time invariant continuous systems. In the General PSO, the movement of a particle is governed by three behaviors namely inertia, cognitive and social. The cognitive behavior helps the particle to remember its previous visited best position. In Modified PSO technique split the cognitive behavior into two sections like previous visited best position and also previous visited worst position. This modification helps the particle to search the target very effectively. MPSO approach is proposed to formulate the higher order model. The method based on the minimization of error between the transient responses of original higher order model and the reduced order model pertaining to the unit step input. The results obtained are compared with the earlier techniques utilized, to validate its ease of computation. The proposed method is illustrated through numerical example from literature.

Keywords—Continuous System, Model Order Formulation, Modified Particle Swarm Optimization, Single Input Single Output, Transfer Function Approach

I. INTRODUCTION

RECENT developments in modeling of complex physical and technical process lead to a growing interest in model order formulation of large scale dynamical systems. Model order formulation is the process of deriving the lower order model from the higher order model. Model order formulation approximates the complex system by simple one. Modeling is the mathematical description of dynamic characters of a physical system. For achieving the higher accuracy while modeling the complex systems, it can be noted that the system order is also increased. Higher order model are difficult to handle because it has higher computational complexities and implementation difficulties. The exact analysis of higher order system is both tedious, costly and it is too complicated to be used in real problems. The use of MOF makes it easier to implement in analysis, design and simulation of the controllers, compensators, state variable controllers and observers for the stabilization of the output response of the given systems. The main aim of the formulation is to find the

best possible approximation of the output of the original system. Several methods have been proposed for reduction of linear systems in different fields like control engineering, micro-systems and applied mathematics.

During the past four decades, numerous impressive varieties of new techniques have been developed for obtaining lower order models from higher order linear system. The first approach is based upon the classical mathematical concept such as the Pade approximation [1], continued fraction method [2], and the time moment matching method [3]. These approaches have been recognized to be a powerful method to obtain a reduced order models, but these methods have the disadvantage that the reduced order model may be unstable although the original system is stable. Second group of reduction techniques are based on some criteria of stability (such as the Routh stability criterion, Hurwitz polynomial, Mihailov criterion, etc.). The absolute stability of these methods achieved only by the cost of series loss of accuracy [4], [5].

Differentiation method was introduced by Gutman [6], wherein, the reciprocal of numerator and denominator polynomial of high order transfer function are differentiated many times to produce the coefficients of the reduced transfer function. The method is computationally simple and is applicable to unstable systems. The drawback of this method is that steady state does not match always with the original higher order system. Pal [7] has developed a system reduction methodology using the continued fraction approach and Routh-Hurwitz array, in which the initial transient response of the reduced order model might not match with that of the higher order system, as only the first few time moments are considered depending upon the order of the reduced model. The viability and limitations of similar methods has been discussed by Shamash [8]. Each of these methods has both advantages and disadvantages when tried on a particular system. In spite of several methods available, no approach gives the best results for all systems.

Now a days, most practicing research field has been “Heuristics from Nature”, an area utilizing analogies with nature or social systems. Several modern heuristic tools have evolved in the last two decades that facilitates solving optimization problems that were previously difficult or impossible to solve. These tools include Evolutionary Computation, Simulated Annealing, Tabu Search, Genetic Algorithm, Particle Swarm Optimization and etc. Among these heuristic techniques, Genetic Algorithm (GA) and Particle Swarm Optimization (PSO) techniques appeared as promising algorithms for handling the optimization problems.

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Instead of this, Particle Swarm Optimization is mostly used because of the following features [9] (i) The method is based on swarms such as fish schooling and a flock of birds (ii) PSO is based on a simple concept and its computation time is short and it requires few memories (iii) All particles in PSO are kept as members of the population through the course of the run (iv) In PSO, there is no selection operation and no crossover operation.

In this paper a simple scheme is proposed for deriving a basic second order model, and to obtain a fine tuned second order system depicting the original characteristics of the higher order model, Modified Particle Swarm Optimization (MPSO) algorithm is proposed. The procedure discussed for linear time invariant SISO continuous systems. The main objective is to minimize the Integral Squared Error (ISE) of the unit step time response under the constraints of maintaining the response characteristics of the original system. The robustness of the proposed scheme is compared with other performance indices like, Integral of time of squared error (ITSE) and Integral of time of absolute error (ITAE).

