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Newton-Raphson State Estimation Solution Employing Systematically Constructed Jacobian Matrix

Nursyarizal Mohd Nor, Ramiah Jegatheesan, and Perumal Nallagownden

***Abstract*—**Newton-Raphson State Estimation method using bus admittance matrix remains as an efficient and most popular method to estimate the state variables. Elements of Jacobian matrix are computed from standard expressions which lack physical significance. In this paper, elements of the state estimation Jacobian matrix are obtained considering the power flow measurements in the network elements. These elements are processed one-by-one and the Jacobian matrix H is updated suitably in a simple manner. The constructed Jacobian matrix H is integrated with Weight Least Square method to estimate the state variables. The suggested procedure is successfully tested on IEEE standard systems.

***Keywords*—**State Estimation (SE), Weight Least Square (WLS), Newton-Raphson State Estimation (NRSE), Jacobian matrix H.

I. Introduction

STATE Estimation (SE) is the task of providing consistent load flow results for an entire power system. The aim of the state estimation is to get the best estimate of the system state by processing a set of real-time redundant measurements available in the Energy Management System (EMS) database. The state of the power system is described by a collection of voltage vectors for a given network topology and parameters. Comprehensive discussion of the state of the art in electric power system state estimation is presented in [1]–[3].

Most of the SE programs in practical use are formulated as over-determined systems of non-linear equations and solved as Weight Least Square (WLS) problems [1], [4]. In WLS method the measured quantities are represented as sum of true values and errors as

$$z = h(x) + e \tag{1}$$

where z is the measurement vector, consisting of real and reactive power flows, bus injection powers and voltage magnitudes; x is the true state variable vector, consisting of bus voltage magnitudes and bus voltage angles; $h(x)$ is the non-linear function that relates the states to the ideal measurements; e is a vector of measurement errors. A state estimate $\hat{x}$ is to be obtained that minimizes the objective function f given by

$$f = \sum_{j=1}^{m} w_j e_j^2 \quad \text{or} \quad \sum_{j=1}^{m} \frac{e_j^2}{\sigma_j^2} \tag{2}$$

and this can be achieved when

$$\sum_{j=1}^{m} 2 w_j e_j \frac{\partial e_j}{\partial x_n} = 0, \tag{3}$$

where 'w_j' is the weighting factor for the respective measurement and n = 1,2,..., number of state variables. This non-linear least squares problem is usually solved iteratively as a sequence of linear least squares problem. At each step of the iteration, a WLS solution to the following noise-corrupted system of linear equation is sought:

$$\hat{e} = z - \hat{z} = z - H\hat{x} = e - H(\hat{x} - x) \tag{4}$$

In (4) e is the measurement residual vector, the difference between the actual measurement vector and the value of $h(x)$ at the current iteration, $\hat{x} - x$ is the difference between the updated state and the current state, H is the Jacobian of $h(x)$ in (1) at the current iteration.

The SE Jacobian H, is not a square matrix. The H matrix always has $(2N - 1)$ columns, where N is equal to number of buses. The number of rows in H matrix is equal to number of measurements available. For full measurement set, number of rows will be equal to $(3N + 4B)$ where B is number of lines. The elements of H represent the partial derivates of bus voltage magnitudes, bus powers and line flows with respect to state variables δ and V. The general structure of H matrix is

$$H = \begin{bmatrix} H_{V,\delta} & H_{V,V} \\ H_{p_{ij},\delta} & H_{p_{ij},V} \\ H_{p_{ji},\delta} & H_{p_{ji},V} \\ H_{q_{ij},\delta} & H_{q_{ij},V} \\ H_{q_{ji},\delta} & H_{q_{ji},V} \\ H_{P,\delta} & H_{P,V} \\ H_{Q,\delta} & H_{Q,V} \end{bmatrix} \tag{5}$$

where $H_{V,\delta}$, $H_{V,V}$, , $H_{pij,\delta}$, $H_{pij,V}$, $H_{pji,\delta}$, $H_{pji,V}$, $H_{qij,\delta}$, $H_{qij,V}$, $H_{qji,\delta}$, $H_{qji,V}$, $H_{P,\delta}$, $H_{P,V}$, $H_{Q,\delta}$ and $H_{Q,V}$ are the sub-matrices of Jacobian matrix. The first suffix indicates the available measurement and the second suffix indicates the variable on which the partial

N. M. Nor is with the Department of Electrical and Electronics Engineering, University Technology PETRONAS, 31750 Tronoh, Perak, Malaysia (e-mail: nursyarizal_mnor@petronas.com.my).
R. Jegatheesan is with the Department of Electrical and Electronics Engineering, University Technology PETRONAS, 31750 Tronoh, Perak, Malaysia (e-mail: ramiah_j@petronas.com.my).
N. Perumal is with the Department of Electrical and Electronics Engineering, University Technology PETRONAS, 31750 Tronoh, Perak, Malaysia (e-mail: perumal@petronas.com.my).

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derivatives are obtained. The constructional details of the SE sub-matrices are discussed in Section III.

Fast Decoupled State Estimator (FDSE) [6], [7] is based on assumptions that in practical power system networks under steady-sate, real power flows are less sensitive to voltage magnitudes and are very sensitive to voltage phase angles, while reactive power flows are less sensitive to voltage phase angles and are very sensitive to voltage magnitudes. Using these properties, the sub-matrices $H_{P,V}$, $H_{pij,V}$, $H_{pji,V}$, $H_{Q,\delta}$, $H_{qij,\delta}$ and $H_{qji,\delta}$ are neglected. Because of the approximations made, the corrections on the voltages computed in each iteration are less accurate. This results in poor convergence characteristic. Newton-Raphson State Estimator (NRSE) method [6]–[9] that was subsequently introduced became more popular because of exact problem formulation and very good convergence characteristic. In NRSE method, elements of Jacobian matrix are computed from the standard expressions which are functions of bus voltages, bus powers and the elements of bus admittance matrix.

Nowadays, with the advent of fast computers, even huge amount of complex calculations can be carried out very efficiently in much lesser time. Therefore, there is no need to go for approximate models. In this paper, an attempt is made to introduce more physical meaning for the elements of the SE Jacobian matrix *H*. Bus admittance matrix of transmission network does not find place in computing the elements of the *H* matrix. The power flows in the network elements are taken as the basic components in constructing the *H* matrix. Network elements are added one-by-one and the *H* matrix is updated in a simple manner. Resulting final *H* matrix is exactly same as that obtained in NRSE method.

II. Power Flows in Transmission Network Elements

The transmission network consists of transmission lines, transformers and shunt parameters. In NRSE method the transmission network is represented by the bus admittance matrix and the elements of the *H* matrix are computed using the elements of bus admittance matrix. Alternatively, in this paper, the elements of the *H* matrix are obtained considering the power flows in the transmission network elements.

Consider the general transmission network element between buses *i* and *j*, as shown in Fig. 1.

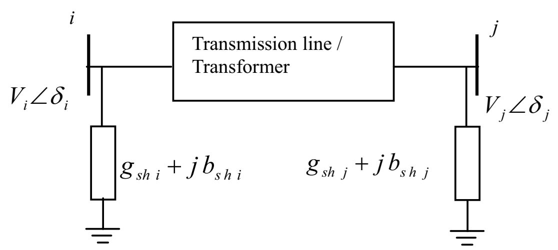

Fig. 1 Transmission network element between buses *i* and *j*

Here the transmission line is represented by the series impedance $r_{ij}+j\,x_{ij}$ or by the corresponding