II. DESCRIPTION OF THE PROBLEM

A. Higher Order Transfer Function

Consider an n^{th} order linear time invariant continuous system represented by,

$$G(s) = \frac{N(s)}{D(s)} = \frac{\sum_{i=0}^{n-1} A_i s^i}{\sum_{i=0}^n a_i s^i} \quad (1)$$

Where, $N(s)$ is the numerator polynomial and $D(s)$ is the denominator polynomial. Also A_i and a_i represents the constant coefficients of the s -terms of the numerator and denominator of $G(s)$. Equation (1) represented the higher order continuous system transfer function.

B. Lower Order Transfer Function

To find a m^{th} lower order model for the continuous system $R^m(s)$, where $m < n$ in the following form represented by (2), such that the formulated lower order model retains the characteristics of the original system and approximates its response as closely as possible for the same type of inputs with minimum error indices.

$$R^m(s) = \frac{N^m(s)}{D^m(s)} = \frac{\sum_{i=0}^{m-1} B_i s^i}{\sum_{i=0}^m b_i s^i} \quad (2)$$

where $N^m(s)$ and $D^m(s)$ are the numerator polynomial and denominator polynomial of the formulated lower order model respectively. Also B_i and b_i represent the constant coefficients of the s -terms of the numerator and denominator of $R^m(s)$. Equation (2) represented the lower order transfer function.

C. Performance Index

The performance of the Lower order model is verified by the performance index criterion. In modern complex control systems the different performance index criterions are used to measure the systems performance and give the useful information about the density of the estimation. Integral squared error (ISE), Integral of time of squared error (ITSE), Integral of time of absolute error (ITAE), ITSE and ITAE are the useful performance index criterions in modern control engineering. ISE is mostly employed for the performance estimation because of ease of implementation. These errors are given by following expressions [10],

$$ISE = \int_0^t [Y(t) - y(t)]^2 dt \quad (3)$$

$$ITSE = \int_0^t [Y(t) - y(t)]^2 \cdot t \cdot dt \quad (4)$$

$$ITAE = \int_0^t |Y(t) - y(t)| \cdot t \cdot dt \quad (5)$$

The main objective of the model formulation problem is to minimize the integral squared error of the unit input time response of the lower order system and maintaining the characteristics of the original system. $Y(t)$ and $y(t)$ are the input time response of the given higher order system and lower order system respectively. A new lower order model system can be derived from the given higher order system, which depict the characteristics of the original higher order system.

III. OVERVIEW OF PARTICLE SWARM OPTIMIZATION

The particle swarm optimization (PSO) technique appeared as a promising algorithm for handling the optimization problems. PSO is a population-based stochastic optimization technique, inspired by social behavior of bird flocking or fish schooling [11]. PSO is inspired by the ability of flocks of birds, schools of fish, and herds of animals to adapt to their environment, find rich sources of food, and avoid predators by implementing an information sharing approach. PSO technique was invented in the mid 1990s while attempting to simulate the choreographed, graceful motion of swarms of birds as part of a socio cognitive study investigating the notion of collective intelligence in biological populations.

The basic idea of the PSO is the mathematical modeling and simulation of the food searching activities of a swarm of birds (particles). In the multi dimensional space where the optimal solution is sought, each particle in the swarm is moved towards the optimal point by adding a velocity with its position. The velocity of a particle is influenced by three components namely, inertial momentum, cognitive and social. The inertial component simulates the inertial behavior of the bird to fly in the previous direction. The cognitive component models the memory of the bird about its previous best

position, and the social component models the memory of the bird about the best position among the particles.

PSO procedures based on the above concept can be described as follows. Namely, bird flocking optimizes a certain objective function. Each agent knows its best value so far (*pbest*) and its XY position. Moreover, each agent knows the best value in the group (*gbest*) among *pbests*. Each agent tries to modify its position using the current velocity and the distance from the *pbest* and *gbest*. Based on the above discussion, the mathematical model for PSO is as follows [12]-[14],

Velocity update equation is given by

$$V_{i+1} = \omega \times V_i + C_1 \times r_1 \times (P_{best_i} - S_i) + C_2 \times r_2 \times (g_{best_i} - S_i) \quad (6)$$

Position update equation is given by

$$S_{i+1} = S_i + V_{i+1} \quad (7)$$

Each particle tries to modify its velocity and position and based on (6) and (7) and reaches the target.