Many methods of SE are introduced since 1960 [1], [4], [5]. Several assumptions are made in the different methods of SE. admittance $g_{ij}+j\,b_{ij}$. Transformer with series impedance $r_{ij}+j\,x_{ij}$ and off-nominal tap setting "*a*" with tap setting facility at bus *i* is represented by the series admittance $\frac{1}{a}\left(g_{ij}+j\,b_{ij}\right)$ and shunt admittances $\left(\frac{1-a}{a^2}\right)\left(g_{ij}+j\,b_{ij}\right)$ and $\left(\frac{a-1}{a}\right)\left(g_{ij}+j\,b_{ij}\right)$ at buses *i* and *j* respectively. Half line charging admittance and external shunt admittance if any, are added together and represented as $g_{sh\,i}+j\,b_{sh\,i}$ and $g_{sh\,j}+j\,b_{sh\,j}$ at buses *i* and *j* respectively. For such a general transmission network element, the real and reactive power flows are given by the following expressions.

$$p_{ij}=V_i^2\left(\frac{g_{ij}}{a^2}+g_{sh\,i}\right)-\frac{V_iV_j}{a}\left(g_{ij}\cos\delta_{ij}+b_{ij}\sin\delta_{ij}\right) \tag{6}$$

$$p_{ji}=V_i^2\left(g_{ij}+g_{sh\,j}\right)-\frac{V_iV_j}{a}\left(g_{ij}\cos\delta_{ij}-b_{ij}\sin\delta_{ij}\right) \tag{7}$$

$$q_{ij}=-V_i^2\left(\frac{b_{ij}}{a^2}+b_{sh\,i}\right)-\frac{V_iV_j}{a}\left(g_{ij}\sin\delta_{ij}-b_{ij}\cos\delta_{ij}\right) \tag{8}$$

$$q_{ji}=-V_j^2\left(b_{ij}+b_{sh\,j}\right)+\frac{V_iV_j}{a}\left(g_{ij}\sin\delta_{ij}+b_{ij}\cos\delta_{ij}\right) \tag{9}$$

where

$$\delta_{ij}=\delta_i-\delta_j \tag{10}$$

All the line flows computed from (6) to (9) are stored in the real power and reactive power matrix as in (11) and (12) from which bus powers can be calculated.

$$P=\begin{bmatrix} 0 & p_{12} & p_{13} & \cdots & p_{1N} \\ p_{21} & 0 & p_{23} & \cdots & p_{2N} \\ p_{31} & p_{32} & 0 & \cdots & p_{3N} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ p_{N1} & p_{N2} & p_{N3} & \cdots & 0 \end{bmatrix} \tag{11}$$

$$Q=\begin{bmatrix} 0 & q_{12} & q_{13} & \cdots & q_{1N} \\ q_{21} & 0 & q_{23} & \cdots & q_{2N} \\ q_{31} & q_{32} & 0 & \cdots & q_{3N} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ q_{N1} & q_{N2} & q_{N3} & \cdots & 0 \end{bmatrix} \tag{12}$$

The real and reactive power flows in line *i-j* depend on δ_i, δ_j, V_i and V_j. The partial derivatives of p_{ij}, p_{ji}, q_{ij} and q_{ji} with respect to δ_i, δ_j, V_i and V_j can be derived from (6) to (9).

III. Construction of SE Jacobian Matrix, *H*

All the elements of *H* matrix are partial derivatives of available measurements with respect to δ and *V*. The elements of sub-matrices $H_{V,\delta}$, $H_{V,V}$ are given by:

$$H_{V_i,\delta_j} = \frac{\partial V_i}{\partial \delta_j} = 0 \quad \Big|_{\text{for all } i \text{ and } j} \tag{13}$$

$$\left.\begin{aligned} H_{V_i,V_j} &= \frac{\partial V_i}{\partial V_j} = 0 \quad \Big|_{i \neq j} \\ H_{V_i,V_i} &= \frac{\partial V_i}{\partial V_i} = 1 \end{aligned}\right\} \tag{14}$$

If at particular bus, the voltage meter is not available, the row corresponding to that particular bus will be deleted.

Using (6) to (9) the expression for the partial derivatives of p_{ij}, p_{ji}, q_{ij} and q_{ji} with respect to δ_i, δ_j, V_i and V_j are obtained. Thus

$$\frac{\partial p_{ij}}{\partial \delta_i} = \frac{V_i V_j}{a}(g_{ij}\sin\delta_{ij} - b_{ij}\cos\delta_{ij}) \tag{15}$$

$$\frac{\partial p_{ij}}{\partial \delta_j} = -\frac{V_i V_j}{a}(g_{ij}\sin\delta_{ij} - b_{ij}\cos\delta_{ij}) \tag{16}$$

$$\frac{\partial p_{ij}}{\partial V_i} = 2V_i\,(\frac{g_{ij}}{a^2} + g_{sh\,i}) - \frac{V_j}{a}(g_{ij}\cos\delta_{ij} + b_{ij}\sin\delta_{ij}) \tag{17}$$

$$\frac{\partial p_{ij}}{\partial V_j} = -\frac{V_i}{a}(g_{ij}\cos\delta_{ij} + b_{ij}\sin\delta_{ij}) \tag{18}$$

$$\frac{\partial p_{ji}}{\partial \delta_i} = \frac{V_i V_j}{a}(g_{ij}\sin\delta_{ij} + b_{ij}\cos\delta_{ij}) \tag{19}$$

$$\frac{\partial p_{ji}}{\partial \delta_j} = -\frac{V_i V_j}{a}(g_{ij}\sin\delta_{ij} + b_{ij}\cos\delta_{ij}) \tag{20}$$

$$\frac{\partial p_{ji}}{\partial V_i} = -\frac{V_j}{a}(g_{ij}\cos\delta_{ij} - b_{ij}\sin\delta_{ij}) \tag{21}$$

$$\frac{\partial p_{ji}}{\partial V_j} = 2V_j\,(g_{ij} + g_{sh\,j}) - \frac{V_i}{a}(g_{ij}\cos\delta_{ij} - b_{ij}\sin\delta_{ij}) \tag{22}$$

$$\frac{\partial q_{ij}}{\partial \delta_i} = -\frac{V_i V_j}{a}(g_{ij}\cos\delta_{ij} + b_{ij}\sin\delta_{ij}) \tag{23}$$

$$\frac{\partial q_{ij}}{\partial \delta_j} = \frac{V_i V_j}{a}(g_{ij}\cos\delta_{ij} + b_{ij}\sin\delta_{ij}) \tag{24}$$

$$\frac{\partial q_{ij}}{\partial V_i} = -2V_i\,(\frac{b_{ij}}{a^2} + b_{sh\,i}) - \frac{V_j}{a}(g_{ij}\sin\delta_{ij} - b_{ij}\cos\delta_{ij}) \tag{25}$$

$$\frac{\partial q_{ij}}{\partial V_j} = -\frac{V_i}{a}(g_{ij}\sin\delta_{ij} - b_{ij}\cos\delta_{ij}) \tag{26}$$

$$\frac{\partial q_{ji}}{\partial \delta_i} = \frac{V_i V_j}{a}(g_{ij}\cos\delta_{ij} - b_{ij}\sin\delta_{ij}) \tag{27}$$

$$\frac{\partial q_{ji}}{\partial \delta_j} = -\frac{V_i V_j}{a}(g_{ij}\cos\delta_{ij} - b_{ij}\sin\delta_{ij}) \tag{28}$$

$$\frac{\partial q_{ji}}{\partial V_i} = \frac{V_j}{a}(g_{ij}\sin\delta_{ij} + b_{ij}\cos\delta_{ij}) \tag{29}$$

$$\frac{\partial q_{ji}}{\partial V_j} = -2V_j\,(b_{ij} + b_{sh\,j}) + \frac{V_i}{a}(g_{ij}\sin\delta_{ij} + b_{ij}\cos\delta_{ij}) \tag{30}$$

To construct the H matrix, initially all its elements are set to zero. Network elements are considered one-by-one. For the element between buses i-j, the partial derivatives of line flows with respect to δ_i, δ_j, V_i and V_j are computed using (15) to (30). These values are simply added to the corresponding elements of sub-matrices $H_{pij,\,\delta i}$, $H_{pij,\,\delta j}$, $H_{pij,\,Vi}$, $H_{pij,\,Vj}$, $H_{pji,\,\delta i}$, $H_{pji,\,\delta j}$, $H_{pji,\,Vi}$, $H_{pji,\,Vj}$, $H_{qij,\,\delta i}$, $H_{qij,\,\delta j}$, $H_{qij,\,Vi}$, $H_{qij,\,Vj}$, $H_{qji,\,\delta i}$, $H_{qji,\,\delta j}$, $H_{qji,\,Vi}$ and $H_{qji,\,Vj}$.