Where,

- V_i = Velocity of particle
- S_i = Current position of the particle
- ω = Inertia weight
- C_1 = Cognition acceleration coefficient
- C_2 = Social acceleration coefficient
- P_{best_i} = Own best position of particle
- g_{best_i} = Global best position among the group of particles
- r_1, r_2 = Uniformly distributed random numbers in the range [0 to 1]

IV. MODIFIED PARTICLE SWARM OPTIMIZATION

In this new proposed modified PSO having better optimization result compare to general PSO by splitting the cognitive component of the general PSO into two different component. The first component can be called good experience component. This means the bird has a memory about its previously visited best position. This is similar to the general PSO method. The second component is given the name by bad experience component. The bad experience component helps the particle to remember its previously visited worst position. To calculate the new velocity, the bad experience of the particle also taken into consideration.

The new velocity update equation is given by

$$V_{i+1} = \omega \times V_i + C_{1g} \times r_1 \times (P_{best_i} - S_i) + C_{1b} \times r_2 \times (S_i - P_{worst_i}) + C_2 \times r_3 \times (g_{best_i} - S_i) \quad (8)$$

C_{1g} = Acceleration coefficient, which accelerate the

particle towards its best position

C_{1b} = Acceleration coefficient, which accelerate the particle away from its worst position

P_{worst_i} = Worst position of the particle i

r_1, r_2, r_3 = Uniformly distributed random numbers in the range [0 to 1]

The positions are updated using (8). The inclusion of the worst experience component in the behavior of the particle gives the additional exploration capacity to the swarm. By using the bad experience component; the particle can bypass its previous worst position and try to occupy the better position. Fig. 1 shows the concept of MPSO searching points.

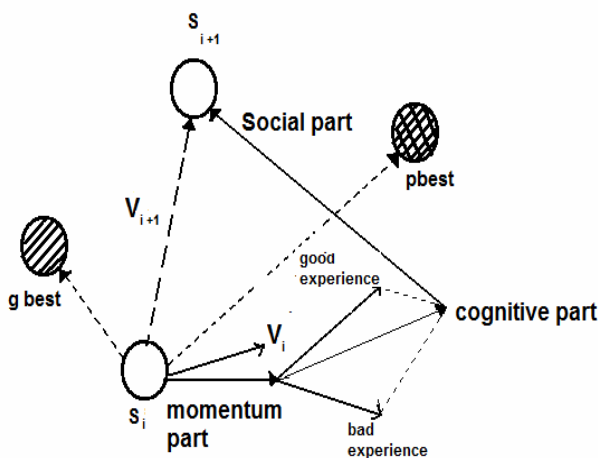


Fig. 1 Concept of MPSO search point

The algorithmic steps for the modified PSO is as follows

- Step 1 Select the number of particles, generations, tuning accelerating coefficients C_{1g} , C_{1b} , and C_2 and random numbers r_1, r_2, r_3 to start the optimal solution searching
- Step 2 Initialize the particle position and velocity
- Step 3 Select particles individual best value for each generation
- Step 4 Select the particles global best value, i.e. particle near to the target among all the particles is obtained by comparing all the individual best values
- Step 5 Select the particles individual worst value, i.e. Particle too away from the target
- Step 6 Update particle individual best (*pbest*), global best (*gbest*), particle worst (*Pworst*) in the velocity equation (8) and obtain the new velocity
- Step 7 Update new velocity value in the equation (7) and obtain the position of the particle
- Step 8 Find the optimal solution with minimum ISE by the updated new velocity and position

The flowchart for the proposed scheme is as shown in Fig. 2.