Sub-matrices $H_{P,\delta}$, $H_{P,V}$, $H_{Q,\,\delta}$ and $H_{Q,V}$ are now considered. Partial derivatives of bus powers can be expressed in terms of partial derivatives of line flows. To illustrate this let i-j, i-k and i-m be the elements connected at bus i. Then the bus powers P_i and Q_i are given by

$$P_i = p_{ij} + p_{ik} + p_{im} \tag{31}$$

$$Q_i = q_{ij} + q_{ik} + q_{im} \tag{32}$$

Therefore

$$\frac{\partial P_i}{\partial \delta_i} = \frac{\partial p_{ij}}{\partial \delta_i} + \frac{\partial p_{ik}}{\partial \delta_i} + \frac{\partial p_{im}}{\partial \delta_i} \tag{33}$$

$$\frac{\partial Q_i}{\partial \delta_i} = \frac{\partial q_{ij}}{\partial \delta_i} + \frac{\partial q_{ik}}{\partial \delta_i} + \frac{\partial q_{im}}{\partial \delta_i} \tag{34}$$

Similar expressions can be written for other partial derivatives of P_i and Q_i with respect to δ_j, V_i and V_j. Likewise considering bus powers P_j and Q_j, partial derivatives of P_j and Q_j can also be obtained in terms of partial derivatives of line flows in the lines connected to bus j. It is to be noted that the partial derivatives of the line flows contribute to the partial derivatives of bus powers. Table I shows a few partial derivative of line flows and the corresponding partial derivative of bus powers to which it contributes.

TABLE I
PARTIAL DERIVATIVES OF LINE FLOWS AND THE CORRESPONDING PARTIAL DERIVATIVES OF BUS POWERS

Partial Derivatives of	
Line flows	Bus Power
$\frac{\partial p_{ij}}{\partial \delta_i}, \frac{\partial p_{ji}}{\partial \delta_j}, \frac{\partial q_{ij}}{\partial V_i}, \frac{\partial q_{ji}}{\partial V_j}$	$\frac{\partial P_i}{\partial \delta_i}, \frac{\partial P_j}{\partial \delta_j}, \frac{\partial Q_i}{\partial V_i}, \frac{\partial Q_j}{\partial V_j}$

The partial derivatives of $\frac{\partial p_{ij}}{\partial \delta_i}, \frac{\partial p_{ij}}{\partial \delta_j}, \frac{\partial p_{ij}}{\partial V_i}, \frac{\partial p_{ij}}{\partial V_j}$ will contribute to $\frac{\partial p_i}{\partial \delta_i}, \frac{\partial p_i}{\partial \delta_j}, \frac{\partial p_i}{\partial V_i}, \frac{\partial p_i}{\partial V_j}$ respectively. Similar results are true for p_{ji}, q_{ij} and q_{ji}. Those values will be added to the corresponding elements of $H_{P,\delta}$, $H_{P,V}$, $H_{Q,\delta}$ and $H_{Q,V}$. This process is repeated for all the network elements. Once all the network elements are added, we get the final H matrix.

IV. COMPUTING AND RECORDING ONLY THE REQUIRED PARTIAL DERIVATIVES ALONE

The H matrix will have $3N$+$4B$ number of rows if all possible measurements are available in the network. However, in practice, number of available measurements will be much less. Instead of computing elements of rows corresponding to unavailable measurements and then deleting them, proper logics can be adopted to compute and record only the required partial derivatives alone. When line i-j is processed, it may not be always necessary to compute all the 16 partial derivatives given by (15) to (30). The partial derivatives $\frac{\partial p_{ij}}{\partial \delta_i}, \frac{\partial p_{ij}}{\partial \delta_j}, \frac{\partial p_{ij}}{\partial V_i}$ and $\frac{\partial p_{ij}}{\partial V_j}$ are to be computed only when p_{ij} or P_i or both p_{ij} and P_i are in the available measurement list. Thus following three cases are possible.

CASE 1: p_{ij} is an available measurement. The four partial derivatives are entered in row corresponding to p_{ij}.

CASE 2: P_i is an available measurement. The four partial derivatives are added to previous values in the row corresponding to P_i.

CASE 3: p_{ij} and P_i are available measurements. The four partial derivatives are entered in the row corresponding to p_{ij} and added to previous values in the row corresponding to P_i.

Such logics are to be followed for $\frac{\partial p_{ji}}{\partial \delta_i}$, $\frac{\partial p_{ji}}{\partial \delta_j}$, $\frac{\partial p_{ji}}{\partial V_i}$, $\frac{\partial p_{ji}}{\partial V_j}$; $\frac{\partial q_{ij}}{\partial \delta_i}$, $\frac{\partial q_{ij}}{\partial \delta_j}$, $\frac{\partial q_{ij}}{\partial V_i}$, $\frac{\partial q_{ij}}{\partial V_j}$ and $\frac{\partial q_{ji}}{\partial \delta_i}$, $\frac{\partial q_{ji}}{\partial \delta_j}$, $\frac{\partial q_{ji}}{\partial V_i}$, $\frac{\partial q_{ji}}{\partial V_j}$ also.

V. Test Results

The three bus power system [6] as shown in Fig. 2 is used to illustrate the construction of H Jacobian matrix. In this system bus 1 is the slack bus and the tap setting "a" for all lines are 1. With the network data as listed in Table II and the available measurements as listed in Table III, the Jacobian matrix H is constructed as discussed in Section IV, taking the initial bus voltages as $V_1 = V_2 = V_3 = 1.0\angle 0^0$.

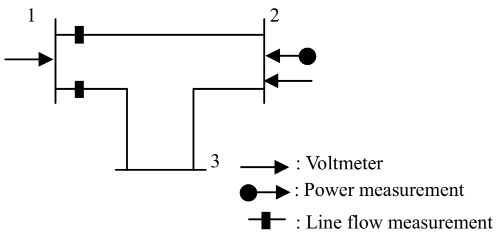

Fig. 2 Single-line diagram and measurement configuration of a 3-bus power system

TABLE II
NETWORK DATA

Line From Bus	Line To Bus	R (pu)	X (pu)	Total Line Charging Susceptance B (pu)
1	2	0.01	0.03	0
1	3	0.02	0.05	0
2	3	0.03	0.08	0

TABLE III
AVAILABLE MEASUREMENTS FOR THE THREE BUS SYSTEM

Measurements	Value (pu)	Weightage
V_1	1.006	62500
V_2	0.968	62500
p_{12}	0.888	15625
p_{13}	1.173	15625
q_{12}	0.568	15625
q_{13}	0.663	15625
P_2	-0.501	10000
Q_2	-0.286	10000

Noting that V_1 and V_2 are available measurements, the sub-matrices of $H_{V,\delta}$, $H_{V,V}$ are obtained as

$$\begin{bmatrix} H_{V_1,\delta} & H_{V_1,V} \\ H_{V_2,\delta} & H_{V_2,V} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

where δ spans from δ_1, δ_2 to δ_3 and V spans from V_1, V_2 to V_3.

To illustrate all the stages of constructing the other sub-matrices of the network elements are added one by one as shown below.