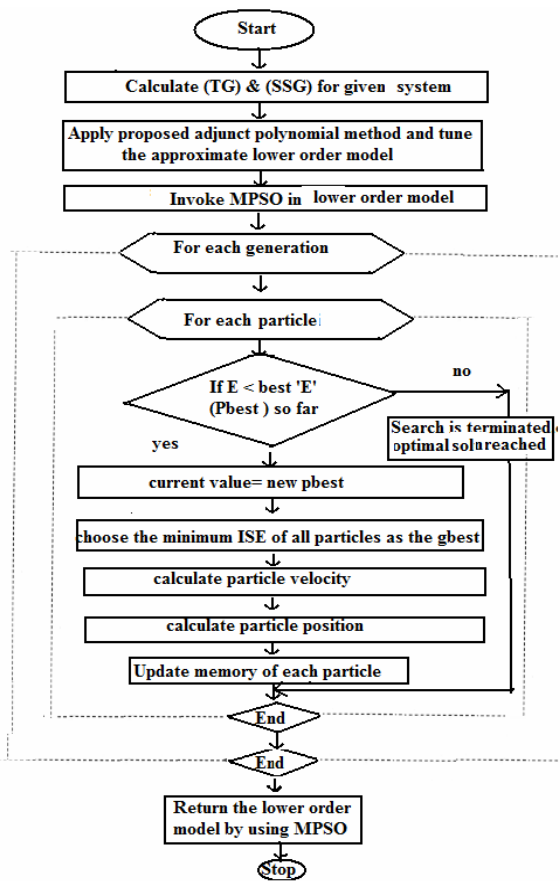


Fig. 2 Flowchart for model reduction of linear time invariant continuous systems

V. NUMERICAL EXAMPLE

Let us consider linear time invariant continuous system represented in the form of transfer function given in [17] as,

$$G(s) = \frac{18s^7 + 514s^6 + 5982s^5 + 36380s^4 + 122664s^3 + 222088s^2 + 185760s + 40320}{s^8 + 36s^7 + 546s^6 + 4536s^5 + 22449s^4 + 67284s^3 + 118124s^2 + 109584s + 40320} \tag{9}$$

Step-1

Calculate the transient gain (TG) and steady state gain (SSG) for the given higher order system in (9).

$$TG = \frac{18}{1} = 18 \tag{10}$$

$$SSG = \frac{40320}{40320} = 10$$

Step- 2

Applying Adjunct polynomial scheme, [Appendix] to G(s)

in (9) to get approximated second order model R(s),

$$R(s) = \frac{185760s + 40320}{118124s^2 + 109584s + 40320} \tag{11}$$

Step-3

On scaling (11),

$$R(s) = \frac{s + 0.2170}{s^2 + 0.9277s + 0.3413} \tag{12}$$

Step-4

To maintain TG and SSG, use (10) in (12). Equation (13) is tuned to maintain the transient and steady state gain and the result, R(s) becomes,

$$R(s) = \frac{18s + 0.3413}{s^2 + 0.9277s + 0.3413} \tag{13}$$

Step-5

The MPSO algorithm is now invoked to search the values of 's' term (0.9277) and the constant term (0.3413) of the denominator in R(s) represented by (13), so the characteristics of second order model matches the given higher order system given by (9). MPSO determines a better reduced second order model with the least integral square error.

Step-6

The transfer function of the reduced second order model obtained using MPSO scheme is,

$$R(s) = \frac{18s + 5.2414}{s^2 + 7.5077s + 5.2414} \tag{14}$$

TABLE I
COMPARISON OF PERFORMANCE INDEX

Method	Reduced order Model	ISE	ITSE	ITAE
Prasad and Pal [16]	$\frac{17.98561s + 500}{s^2 + 13.24571s + 500}$	1.0344	1.2209	2.4990
Shamash [17]	$\frac{6.7786s + 2}{s^2 + 3s + 2}$	0.0396	0.0863	0.8530
Mukherje et al [18]	$\frac{11.3909s + 4.4357}{s^2 + 4.2122s + 4.4357}$	0.0280	0.0661	0.9287
PSO [11]	$\frac{18s + 0.6169}{s^2 + 5.0369s + 0.6169}$	20.2572	108.57	169.84
Proposed MPSO	$\frac{18s + 5.2414}{s^2 + 7.5077s + 5.2414}$	0.0091	0.0178	0.6086