Iteration 1

Element 1-2 is added. The line flow measurements corresponding to this element are p_{12}, p_{21}, q_{12} and q_{21}. All these measurements are categorized according to the three different cases as in Section IV. The p_{12} will be categorized as CASE 1 since this measurement is one of the available measurements and P_1 is not an available measurement. Similarly, q_{12} is also categorized as CASE 1.However, p_{21} and q_{21} are categorized as CASE 2 since these measurements will contribute to P_2 and Q_2 respectively; but they are not listed as the available measurements. The new constructed sub-matrices are:

$$\begin{bmatrix} H_{p_{12},\delta} & H_{p_{12},V} \\ H_{p_{13},\delta} & H_{p_{13},V} \\ H_{q_{12},\delta} & H_{q_{12},V} \\ H_{q_{13},\delta} & H_{q_{13},V} \\ H_{P_2,\delta} & H_{P_2,V} \\ H_{Q_2,\delta} & H_{Q_2,V} \end{bmatrix} = \begin{bmatrix} 30 & -30 & 0 & 10 & -10 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -10 & 10 & 0 & 30 & -30 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -30 & 30 & 0 & -10 & 10 & 0 \\ 10 & -10 & 0 & -30 & 30 & 0 \end{bmatrix}$$

Element 1-3 is added. The line flow measurements corresponding to this element are p_{13}, p_{31}, q_{13} and q_{31}. Now, p_{13} and q_{13} will be categorized as CASE 1 since these measurements are listed as the available measurements and P_1 and Q_1 are not the available measurements. However it is not necessary to compute the partial derivatives of p_{31} and q_{31} as they and P_3 and Q_3 are not in available measurements. With this, the constructed sub-matrices are:

$$\begin{bmatrix} H_{p_{12},\delta} & H_{p_{12},V} \\ H_{p_{13},\delta} & H_{p_{13},V} \\ H_{q_{12},\delta} & H_{q_{12},V} \\ H_{q_{13},\delta} & H_{q_{13},V} \\ H_{P_2,\delta} & H_{P_2,V} \\ H_{Q_2,\delta} & H_{Q_2,V} \end{bmatrix} = \begin{bmatrix} 30 & -30 & 0 & 10 & -10 & 0 \\ 17.24 & 0 & -17.24 & 6.89 & 0 & -6.89 \\ -10 & 10 & 0 & 30 & -30 & 0 \\ -6.89 & 0 & 6.89 & 17.24 & 0 & -17.24 \\ -30 & 30 & 0 & -10 & 10 & 0 \\ 10 & -10 & 0 & -30 & 30 & 0 \end{bmatrix}$$

Element 2-3 is added. Following similar logics, p_{23} and q_{23} will fall under CASE 2 and the partial derivatives of p_{32} and q_{32} are not required. The constructed sub-matrices are:

$$\begin{bmatrix} H_{p_{12},\delta} & H_{p_{12},V} \\ H_{p_{13},\delta} & H_{p_{13},V} \\ H_{q_{12},\delta} & H_{q_{12},V} \\ H_{q_{13},\delta} & H_{q_{13},V} \\ H_{P_2,\delta} & H_{P_2,V} \\ H_{Q_2,\delta} & H_{Q_2,V} \end{bmatrix} = \begin{bmatrix} 30 & -30 & 0 & 10 & -10 & 0 \\ 17.24 & 0 & -17.24 & 6.89 & 0 & -6.89 \\ -10 & 10 & 0 & 30 & -30 & 0 \\ -6.89 & 0 & 6.89 & 17.24 & 0 & -17.24 \\ -30 & 40.96 & -10.96 & -10 & 14.11 & -4.11 \\ 10 & -14.11 & 4.11 & -30 & 40.96 & -10.96 \end{bmatrix}$$

The final H matrix will be the combination of all the sub-matrices with the column corresponding to slack bus being deleted. Thus the constructed Jacobian matrix H in the first iteration is

$$H = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ -30 & 0 & 10 & -10 & 0 \\ 0 & -17.24 & 6.89 & 0 & -6.89 \\ 10 & 0 & 30 & -30 & 0 \\ 0 & 6.89 & 17.24 & 0 & -17.24 \\ 40.96 & -10.96 & -10 & 14.11 & -4.11 \\ -14.11 & 4.11 & -30 & 40.96 & -10.96 \end{bmatrix}$$

Using the above H matrix, state variables are updated as $V_1 = 0.9997\angle 0^0; V_2 = 0.9743\angle -0.021^0; V_3 = 0.9428\angle -0.045^0$.

All the above stages are repeated until the convergence is obtained in iteration 3 with the final state variables values as $V_1 = 0.9996\angle 0^0; V_2 = 0.9741\angle -0.022^0; V_3 = 0.9439\angle -0.048^0$. These estimates are same as obtained in NRSE method.

The suggested procedure is tested on 5-bus system [9] and IEEE 14-bus system also. Table IV shows the state variables results obtained through the proposed algorithm and existing method [10] for IEEE 14-bus system. The results agree with those obtained by NRSE method.

TABLE IV
COMPARISON RESULTS – IEEE 14-BUS SYSTEM

Actual values		NRSE		Proposed	
V (pu)	θ (rad)	V (pu)	θ (rad)	V (pu)	θ (rad)
1.06	0	1.06	0	1.06	0
1.045	-0.0871	1.0451	-0.0871	1.0451	-0.0871
1.01	-0.2227	1.0101	-0.22262	1.0101	-0.22262
1.012	-0.1785	1.0123	-0.17854	1.0123	-0.17854
1.016	-0.1527	1.0163	-0.15279	1.0163	-0.15279
1.07	-0.2516	1.0707	-0.25188	1.0707	-0.25188
1.049	-0.2309	1.0498	-0.23095	1.0498	-0.23095
1.09	-0.2309	1.0905	-0.23095	1.0905	-0.23095
1.033	-0.2585	1.0332	-0.25855	1.0332	-0.25855
1.032	-0.2622	1.0323	-0.26224	1.0323	-0.26224
1.047	-0.259	1.0479	-0.2592	1.0479	-0.2592
1.053	-0.2664	1.0543	-0.26667	1.0543	-0.26667
1.047	-0.2671	1.0476	-0.26727	1.0476	-0.26727
1.021	-0.2802	1.0212	-0.28024	1.0212	-0.28024

VI. CONCLUSION

WLS method embedded with NRSE to calculate the bus power and lines flows is still the best and well accepted method for the SE. Elements of the Jacobian matrix are computed using the elements of the bus admittance matrix. Recognizing that the elements of the Jacobian matrix H are contributed by the partial derivatives of the power flows in the network elements, a simple and meaningful algorithm to construct the Jacobian matrix H is presented. The final Jacobian H matrix is obtained mainly from the partial derivatives of the line flows.

The concept involved in this algorithm is simple to understand. Since it involves repeated procedure, Jacobian matrix can be obtained through simple computer program. The suggested procedure is tested on 3-bus, 5-bus and IEEE 14-bus systems and found to give the correct results.

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Nursyarizal Mohd Nor obtained his B Eng (Hons) in Electrical Engineering from University Technology Malaysia (UTM), Johor. In year 2001 he obtained his MSc in Electrical Power Engineering from The University of Manchester Institute of Science and Technology (UMIST), UK. Presently he is a Ph.D. student under the guidance of Dr. Ramiah Jegatheesan in Universiti Teknologi PETRONAS Tronoh, Perak. His research interests are in Power Economics Operation and Control, Power Quality and Power System Analysis.

Ramiah Jegatheesan joined the teaching profession in 1969. He has obtained Ph.D. from Indian Institute of Technology, Kanpur, India in 1975. His areas of specialization are ‘Analysis and optimization of large scale power systems’ and ‘State estimation’. He has served in Anna University, Chennai, India for more than three decades. He has several publications at his credit. He is the author of two books on ‘Circuit Theory’. Since 2004, he is working in Malaysia. Presently he is working as Professor in the Department of Electrical and Electronics Engineering at Universiti Teknologi Petronas.

Ir. N. Perumal was born in Trolak, Perak in 1951. He obtained his B.E (Hons) in Electrical & Electronics Engineering from Portsmouth Polytechnic, U.K and M.Sc from University of Wales, U.K. His employment experience includes Polytechnic Ungku Omar, Conso Light Sdn Bhd and Universiti Teknologi PETRONAS. His special area of interest is electrical power system. He is a member of the Institution of Engineers Malaysia and is a Professional Engineer registered with the Board of Engineers Malaysia.