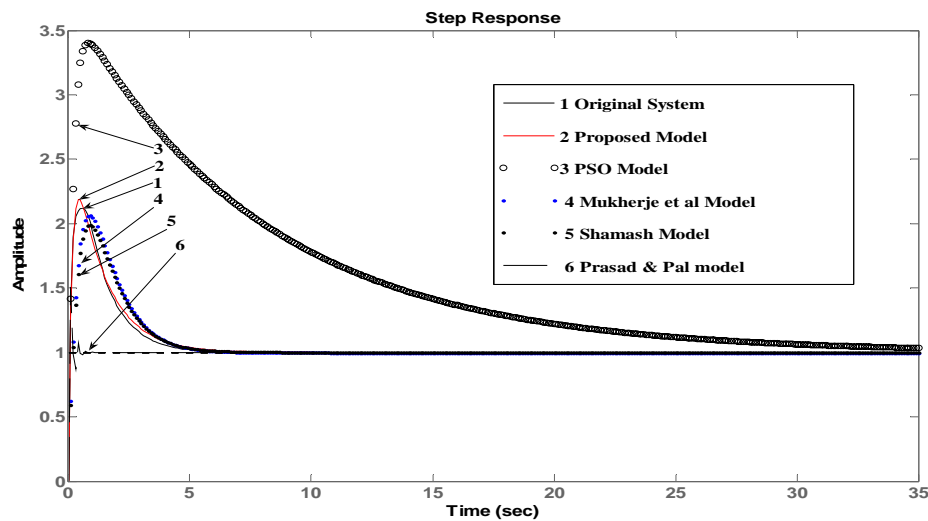


Fig. 3 Step response of original 8th order system with proposed methods and other conventional method

The performance comparison of the proposed approach for order reduction techniques for a unit step input is given in Table I. The unit step response of the original eight order system and reduced system models are shown in Fig. 3. In Fig. 3, the unit step response of the original system is shown in black solid line and the unit step response of the PSO model is shown in black circle symbols. MPSO model response is represented in red line. Mukherje et al model is in blue dotted symbols, Shamash models in black dotted symbol and Prasad and Pal model represented by black dashed line. It can be seen that the proposed MPSO reduced model as given by (14) is closely matching with that of the original higher order model compared with the PSO model.

VI. DISCUSSION

The considered eight order linear time invariant continuous system [17] has a steady state gain of 18 and transient gain 1. Initially the approximate lower order method was obtained by the adjunct polynomial method. MPSO was used to minimize the performance indices error for the approximate lower order model with the constraints of maintaining the transient and steady state gain of the given higher order system. The MPSO algorithm was coded in Intel Pentium processor 4.0, 2.8 GHz, 256 MB RAM and it took 10 seconds by the CPU for the complete simulation of 40 particles and 100 generations. MPSO algorithm was simulated and its searched for the best second order model and obtained a minimal error value for unit step response. Table 1 show that the proposed Modified particle swarm optimization gives the least error values in comparison with other techniques. From Fig.3, its observed that update the worst experience of the particle in the velocity equation gives better optimal solution compared with the general PSO method.

VII. CONCLUSION

The characteristics of the lower order model closely equate the higher order model obtained by the Modified Particle swarm algorithm. The main advantage of the proposed method is that it is easy of implementation and least elapsed time. The proposed approach can also be used for discrete as well as multi input multi output systems. This can also extended for other evolutionary techniques and hybrid methods and also its extended for further design of controllers and compensators as well as state variable controllers and observers for stabilization process.

APPENDIX

Consider an nth linear time invariant continuous higher order system represented by its transfer function as

$$G(s) = \frac{N(s)}{D(s)} = \frac{\sum_{i=0}^{n-1} A_i s^i}{\sum_{i=0}^n a_i s^i} \quad (15)$$

$$= \frac{A_{n-1}s^{n-1} + A_{n-2}s^{n-2} + \dots + A_3s^3 + A_2s^2 + A_1s + A_0}{a_n s^n + a_{n-1}s^{n-1} + \dots + a_3s^3 + a_2s^2 + a_1s + a_0} \quad (16)$$

The Adjunct Polynomial scheme for obtaining the approximated lower order models from the given higher order system is as follows:

$$\text{First order: } \frac{A_0}{a_1s + a_0} \quad (17)$$

$$\text{Second order: } \frac{A_1s + A_0}{a_2s^2 + a_1s + a_0} \quad (18)$$

$$\text{Third order: } \frac{A_2s^2 + A_1s + A_0}{a_3s^3 + a_2s^2 + a_1s + a_0} \quad (19)$$

$$(n-1)^{\text{th}} \text{ order: } \frac{A_{n-2}s^{n-2} + A_{n-3}s^{n-3} + \dots + A_1s + A_0}{a_{n-1}s^{n-1} + a_{n-2}s^{n-2} + \dots + a_1s + a_0} \quad (20)$$

Equations (15) through (20), gives the lower order models formulated using adjunct polynomial scheme from the given higher order system $G(s)$. Based on the requirement, suitable lower order model can be selected and operates. It should be noted for a higher order system of order 'n', (n-1) lower order models could be formulated. This method of selection of approximate lower order models helps to set the initial values of operating parameters to be used in the Modified Particle Swarm Optimization process.

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